Reinforcement Learning

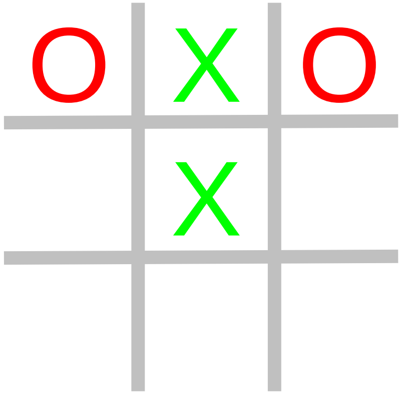
- Introduction
  - MENACE (Michie 1963)
- Formalization
  - Policies
  - Value Function
  - Q-Function
- Model-based Reinforcement Learning
  - Policy Iteration
  - Value Iteration
- Model-free Reinforcement Learning
  - Q-Learning
  - extensions
- Application Examples

Reinforcement Learning

- Goal
  - Learning of policies (action selection strategies) based on feedback from the environment (reinforcement)
    - e.g., game won / game lost
- Applications
  - **Games**
    - Tic-Tac-Toe: MENACE (Michie 1963)
    - Backgammon: TD-Gammon (Tesauro 1995)
    - Schach: KnightCap (Baxter et al. 2000)
  - **Other**
    - Elevator Dispatching
    - Robot Control
    - Job-Shop Scheduling

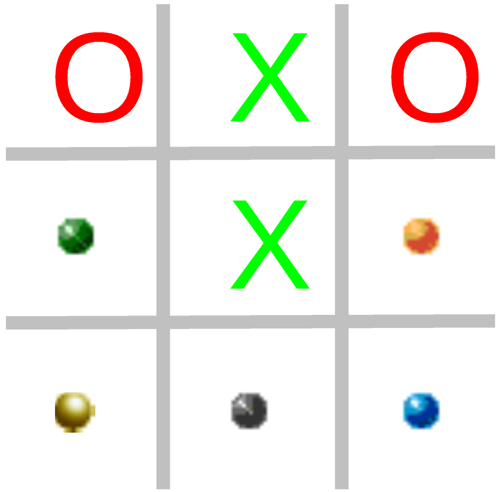
MENACE (Michie, 1963)

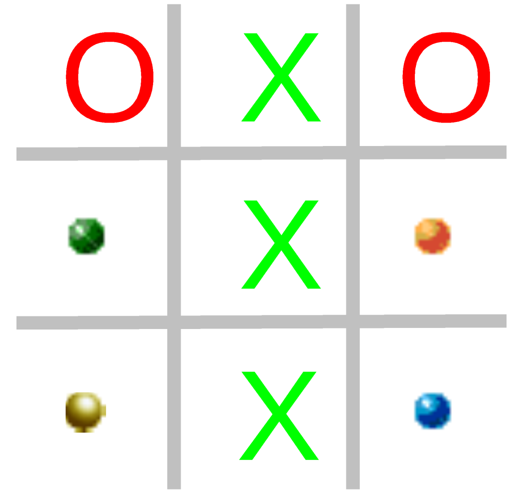
- Learns to play Tic-Tac-Toe
- Hardware:
  - 287 Matchboxes
    (1 for each position)
  - Beads in 9 different colors
    (1 color for each square)



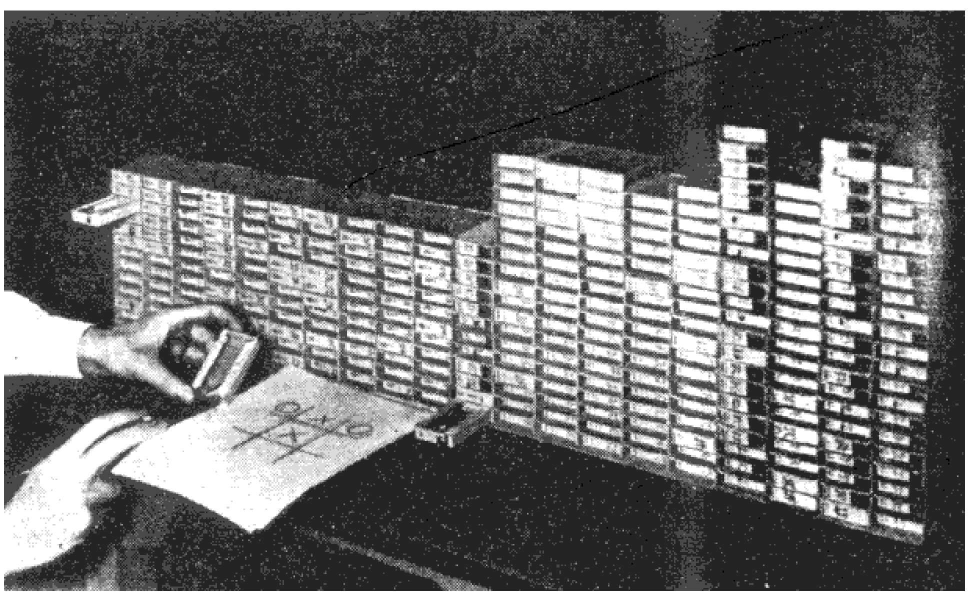
- Playing algorithm:
  - Select the matchbox corresponding to the current position
  - Randomly draw a bead from this matchbox
  - Play the move corresponding to the color of the drawn bead
- Implementation: http://www.codeproject.com/KB/cpp/ccross.aspx

