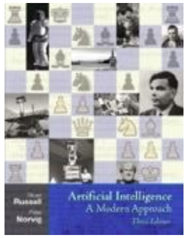
Bayesian Networks

- Syntax
- Semantics
- Parametrized Distributions



Many slides based on Russell & Norvig's slides
Artificial Intelligence: A Modern Approach

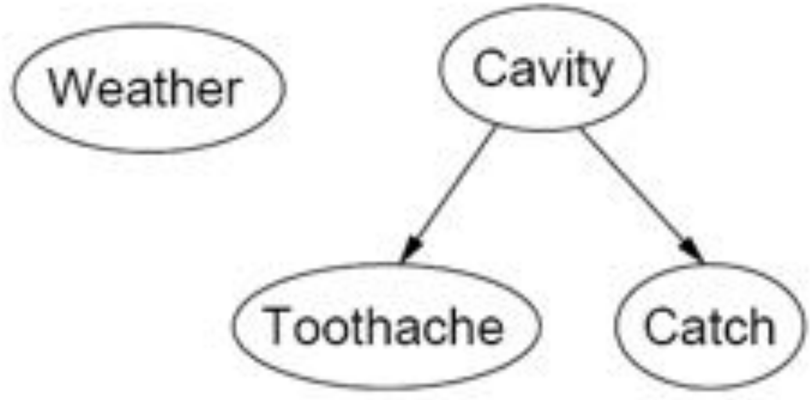
Judea Pearl, UCLA
Turing Award 2012

Bayesian Networks - Structure

- Are a simple, graphical notation for **conditional independence** assertions, hence for **compact specifications of full joint distributions**

A BN is a directed acyclic graph with the following components:

- **Nodes**: one node for each variable
- **Edges**: a directed edge from node N_i to node N_j indicates that variable X_i has a direct influence upon variable X_j

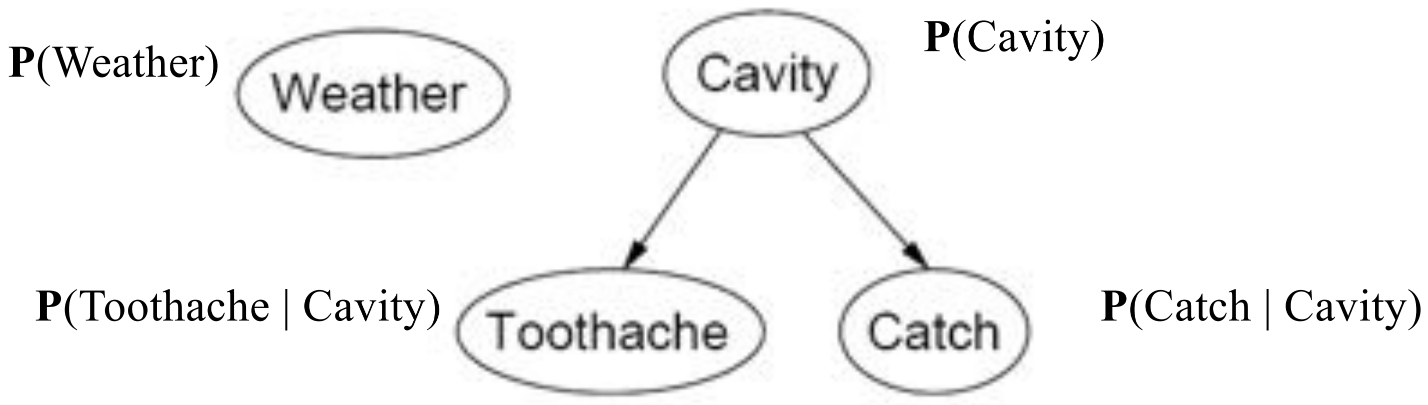


Bayesian Networks - Probabilities

In addition to the structure, we need a **conditional probability distribution** for the random variable of each node given the random variables of its parents.

- i.e. we need $\mathbf{P}(X_i \mid \mathrm{Parents}(X_i))$
- nodes/variables that are not connected are (conditionally) independent:

Example: Weather is independent of Cavity, Toothache is independent of Catch given Cavity



Running Example: Alarm

Situation:

- I'm at work
- John calls to say that in my house the alarm went off but Mary (my neighbor) did not call
- The alarm will usually be set off by burglars but sometimes it may also go off because of minor earthquakes

How do you model this?

Running Example: Alarm

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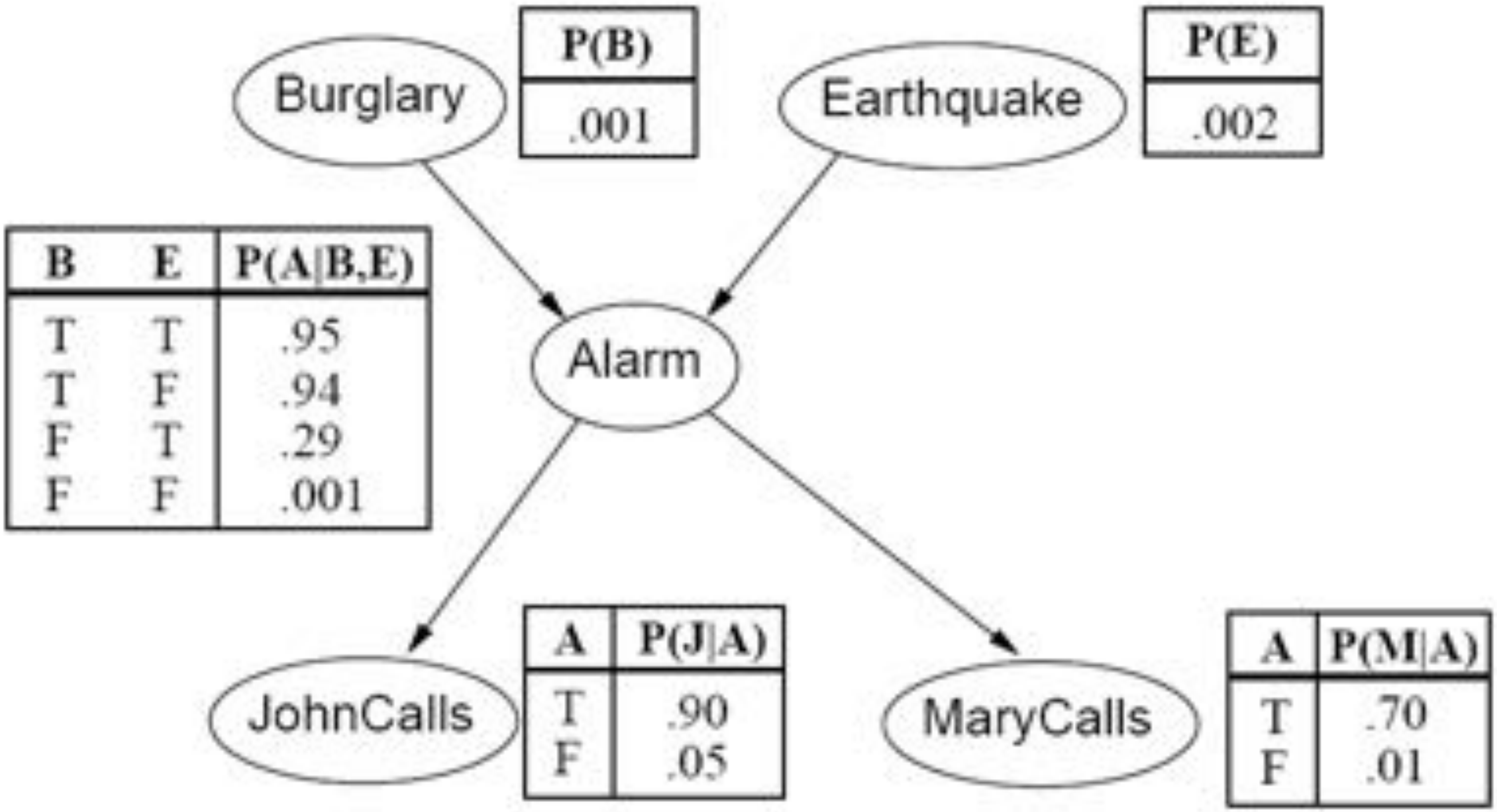
Random Variables:

- *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, *MaryCalls*

Network topology reflects causal knowledge:

