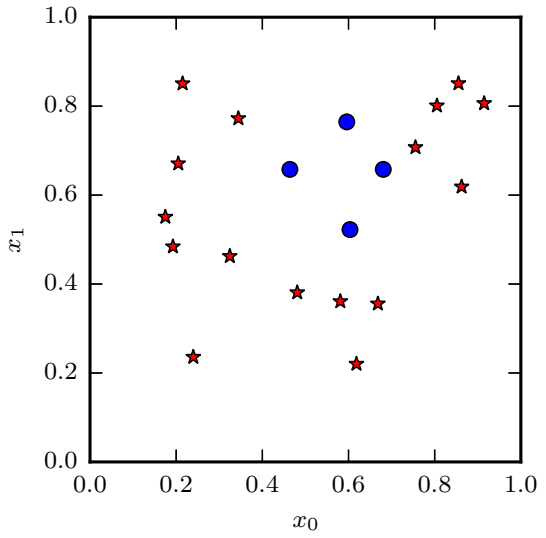
Decision Tree Exercises



1. **Gini Impurity**

   The goal in building a decision tree is to create the smallest possible tree in which each leaf node contains training data from only one class. In evaluating possible splits, it is useful to have a way of measuring the *purity* of a node. The purity describes how close the node is to containing data from only one class. Gini purity is defined as follows[1]:

$$\phi(\mathbf{p}) = \sum_i p_i(1 - p_i)$$

   Where $\mathbf{p} = (p_1, ..., p_n)$ and each p_i is the fraction of elements from class i. This expresses the fractions of incorrect predictions in the node if the class of each element was predicted by randomly selecting a label according to the distribution of classes in the node. This value will be 0 if all elements are from the same class, and it increases as the mix becomes more uniform.

   Calculate the Gini impurity of the following data set:

2. **Tree Construction**

   The decision tree construction algorithm proceeds by recursively splitting the training data into increasingly smaller subsets. When splitting a node in the tree we search across all dimensions and all split points to select the split that results in the greatest decrease in impurity. This goodness-of-split value can be expressed as:

$$\Theta(s, t) = \phi(\mathbf{p}) - P_L\phi(\mathbf{p_L}) - P_R\phi(\mathbf{p_R})$$

   Where s is a possible split, t is the node and P_L and P_R represent the fraction of elements that ended up in the left child and right child respectively. Higher values represent better splits.

   Execute the recursive tree-construction algorithm on the data above and draw the resulting tree. Calculate the impurity of each node and the goodness-of-split for each split.

[1] Notation follows: Breiman, Leo. "Technical note: Some properties of splitting criteria." Machine Learning 24.1 (1996): 41-47.

3. **Classification**

   Classify the following three points using your decision tree.

   $(.4, 1.0)$
   $(.6, 1.0)$
   $(.6, 0)$