X to move





Select the matchbox corresponding to this position



Draw a bead from the matchbox

Play the move that corresponds to the color of the drawn bead

Reinforcement Learning in MENACE

- Initialisation
  - all moves are equally likely, i.e. every box contains an equal number of beads for each possible move / color
- Learning algorithm:
  - Game lost → drawn beads are kept (*negative reinforcement)*
  - Game won → put the drawn bead back and add another one in the same color to this box (*positive reinforcement*)
  - Game drawn → drawn beads are put back (no change)
- This results in
  - Increased likelihood that a successful move will be tried again
  - Decreased likelihood that an unsuccessful move will be repeated

Credit Assignment Problem

- Delayed Reward
  - The learner knows whether it has one or lost not before the end of the game
  - The learner does not know which move(s) are responsible for the win / loss
    - a crucial mistake may already have happened early in the game, and the remaining moves were not so bad (or vice versa)

- Solution in Reinforcement Learning:
  - All moves of the game are rewarded or penalized (adding or removing beads from a box)
  - Over many games, this procedure will converge
    - bad moves will rarely receive a positive feedback
    - good moves will be more likely to be positively reinforced

Reinforcement Learning - Formalization

- Learning Scenario
  - $s \in S$: state space
  - $a \in A$: action space
  - $s_0 \in S_0$: initial states
  - a state transition function $\delta : S \times A \to S$
  - a reward function $r : S \times A \to \mathbb{R}$
- Markov property
  - rewards and state transitions only depend on last state
  - not on how you got into this state
- State and action space can be
  - Discrete: S and/or A is a set
  - Continuous: S and/or A are infinite (not part of this lecture!)
- State transition function can be
  - Stochastic: Next state is drawn according to $\delta(s'|s,a)$
  - Deterministic: Next state is fixed $\delta(s,a) = s'$

Reinforcement Learning - Formalization

- Enviroment:
  - the agent repeatedly chooses an action according to some *policy* $\pi(a|s)$ *or* $\pi(s) = a$
  - this will put the agent in state s into a new state s' according to

    stochastic: $\Pr^{\pi}(s'|s) = \delta(s'|s,a)\pi(a|s)$

    deterministic: $s' = \delta(s, \pi(s))$
  - in some states the agent receives feedback from the environment (reinforcement)

MENACE - Formalization

- Framework
  - states = matchboxes, discrete
  - actions = moves/beads, discrete
  - policy = prefer actions with higher number of beads, stochastic
  - reward = game won/ game lost
    - *delayed* reward: we don't know right away whether a move was good or bad+
  - transition function: choose next matchbox according to rules, deterministic

- Task
  - Find a policy that maximizes the sum of future rewards

More Terminology

- delayed reward
  - reward for actions may not come immediately (e.g., game playing)
  - modeled as: every state s_i gives a reward r_i, but most r_i=0
- trajectory:
  - sequence of state-actions $\langle s_{0,} a_{0,} s_{1,} \ldots, a_{n-1}, s_n \rangle$
  - A deterministic policy and transition function create a unique trajectory, stochastic policies or transition functions may result in different trajectories

Learning Task

Learning goal:

- maximize cumulative reward (return) for the trajectories a policy is generating
  - reward from "now" until the end of time

$$R(\pi) = R(\tau^{\pi}) = \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)$$

  - immediate rewards are weighted higher, rewards further in the future are discounted (discount factor γ)
  - if not discounted, the sum to infinty could be infinite

Learning Task

- How can we compute $R(\tau^\pi)$?

$$
\begin{aligned}
R(\tau^\pi) &= \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \\
&= r(s_0, a_0) + \gamma r(s_1, a_1) + \gamma^2 r(s_2, a_2) \cdots \\
&= r(s_0, \pi(s_0)) + \sum_{t=1}^{\infty} \gamma^t r(\delta(s_{t-1}, \pi(s_{t-1})), \pi(s_t)) \\
&= V^\pi(s_0)
\end{aligned}
$$

  - Sum the observed rewards (with decay)

- Value function = return when starting in state s and following policy π afterwards

Optimal Policies and Value Functions

- Optimal policy
  - the policy with the highest expected value for all states s

$$\pi^*(s) = \arg\max_{\pi} V^{\pi}(s)$$
$$= \arg\max_{a \in A} r(s,a) + \gamma V^{\pi^*}(\delta(s,a))$$

  - Always select the action that maximizes the value function for the next step, when following the optimal policy afterwards
  - But we don't know the optimal policy...