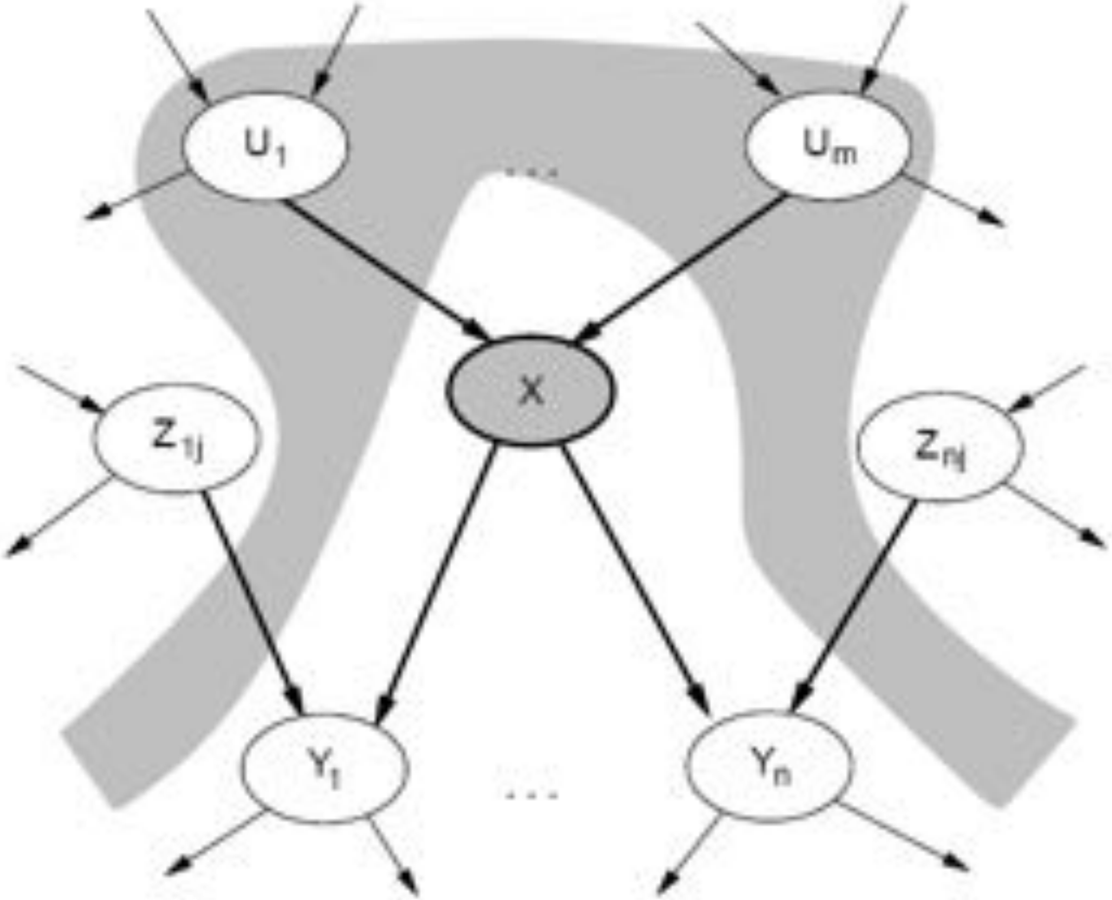
- A burglar can set the alarm off
- An earthquake can set the alarm off
- The alarm can cause Mary to call
- The alarm can cause John to call

Alarm Example



Local Semantics of a BN

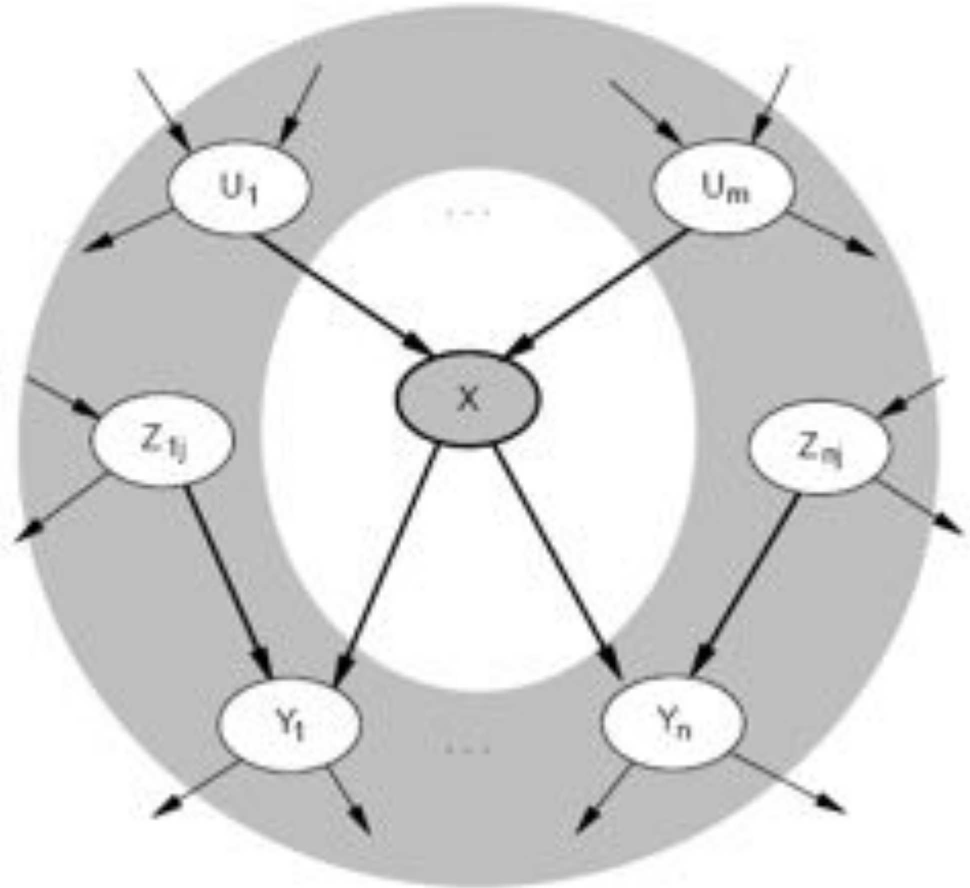
- Each node is is conditionally independent of its nondescendants given its parents
  (sometime called the **local Markov assumption**)



$$\mathbf{P}(X \mid U_1, \ldots, U_m, Z_{1j}, \ldots, Z_{nj}) = \\ = \boldsymbol{P}(X \mid U_1, \ldots, U_m)$$

Markov Blanket

- **Markov Blanket = parents + children + children's parents**



Each node is conditionally independent of all other nodes given its Markov blanket

$$\mathbf{P}(X \mid U_1, \ldots, U_m, Y_1, \ldots, Y_n, Z_{1j}, \ldots, Z_{nj}) =$$
$$= \boldsymbol{P}(X \mid allvariables)$$

Global Semantics of a BN

- The conditional probability distributions define (**factorizes**) the joint probability distribution of the variables of the network

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{Parents}(X_i))$$

- Example:
  - What is the probability that the alarm goes off and both John and Mary call, but there is neither a burglary nor an earthquake?

$$P(j \land m \land a \land \neg b \land \neg e) =$$
$$= P(j \mid a) P(m \mid a) P(a \mid \neg b, \neg e) P(\neg b) P(\neg e)$$
$$= 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 \approx 0.00063$$

(Representation) Theorem

Local Semantics ⇔ Global Semantics

Proof (<=):

- order the variables so that parents always appear after children

$$P(J, M, A, E, B) =$$

- apply chain rule (now each variable is conditioned on its parents and other non-descendants)

$$= P(J \mid M, A, E, B) P(M \mid A, E, B) P(A \mid E, B) P(E \mid B) P(B)$$

- use conditional independence

$$= P(J \mid A) P(M \mid A) P(A \mid E, B) P(E) P(B)$$

Constructing Bayesian Networks („=>“)

Need a method such that a series of locally testable assertions of conditional independence guarantees the required global semantics

1. Choose an ordering of variables $X_1, \ldots, X_n$
2. For $i = 1$ to n
   add X_i to the network
   select parents from $X_1, \ldots, X_{i-1}$ such that
   $\mathbf{P}(X_i | Parents(X_i)) = \mathbf{P}(X_i | X_1, \ldots, X_{i-1})$

This choice of parents guarantees the global semantics:

$$\begin{aligned} \mathbf{P}(X_1, \ldots, X_n) &= \prod_{i=1}^{n} \mathbf{P}(X_i | X_1, \ldots, X_{i-1}) \quad \text{(chain rule)} \\ &= \prod_{i=1}^{n} \mathbf{P}(X_i | Parents(X_i)) \quad \text{(by construction)} \end{aligned}$$

Example

- Suppose we first select the ordering

MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

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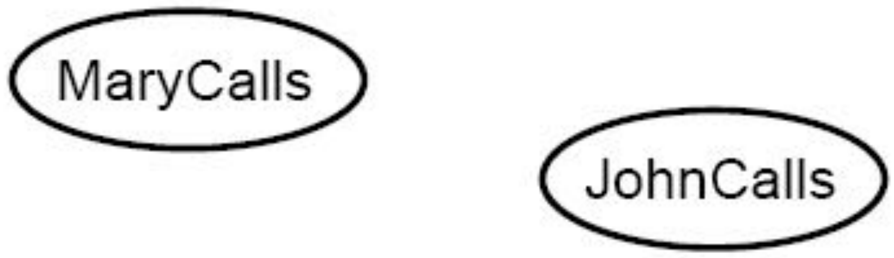

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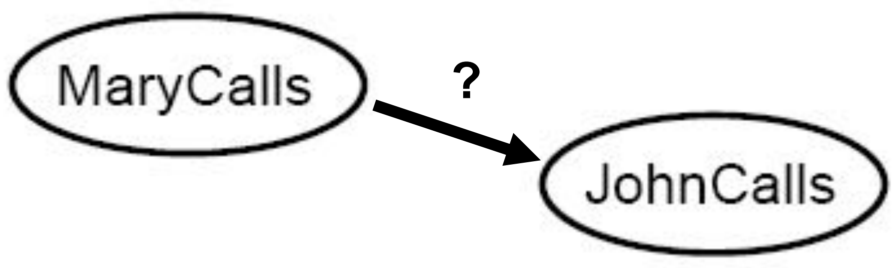
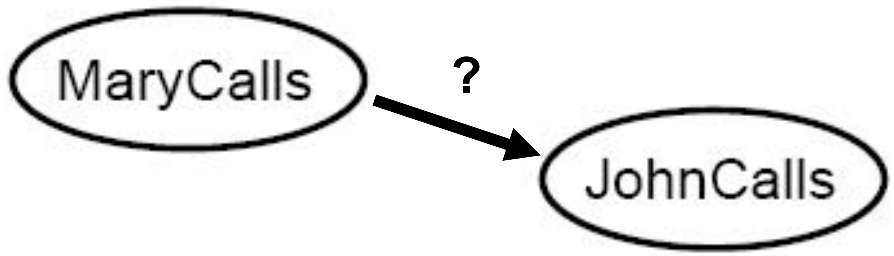


Example

- Suppose we first select the ordering

MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

$$P(J \mid M) = P(J)?$$



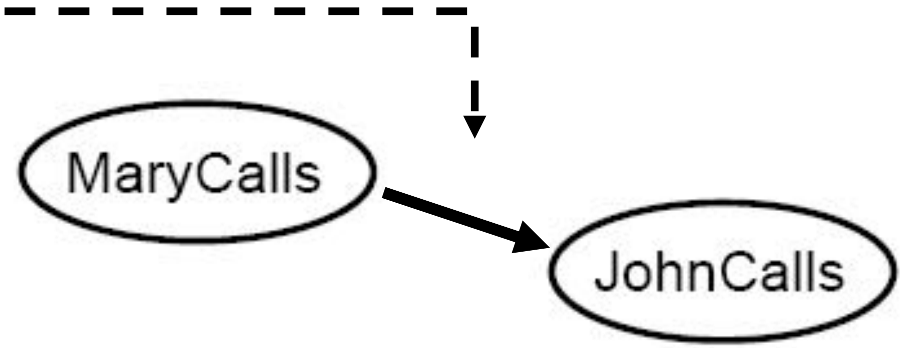
Example

- Suppose we first select the ordering

MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

$$P(J \mid M) = P(J)?$$ ✗

If Mary calls, it is more likely that John calls as well.



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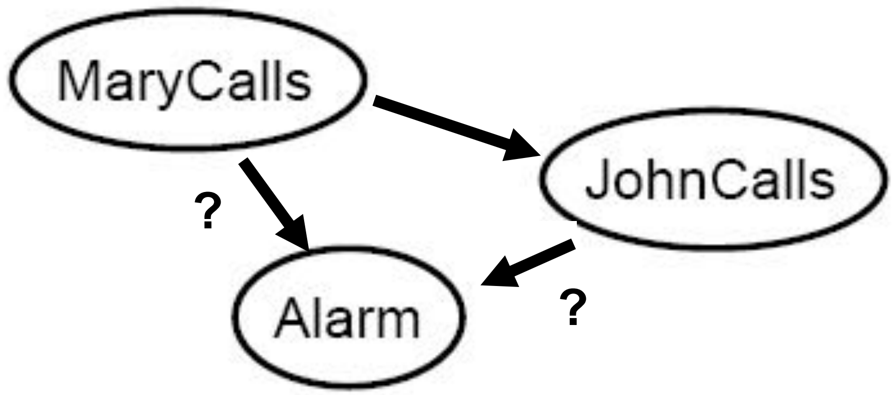

Example

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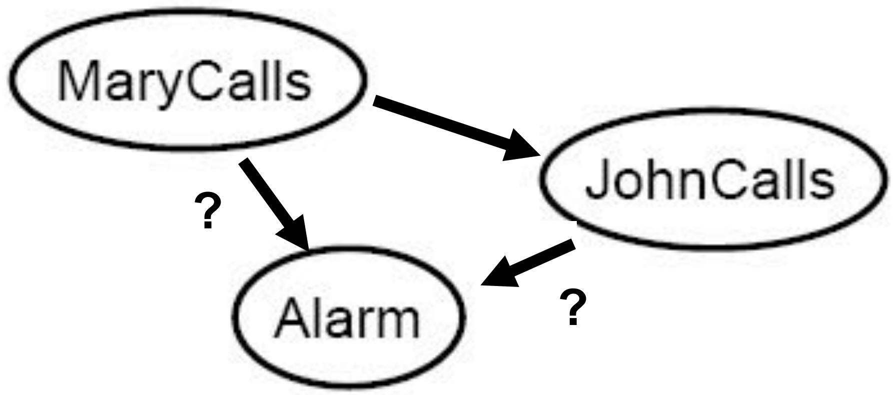


Example

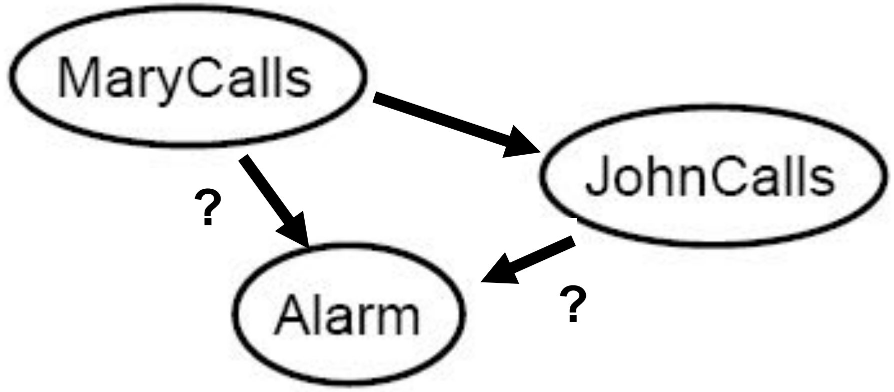
- Suppose we first select the ordering

MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

$P(A \mid J, M) = P(A)?$ ✗

If Mary and John call, the probability that the alarm has gone off is larger than if they don't call.

Node A needs parents J or M



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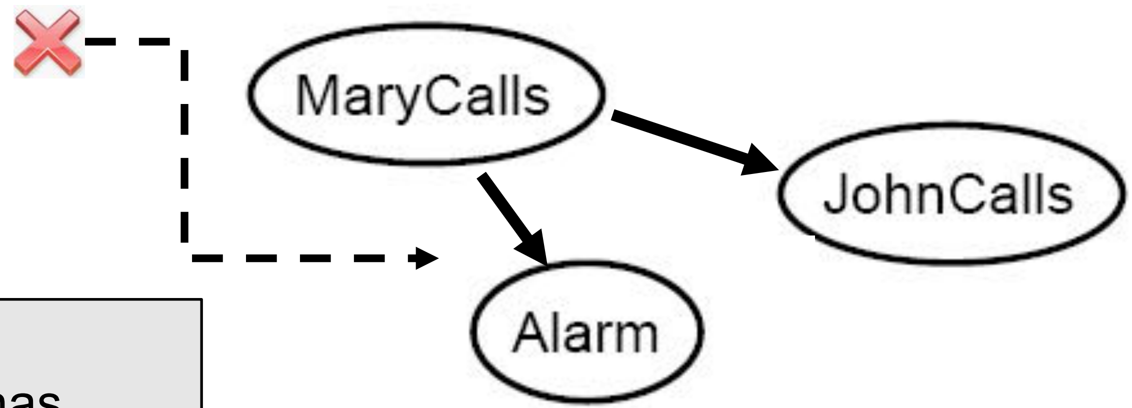

If John and Mary call, the probability that the alarm has gone off is higher than if only John calls.

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MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

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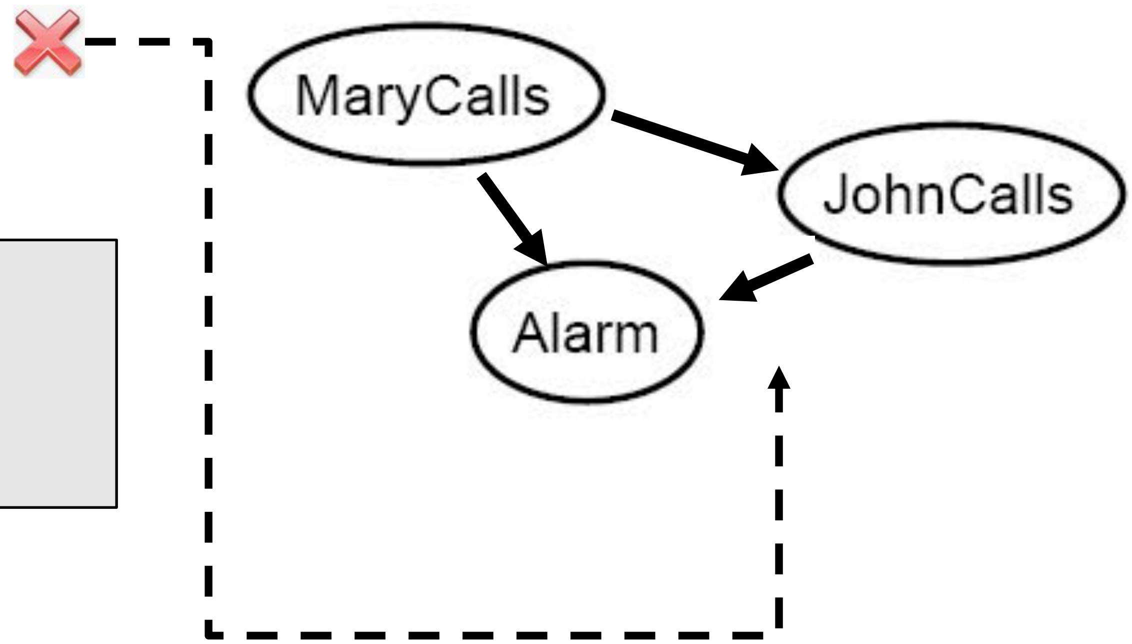


If John and Mary call, the probability that the alarm has gone off is higher than if only Mary calls.