Policy Iteration

- Policy Improvement Theorem
  - if it is true that selecting the first action in each state according to a policy π' and continuing with policy π is better than always following π then π' is a better policy than π
- Policy Improvement
  - always select the action that maximizes the value function of the current policy $\pi'(s) = \arg\max_{a \in A} r(s,a) + \gamma V^{\pi}(\delta(s,a))$
- Policy Evaluation
  - Compute the value function for the new policy
- Policy Iteration
  - Interleave steps of policy evaluation with policy improvement

$$\pi^0(s) \rightarrow V^{\pi^0}(s) \rightarrow \pi^1(s) \rightarrow \cdots \rightarrow \pi^*(s)$$

Policy Evaluation

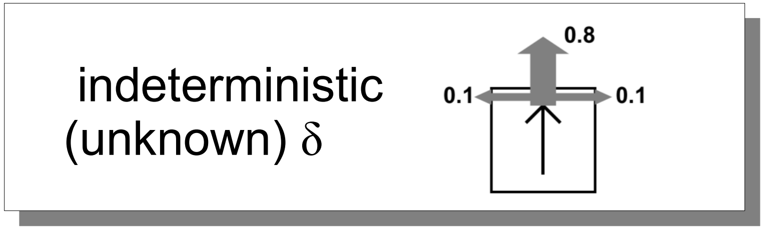
- We need the value of all states, but can only start in s_0
  - Update all states along the trajectory
- We assumed the transition function to be deterministic, that is not realistic in many settings
  - Monte Carlo approximation
  - Create k samples and average

$$V^\pi(s_0) = \mathbb{E}_{s_t} \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)$$

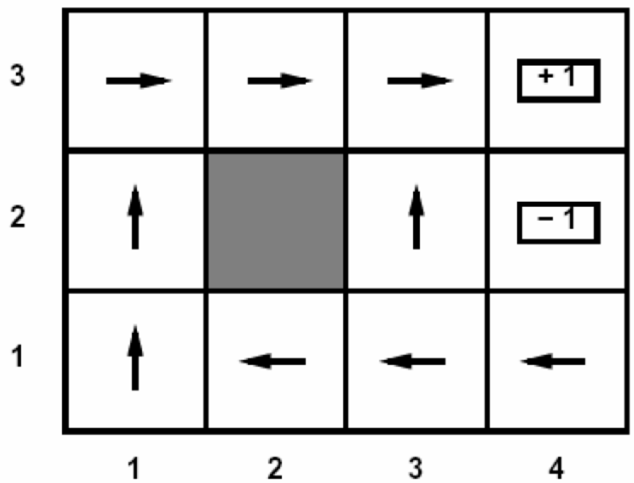
$$= r(s_0, \pi(s_0)) + \sum_{t=1}^{\infty} \gamma^t \mathbb{E}_{s_t} \delta(s_t | s_{t-1}, \pi(s_{t-1})) r(s_t, \pi(s_t))$$

$$= r(s_0, \pi(s_0)) + \frac{1}{k} \sum_{i=0}^{k} \sum_{t=1}^{\infty} \gamma^t r(s_t^i, \pi(s_t^i))$$

Policy Evaluation - Example



- Simplified task
  - we don't know δ
  - we don't know r
  - but we are given a policy π
    - i.e., we have a function that gives us an action in each state



- Goal:
  - learn the value of each state
- Note:
  - here we have no choice about the actions to take
  - we just execute the policy and observe what happens

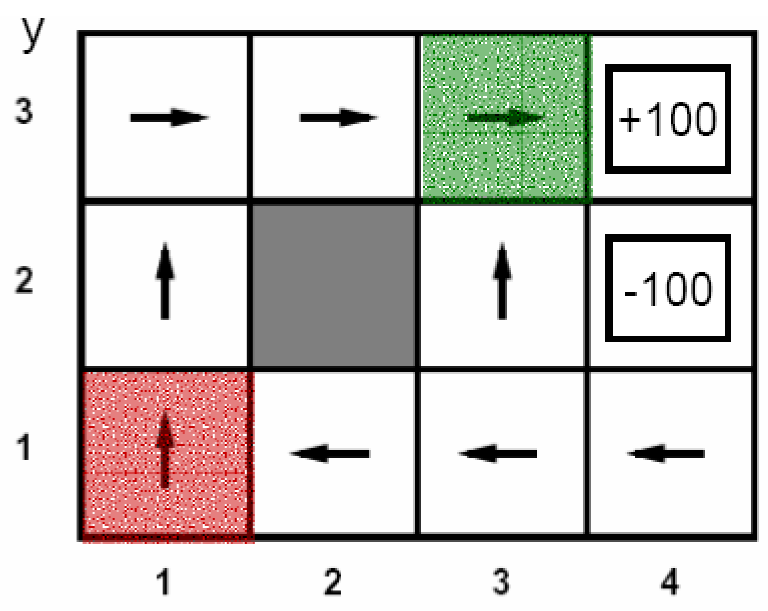
Policy Evaluation – Example

- Episodes:

(1,1) up -1
(1,2) up -1
(1,2) up -1
(1,3) right -1
(2,3) right -1
(3,3) right -1
(3,2) up -1
(3,3) right -1
(4,3) exit +100
(done)

(1,1) up -1
(1,2) up -1
(1,3) right -1
(2,3) right -1
(3,3) right -1
(3,2) up -1
(4,2) exit -100
(done)

Transitions are indeterministic!



$\gamma = 1,$

$$V^{\pi}(1,1) \leftarrow (92 + -106)/2 = -7$$

$$V^{\pi}(3,3) \leftarrow (99 + 97 + -102)/3 = 31.3$$

Policy Improvement

- Compute the value for every state
- Update the policy according to

$$\pi'(s) = \arg\max_{a \in A} r(s,a) + \gamma \underbrace{\mathbb{E}_{s'} \delta(s'|s,a) V^{\pi}(s')}_{\text{expected value of the policy when performing action } a \text{ in state } s \text{ x}}$$

$$= V^{\pi}(s') \text{ for deterministic state transitions } s' = \delta(s,a)$$

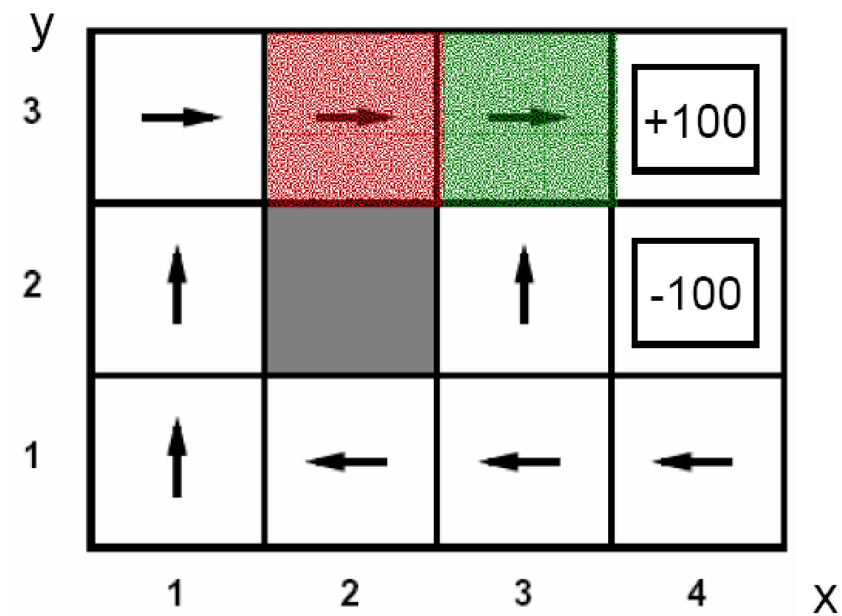
$$= \sum_{s'} \delta(s'|s,a) V^{\pi}(s') \text{ for probabilistic transitions and discrete states}$$

- But here we need the transition function we don't know ?

Simple Approach: Learn the Model from Data

- Episodes:

(1,1) up -1	(1,1) up -1
(1,2) up -1	(1,2) up -1
(1,2) up -1	(1,3) right -1
(1,3) right -1	(2,3) right -1
(2,3) right -1	(3,3) right -1
(3,3) right -1	(3,2) up -1
(3,2) up -1	(4,2) exit -100
(3,3) right -1	(done)
(4,3) exit +100	
(done)	



$\gamma = 1$

$$P((4,3) \mid (3,3), \text{right}) = 1/3$$

$$P((3,3) \mid (2,3), \text{right}) = 2/2$$

But do we really need to learn the transition model?