Example

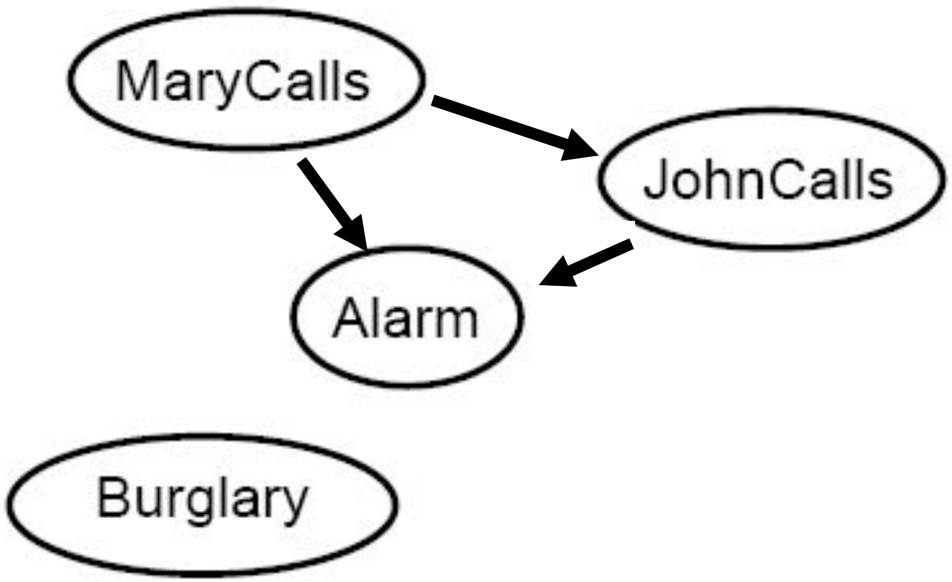
- Suppose we first select the ordering

MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

$$P(B \mid A, J, M) = P(B)?$$ ✗

Knowing whether Mary or John called and whether the alarm went off influences my knowledge about whether there has been a burglary

Node B needs parents A, J or M



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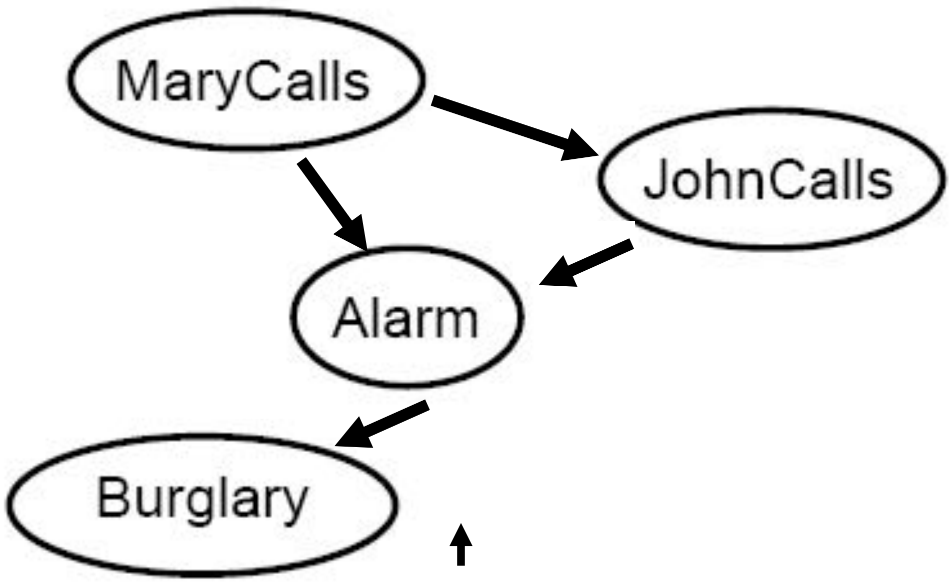
$$P(B \mid A, J, M) = P(B \mid A)? \quad ✓$$



If I know that the alarm has gone off, knowing that John or Mary have called does not add to my knowledge of whether there has been a burglary or not.

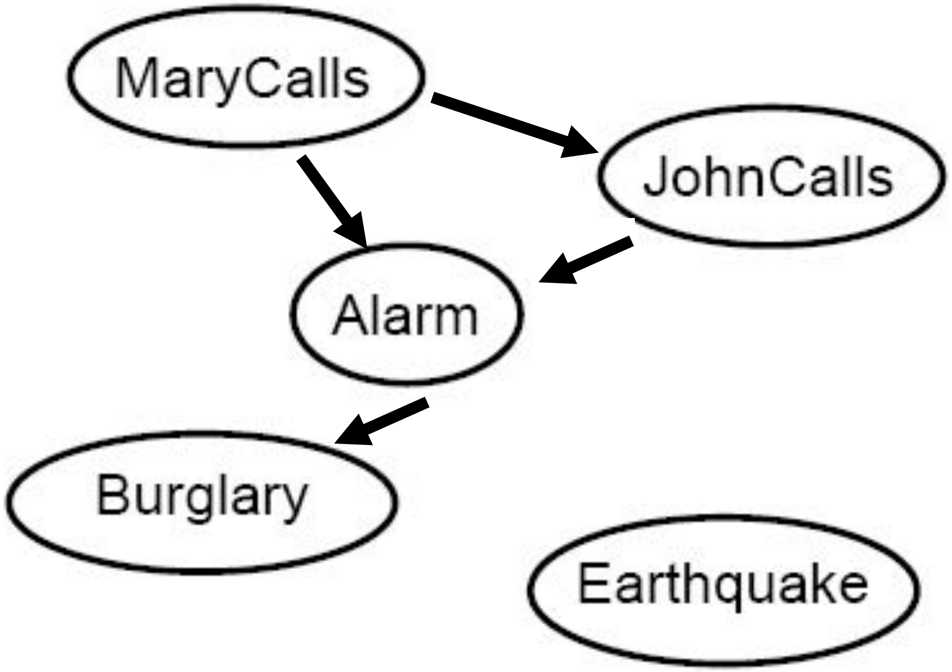
Thus, no edges from M and J, only from A

Example

- Suppose we first select the ordering

MaryCalls, JohnCalls, Alarm, Burglary, Earthquake,

$$P(E \mid B, A, J, M) = P(E \mid A)?$$



Knowing whether there has been an Alarm does not suffice to determine the probability of an earthquake, we have to know whether there has been a burglary as well.

Example

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$$P(E \mid B, A, J, M) = P(E \mid A, B)?$$ ✓

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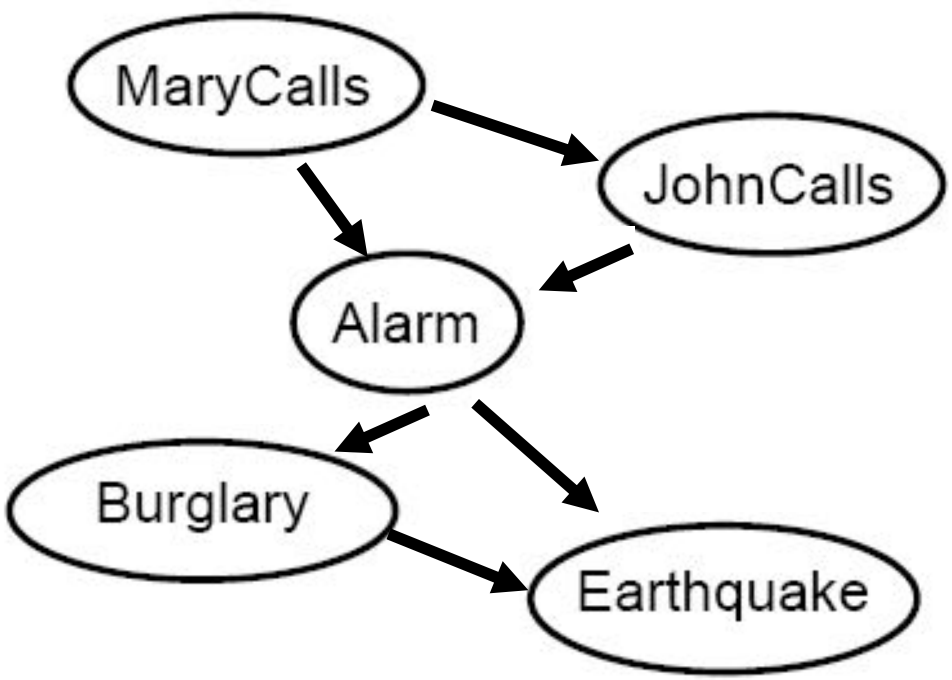

Example - Discussion

- Deciding conditional independence is hard in non-causal direction
- for assessing whether X is conditionally independent of Z ask the question:

If I add variable Z in the condition, does it change the probabilities for X?

- causal models and conditional independence seem hardwired for humans!
- Assessing conditional probabilities is also hard in non-causal direction

- Network is less compact
- more edges and more parameters to estimate
- **Worst possible ordering ?**

Example - Discussion

- Deciding conditional independence is hard in non-causal direction
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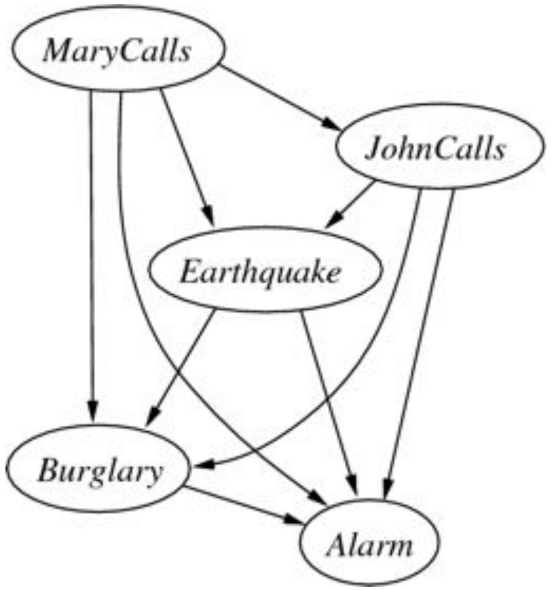
- Network is less compact
- more edges and more parameters to estimate

- Worst possible ordering
- *MaryCalls, JohnCalls Earthquake, Burglary, Alarm*
- → fully connected network



Reasoning with Bayesian Networks

- Belief Functions (margin probabilities)
  - given the probability distribution, we can compute a margin probability at each node, which represents the belief into the truth of the proposition
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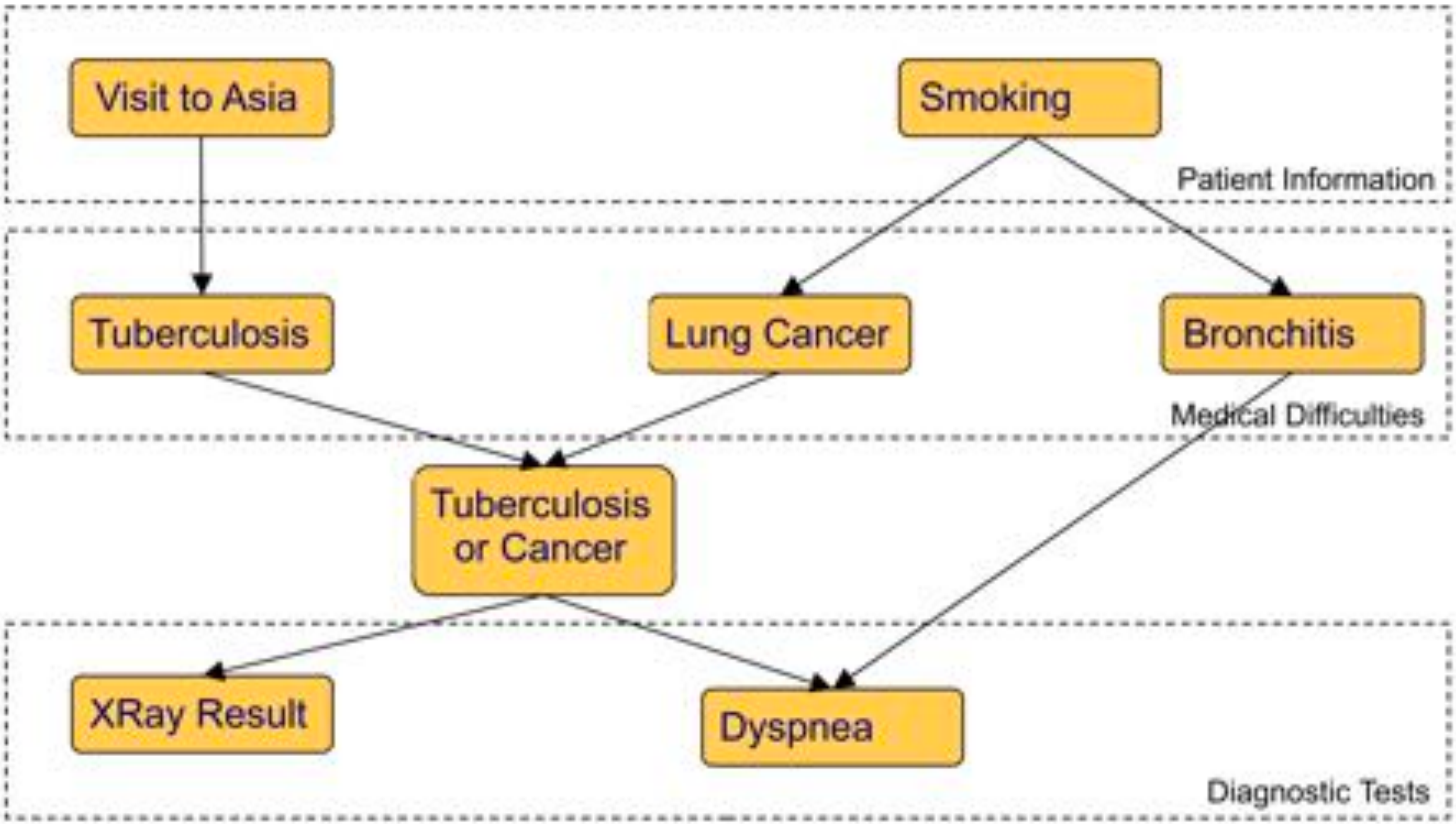
- New evidence can be incorporated into the network by changing the appropriate belief functions
  - this may not only happen in unconditional nodes!
- changes in the belief functions are then propagated through the network
  - propagation happens in forward (along the causal links) and backward direction (against them)
  - e.g., determining a symptom of a disease does not cause the disease, but changes the probability with which we believe that the patient has the disease