Q-function

- the Q-function does not evaluate states, but evaluates state-action pairs
- The Q-function for a given policy π
  - is the cumulative reward for starting in s, applying action a, and, in the resulting state s', play according to π

$$Q^{\pi}(s_0,a_0) = r(s_0,a_0) + \sum_{t=1}^{\infty} \gamma^t \mathbb{E}_{s_t} \delta(s_t|s_{t-1},a_{t-1}) r(s_t,\pi(s_t))$$

$$= r(s_0,a_0) + \frac{1}{k}\sum_{i=0}^{k}\sum_{t=1}^{\infty} \gamma^t r(s_t^i,a_t^i) \mid s_t \sim \delta(s_t|s_{t-1},\pi(s_{t-1}))$$

Estimate of the expected value based on k randomly drawn samples of s_t (instead of the unknown transition function)

- Now we update the policy without the transition function

$$\pi'(s) = \arg\max_a Q^{\pi}(s,a)$$

Exploration vs. Exploitation

- The current approach requires us to evaluate every action
  - We need to sample each state (that is reachable from s_0)
  - We need to compute argmax a over all available actions
- Exhaustive sampling is unrealistic
  - The state/action space may be very large, even infinite (continuous)
  - We approximate an expectation, hence multiple samples for every state/action are required
- We need to decide where to sample the transition function
  - Interesting = visited by the optimal policy
  - But we don't know the optimal policy till the end

Exploration vs. Exploitation

- Exploit
  - Use the action we assume to be the best
  - Approximate the optimal policy

- Explore
  - Optimal action may be wrong due to approximation errors
  - Try a suboptimal action

- Define probabilities for exploration and exploitation
  - Policy evaluation with stochastic policy

$$Q^{\pi}(s_0, a_0) = r(s_0, a_0) + \frac{1}{k} \sum_{i=0}^{k} \sum_{t=1}^{\infty} \gamma^t r(s_t^i, a_t^i) \mid s_t^i \sim \Pr^{\pi}(s_t^i | s_{t-1}^i)$$

  - Well-defined tradeoff can substantially reduce sample counts
  - Most relevant problem for reinforcement learning

Exploration vs. Exploitation

- ϵ-greedy
  - Fixed probability for selecting a suboptimal action

$$\pi'(a|s) = \begin{cases} 1-\epsilon+\frac{\epsilon}{|A|} & \textbf{if } a = \arg\max_{a\in A} Q^\pi(s,a) \\ \frac{\epsilon}{|A|} & \textbf{otherwise} \end{cases}$$

- Soft-Max
  - Action probability related to expected value

$$\pi'(a|s) = \frac{e^{Q^\pi(s,a)/\mathrm{T}}}{\int e^{Q^\pi(s,a)/\mathrm{T}}}$$

- High exploration in the beginning (temperature T high)
- Pure exploitation at the end (temperature T low)
- Tradeoff must change over time

Drawbacks

- Policy Iteration with Monte Carlo evaluation works well in practice with small state spaces
  - Don't learn a policy for each state, but learn the policy as a function
  - Especially well suited for continuous state spaces
  - Amount of function parameters usually much smaller than the amount of states
  - Requires well defined function space

  → Direct Policy Search (not part of this lecture)

- Alternative: Bootstrapping
  - Evaluate policy based on estimates
  - May induce errors
  - But requires much lower amount of samples

Optimal Q-function

- the optimal Q-function is the cumulative reward for starting in s, applying action a, and, in the resulting state s', play optimally (derivation: deterministic policy)

$$\begin{aligned}
Q^*(s_0,a_0) &= r(s_0,a_0) + \sum_{t=1}^{\infty} \gamma^t \mathbb{E}_{s_t} \delta(s_t|s_{t-1},\pi^*(s_{t-1})) r(s_t,\pi^*(s_t)) \\
&= r(s_0,a_0) + \gamma \mathbb{E}_{s_1} \delta(s_1|s_0,a_0) r(s_1,\pi^*(s_1)) + \gamma^2 \mathbb{E}_{s_2} \delta(s_2|s_1,\pi^*(s_1)) r(s_2,\pi^*(s_2)) + \cdots \\
&= r(s_0,a_0) + \gamma \left(\mathbb{E}_{s_1} \delta(s_1|s_0,a_0) r(s_1,\pi^*(s_1)) + \gamma \mathbb{E}_{s_2} \delta(s_2|s_1,\pi^*(s_1)) r(s_2,\pi^*(s_2)) + \cdots \right) \\
&= r(s_0,a_0) + \gamma \mathbb{E}_{s_1} \delta(s_1|s_0,a_0) Q^*(s_1,\pi^*(s_1)) \\
&= r(s_0,a_0) + \gamma \mathbb{E}_{s_1} \delta(s_1|s_0,a_0) \max_{a_1 \in A} Q^*(s_1,a_1)
\end{aligned}$$

- *Bellman equation*: $Q(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} \delta(s'|s,a) \max_{a' \in A} Q(s',a')$
  - the value of the Q-function for the current state s and an action a is the same as the sum of
    - the reward in the current state s for the chosen action a
    - the (discounted) value of the Q-function for the best action that I can play in the successor state s'

Better Approach: Directly Learning the Q-function

- Basic strategy:
  - start with some function $\hat{Q}$, and update it after each step
  - in MENACE: $\hat{Q}$ returns for each box s and each action a the number of beads in the box
- update rule:
  - the Bellman equation will in general not hold for Q
    i.e., the left side and the right side will be different

$$Q(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} \delta(s'|s,a) \max_{a' \in A} Q(s',a')$$