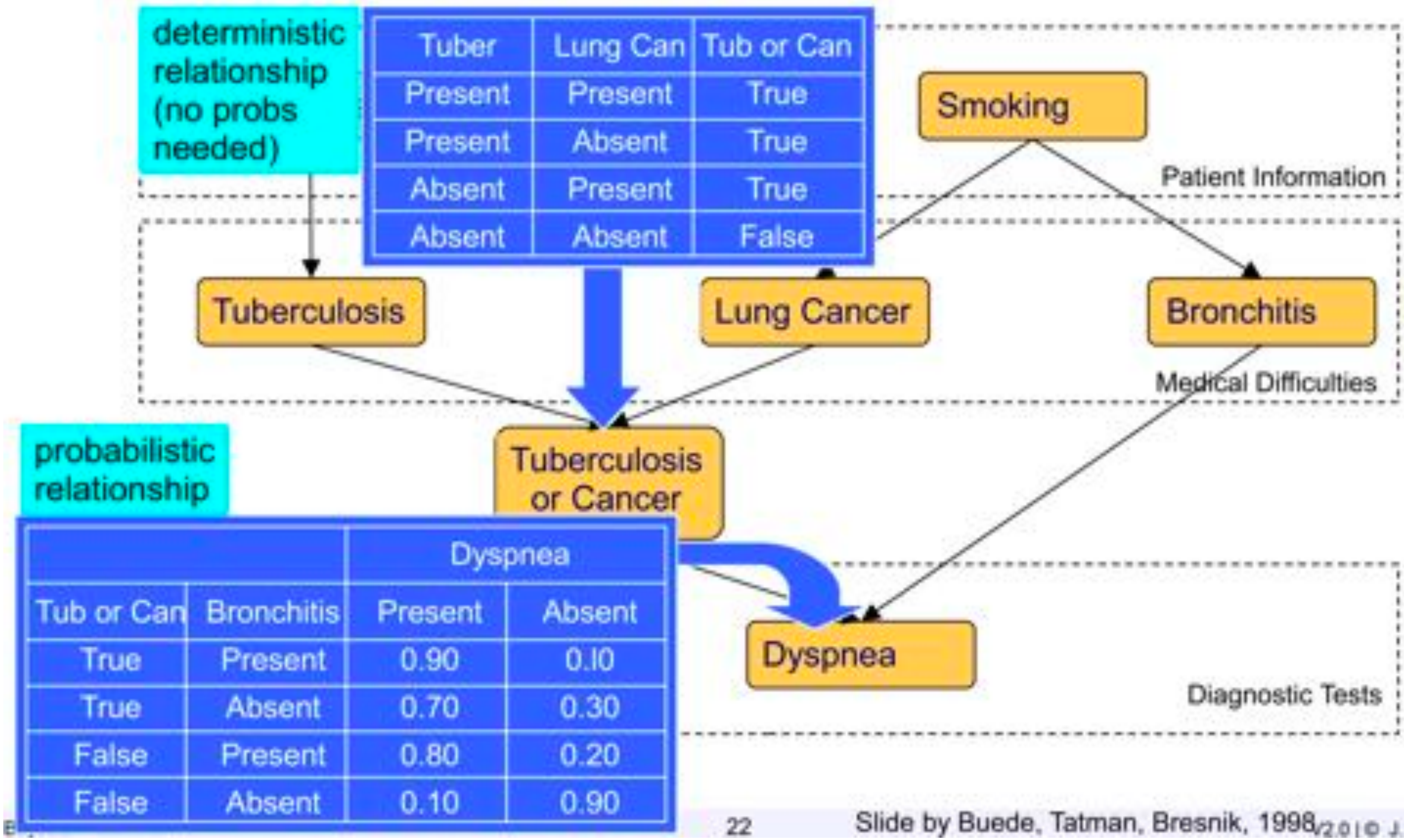
Example: Medical Diagnosis

- Structure of the network



Example: Medical Diagnosis

- Adding Probability Distributions

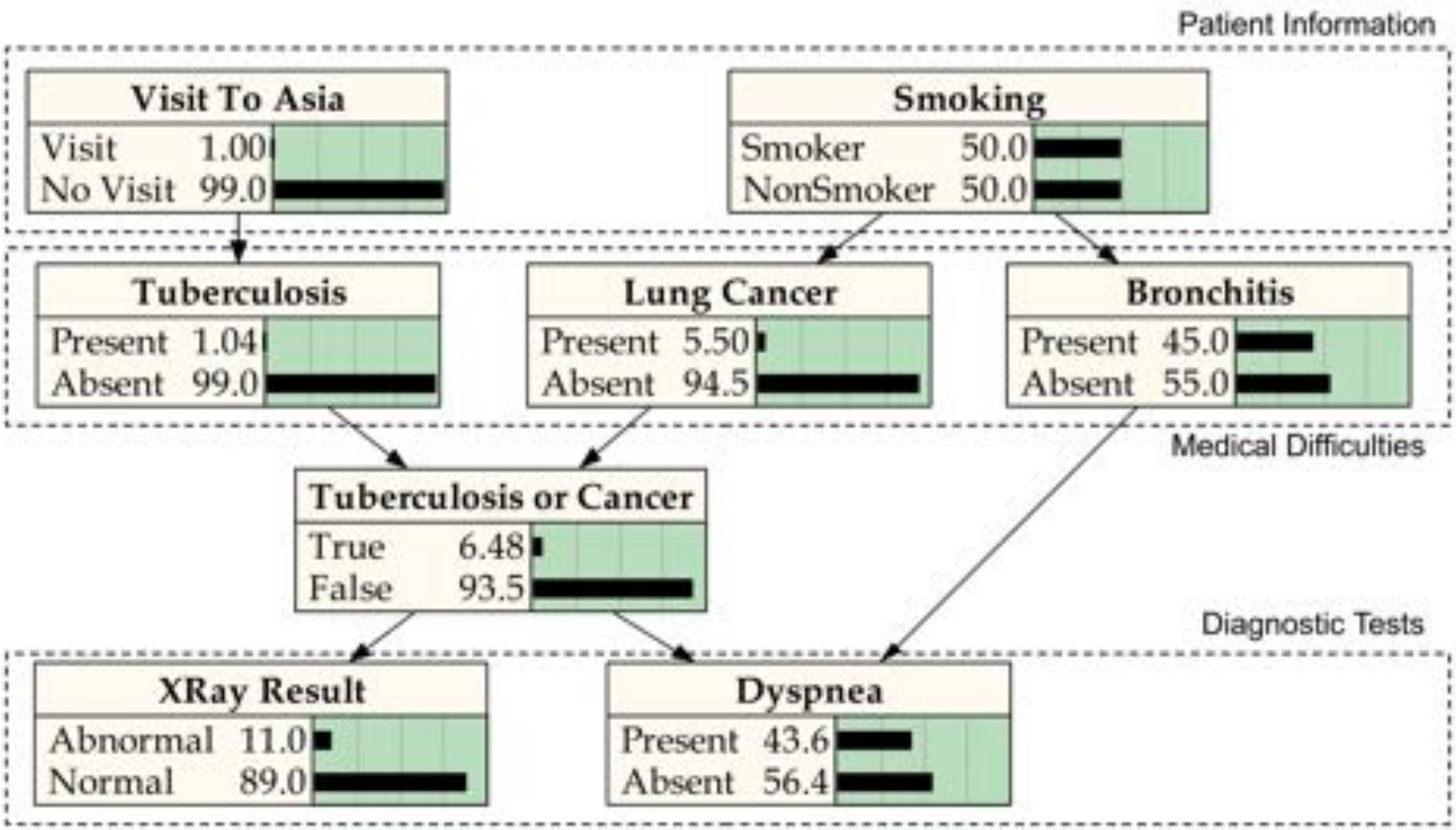


Tuber	Lung Can	Tub or Can
Present	Present	True
Present	Absent	True
Absent	Present	True
Absent	Absent	False

		Dyspnea	
Tub or Can	Bronchitis	Present	Absent
True	Present	0.90	0.l0
True	Absent	0.70	0.30
False	Present	0.80	0.20
False	Absent	0.10	0.90

Example: Medical Diagnosis

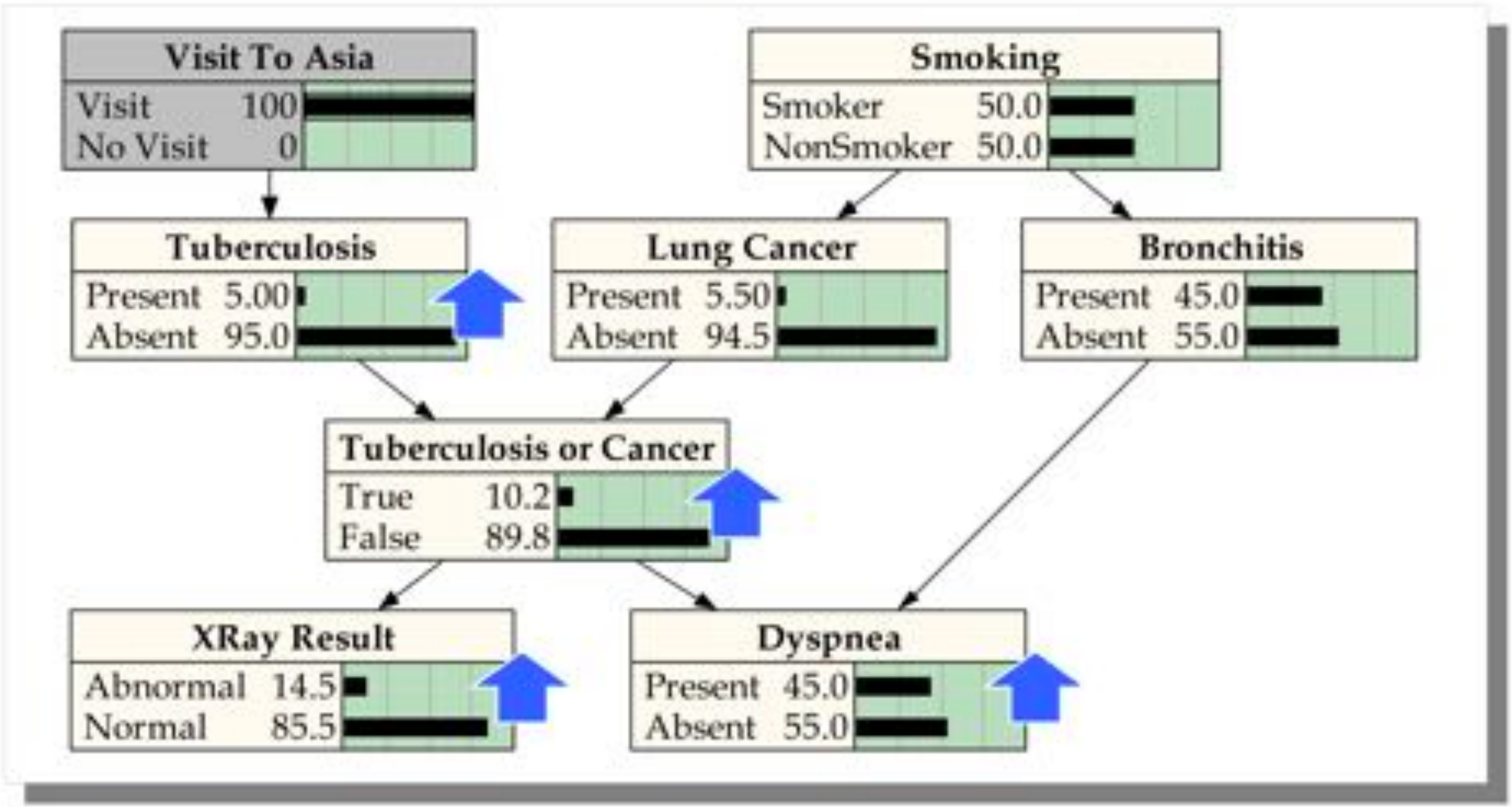
- Belief functions



Slide by Buede, Tatman, Bresnik, 1998

Example: Medical Diagnosis

- Interviewing the patient results in change of probability for variable for “visit to Asia”



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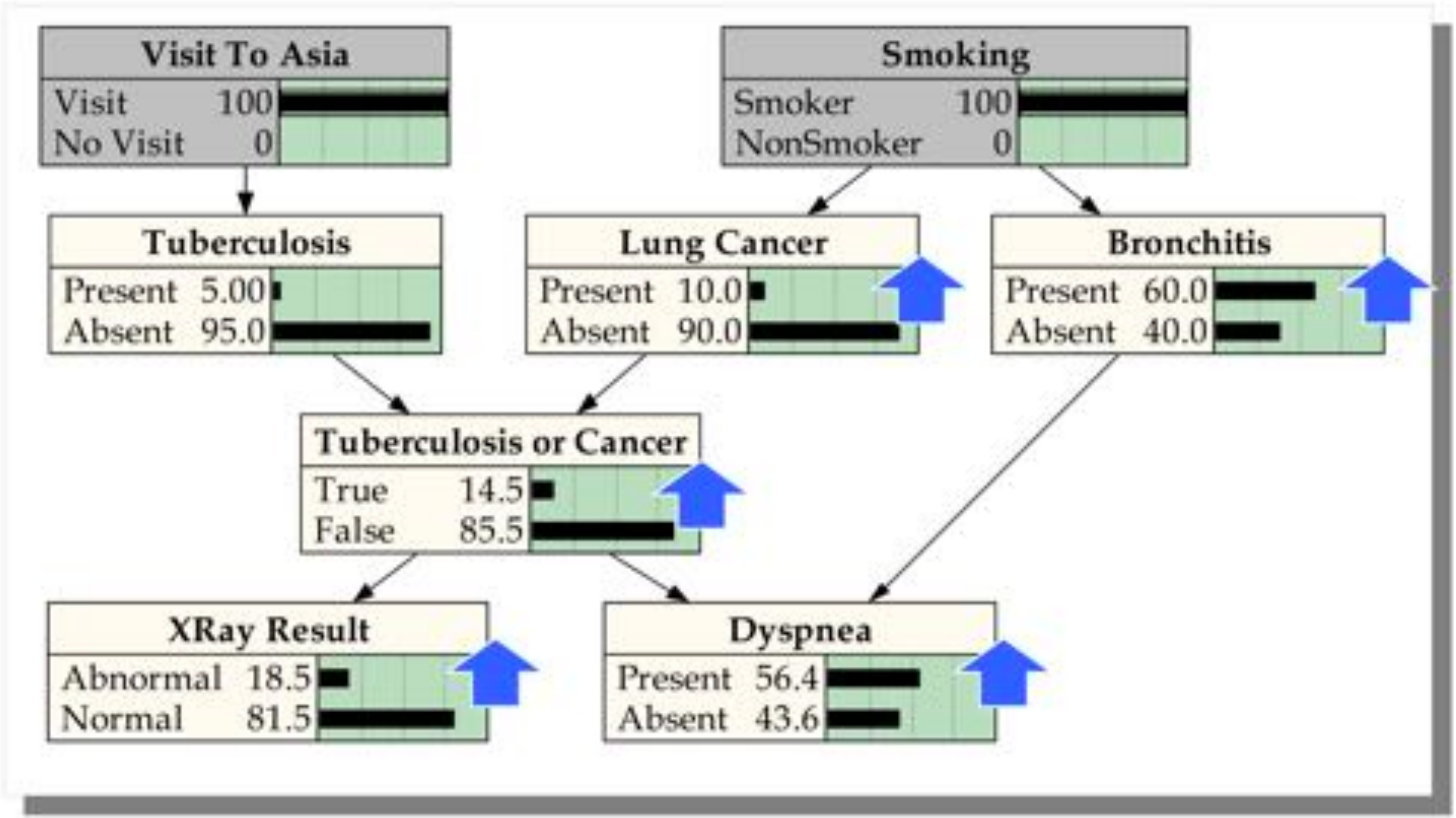
Example: Medical Diagnosis

- Patient is also a smoker...