  - We can not easily compute the expectation
  - But we have multiple samples that contribute to the expectation

Better Approach: Directly Learning the Q-function

- Update Q-Function whenever we observe a transition s,a,r,s'
- Weighted update by a learning rate α

$$\hat{Q}(s,a) \leftarrow (1-\alpha)\hat{Q}(s,a) + \alpha(r(s,a) + \gamma \max_{a' \in A} \hat{Q}(s',a'))$$

$$\leftarrow \hat{Q}(s,a) + \alpha \left(r(s,a) + \gamma \max_{a' \in A} \hat{Q}(s',a') - \hat{Q}(s,a) \right)$$

new Q-value for state s and action a

old Q-value for this state/action pair

predicted Q-value for state s' and action a'

Q-learning (Watkins, 1989)

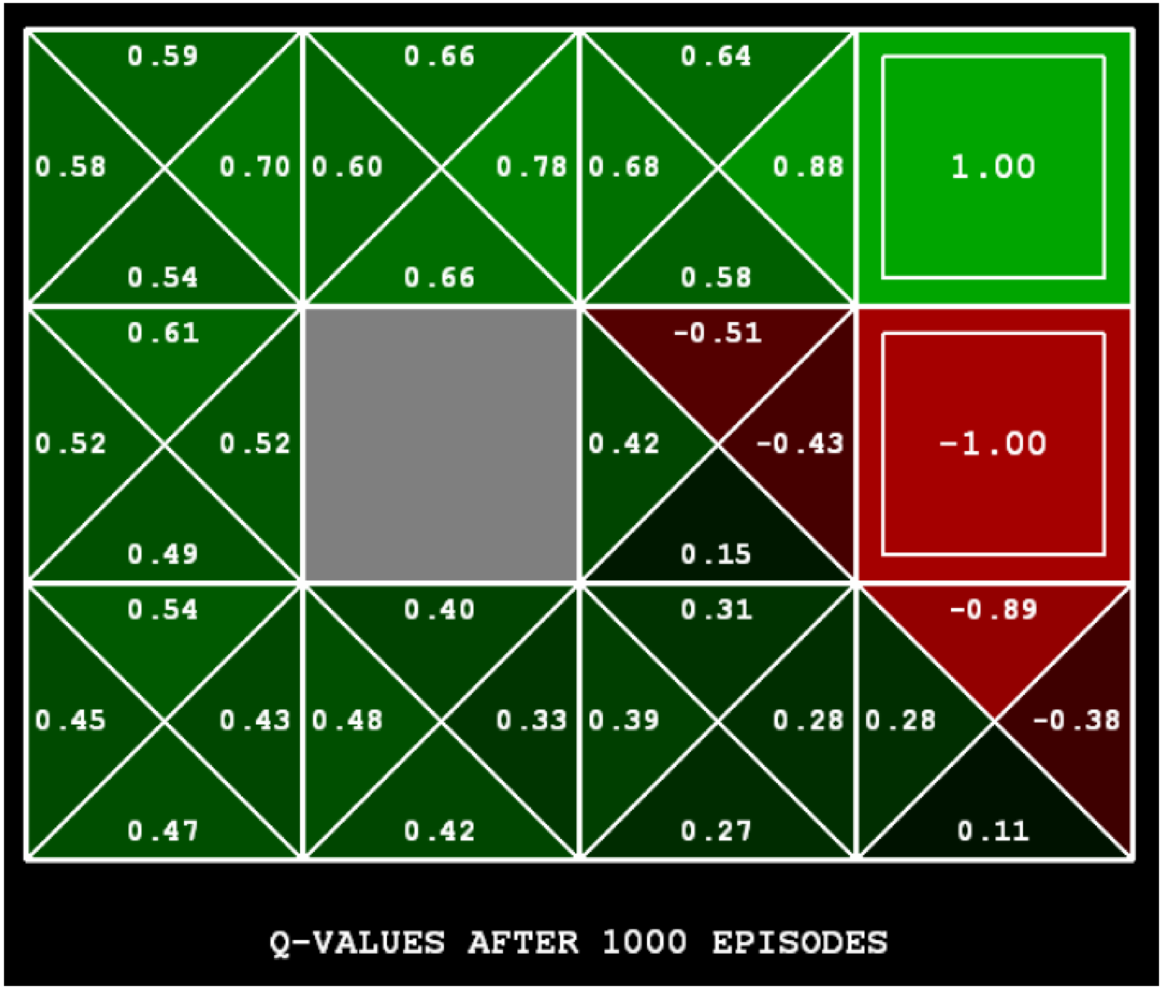
1. initialize all $\hat{Q}(s,a)$ with 0
2. observe current state s
3. loop
   1. select an action a and execute it
   2. receive the immediate reward and observe the new state s'
   3. update the table entry

      $$\hat{Q}(s,a) \leftarrow \hat{Q}(s,a) + \alpha\left[\underline{(r(s,a) + \gamma \max_{a'} \hat{Q}(s',a'))} - \underline{\hat{Q}(s,a)}\right]$$
   4. $s = s'$