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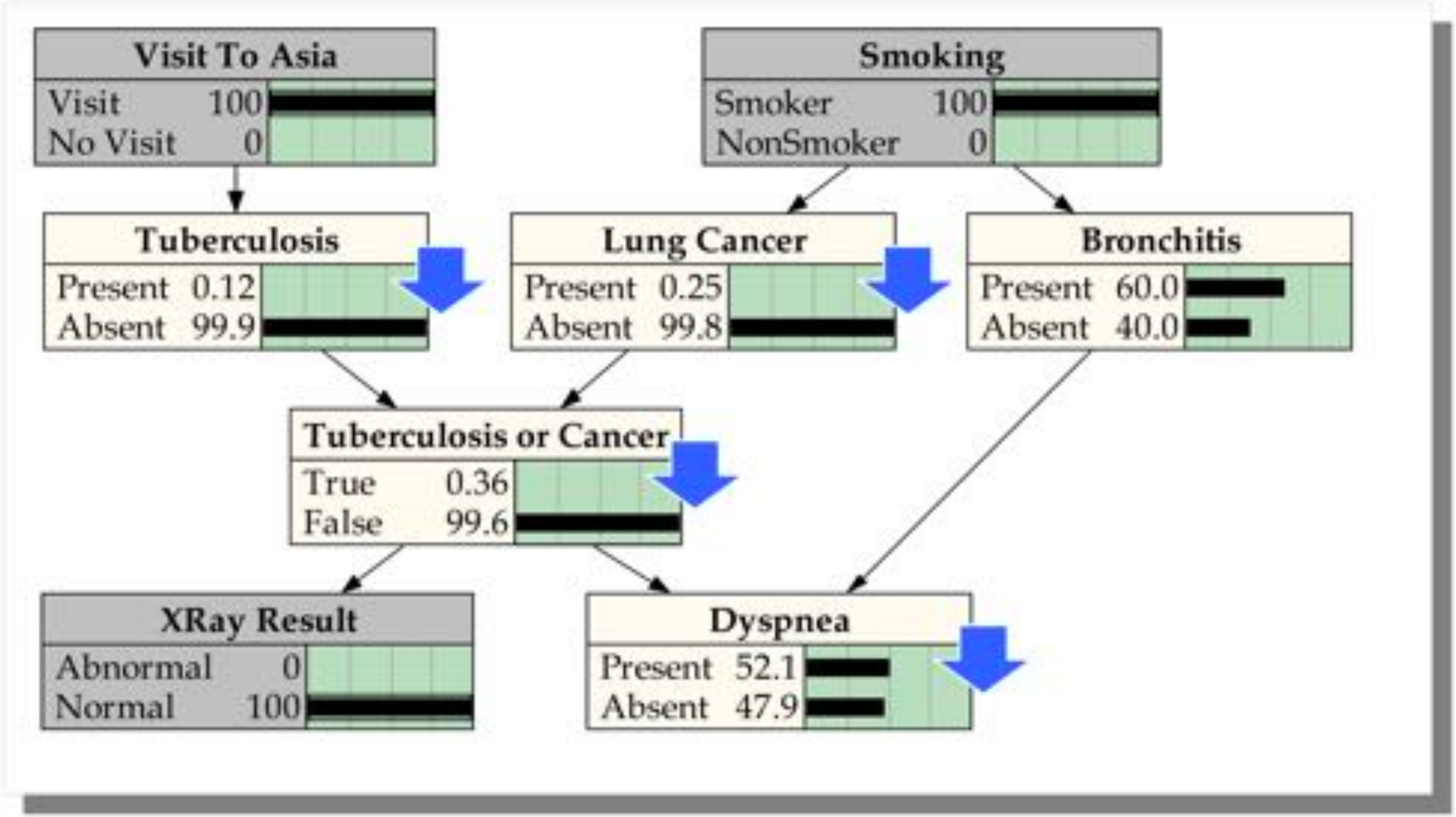
Example: Medical Diagnosis

- but fortunately the X-ray is normal...



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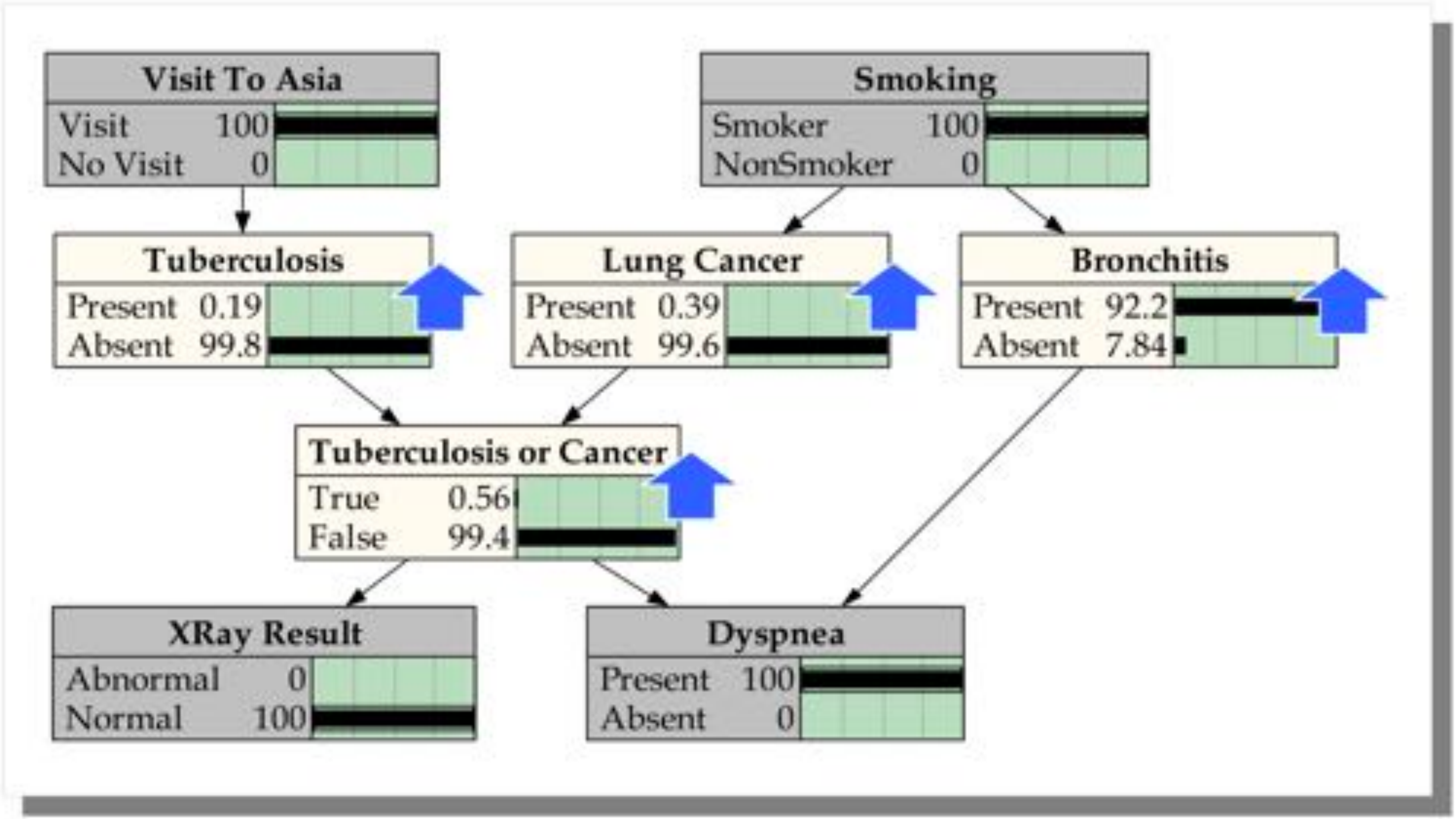
Example: Medical Diagnosis

- but then again patient has difficulty in breathing.