Temporal Difference:
Difference between the estimate of the value of a state/action pair before and after performing the action.
→ Temporal Difference Learning

Example: Maze

- Q-Learning will produce the following values



Miscellaneous

- Weight Decay:
  - α decreases over time, e.g. $\alpha = \frac{1}{1 + visits(s,a)}$
- Convergence:
  it can be shown that Q-learning converges
  - if every state/action pair is visited infinitely often
    - not very realistic for large state/action spaces
    - but it typically converges in practice under less restricting conditions
- Representation
  - in the simplest case, $\hat{Q}(s,a)$ is realized with a look-up table with one entry for each state/action pair
  - a better idea would be to have trainable function, so that experience in some part of the space can be generalized
  - special training algorithms for, e.g., neural networks exist

Drawbacks of Q-Learning

- We still need to compute $\arg\max a$, requiring estimates for all actions
  - $\arg\max a$ is the optimal policy
  - our policy converges to the optimal policy

$\rightarrow$ don't use $\arg\max a$, but the action from the current policy

- perform *on-policy updates*
  - update rule assumes action a' is chosen according to current policy
  - Update whenever observing a sample s,a,r,s',a'

$$\hat{Q}(s,a) \leftarrow \hat{Q}(s,a) + \alpha \left(r(s,a) + \gamma \hat{Q}(s',a') - \hat{Q}(s,a) \right)$$

  - convergence if the policy gradually moves towards a policy that is greedy with respect to the current Q-function

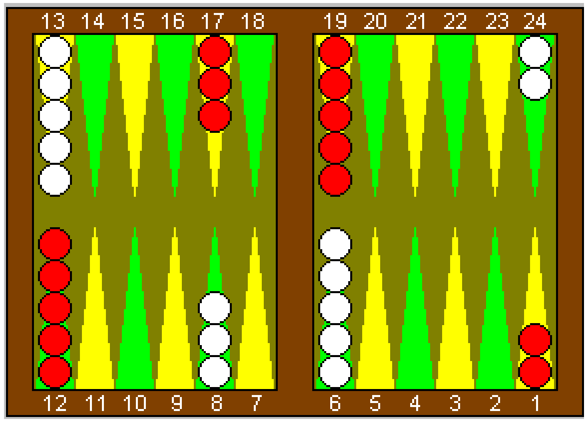
$\rightarrow$ SARSA

Properties of RL Algorithms

- Transition Function
  - Model-based: Assumed to be know or approximated
  - Model-free
- Sampling
  - On-Policy: Samples must be from the policy we want to evaluate
  - Off-Policy: Samples obtained from any policy
- Policy Evaluation
  - Value-based: Computes a state/action value function (this lecture)
  - Direct: Compute expected return for a policy
- Exploration
  - Directed: Method guides to a specific trajectory/state/action
  - Undirected: Method allows random sampling close to the expected maximum

TD-Gammon (Tesauro, 1995)

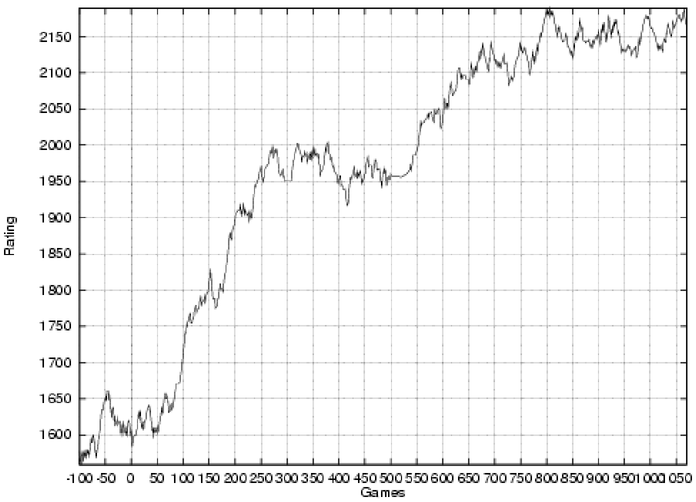
- world champion-calibre backgammon program
  - Developed from beginner to world-champion strength after 1,500,000 training games against itself (!)
  - Lost World championship 1998 in a match over 100 games with a mere 8 points
  - Led to changes in backgammon theory and was used as a popular practice and analysis partner of leading human players



- Improvements over MENACE:
  - Faster convergence because of → Temporal Difference Learning
  - Neural Network instead of boxes and beads allows generalization
  - use of positional characteristics as features

KnightCap (Baxter et al. 2000)

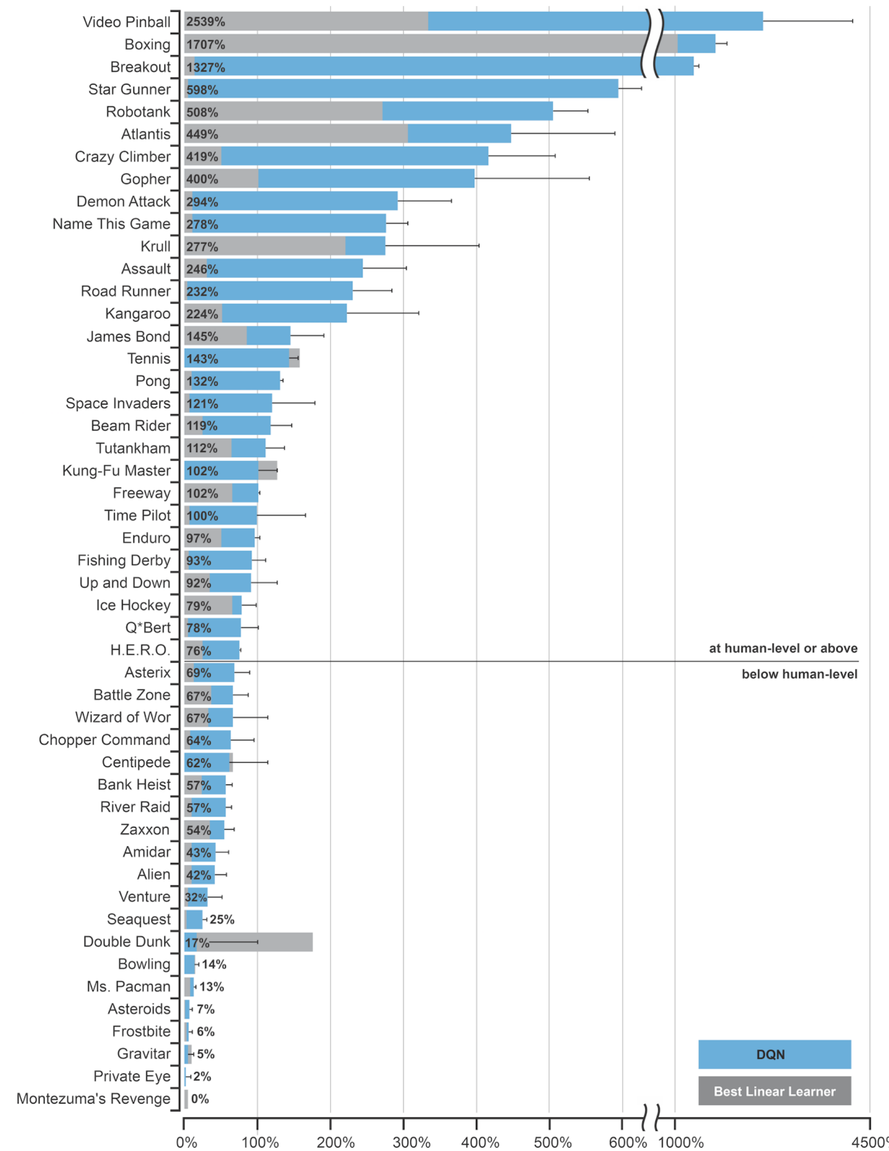
- Learned to play expertly in chess
  - improvement from 1650 Elo (beginner) to 2150 Elo (good club player) in only ca. 1000 games on the internet
    - learning was turned on at game 0 (games before for getting a rating)
    - at game 500, memory was increased, which helped the search



- Improvements over TD-Gammon:
  - Integration of TD-learning with deep searches which are necessary for computer chess
  - self-play training is replaced with training by playing against various partners on the internet

Super Human ATARI playing
(Minh et al. 2013)

- Reinforcement Learning with Deep Learning
- State-of-the-Art
- Better than humans in 29/49 ATARI games
- Extremely high computation times



Reinforcement Learning Resources

- Book
  - On-line Textbook on Reinforcement learning
    - http://www.cs.ualberta.ca/~sutton/book/the-book.html
- More Demos
  - Grid world
    - http://thierry.masson.free.fr/IA/en/qlearning_applet.htm
  - Robot learns to crawl
    - http://www.applied-mathematics.net/qlearning/qlearning.html
- Reinforcement Learning Repository
  - tutorial articles, applications, more demos, etc.
    - http://www-anw.cs.umass.edu/rlr/
- RL-Glue (Open Source RL Programming framework)
    - http://glue.rl-community.org/