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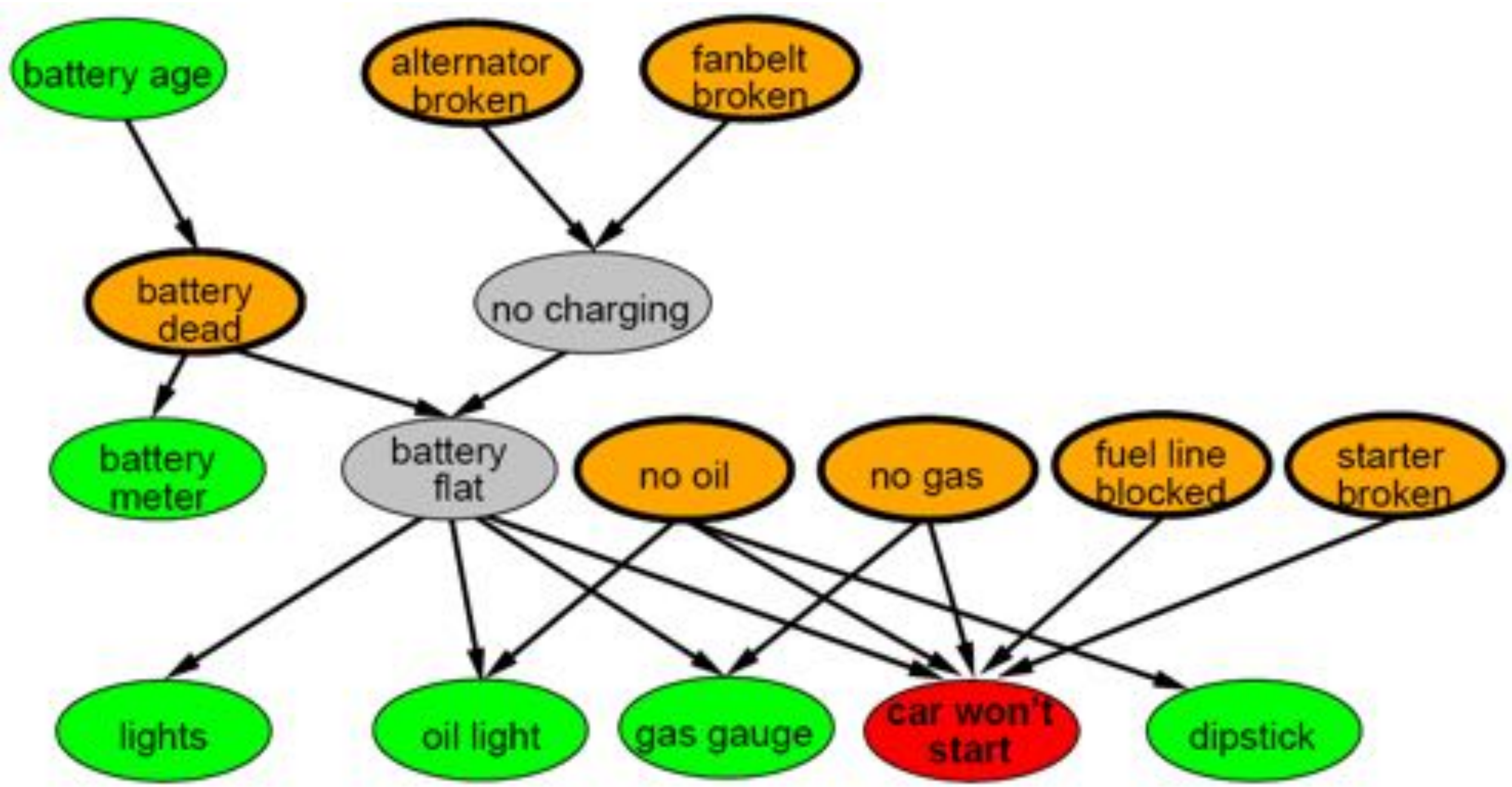
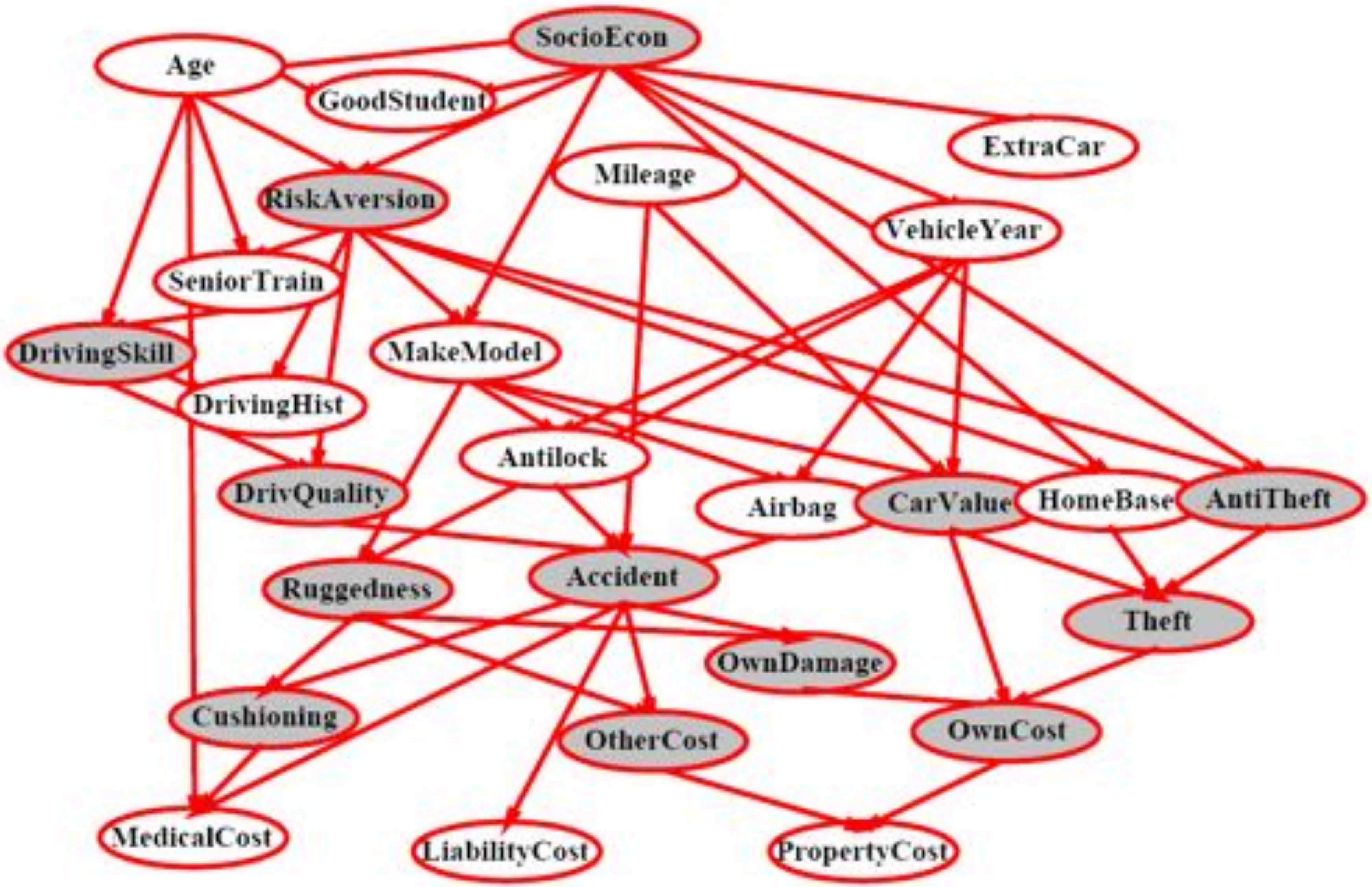
More Complex Example: Car Diagnosis

- Initial evidence: Car does not start
- Test variables
- Variables for possible failures
- Hidden variables: ensure spare structure, reduce parameters

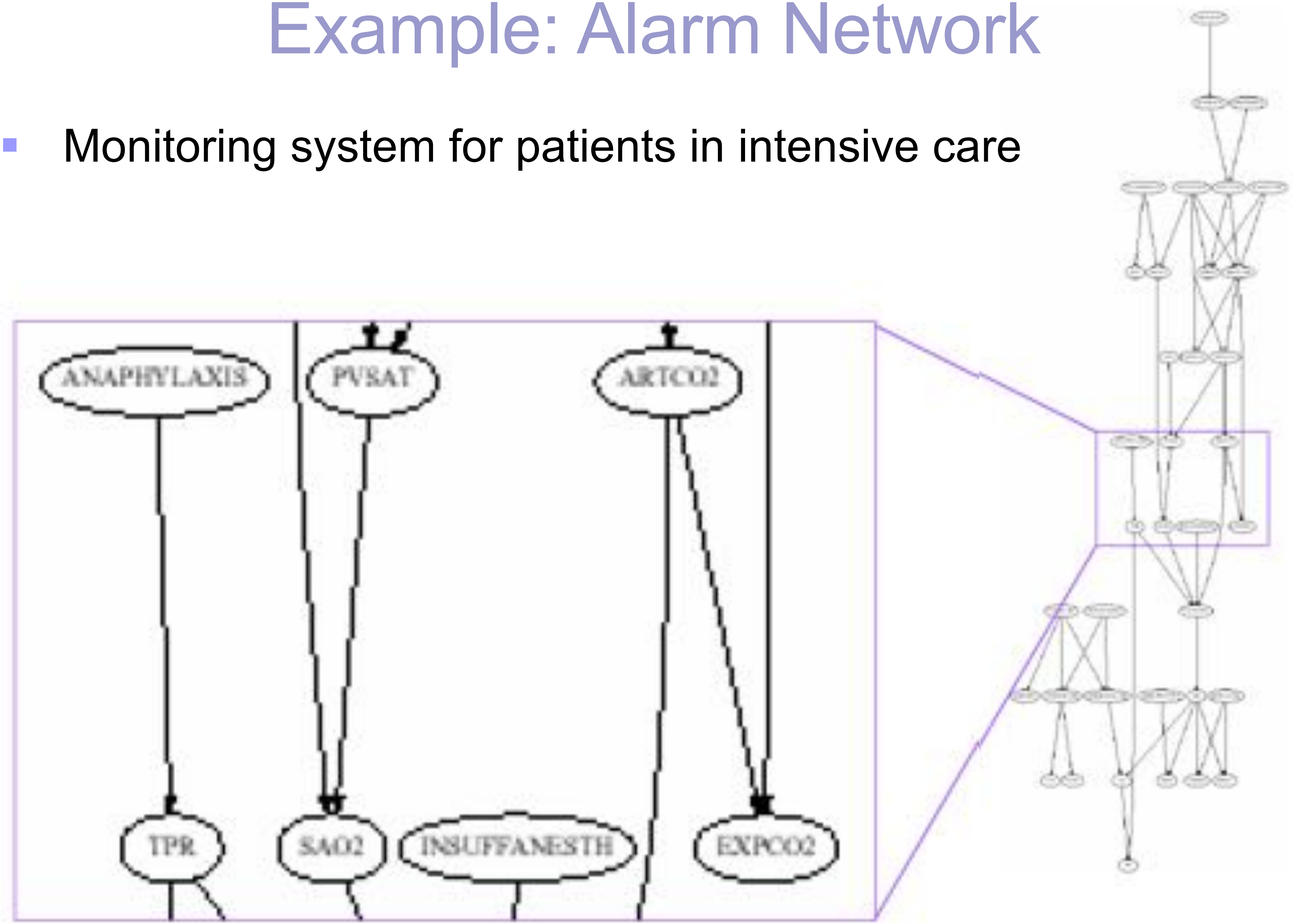


More Complex: Car Insurance



Example: Alarm Network

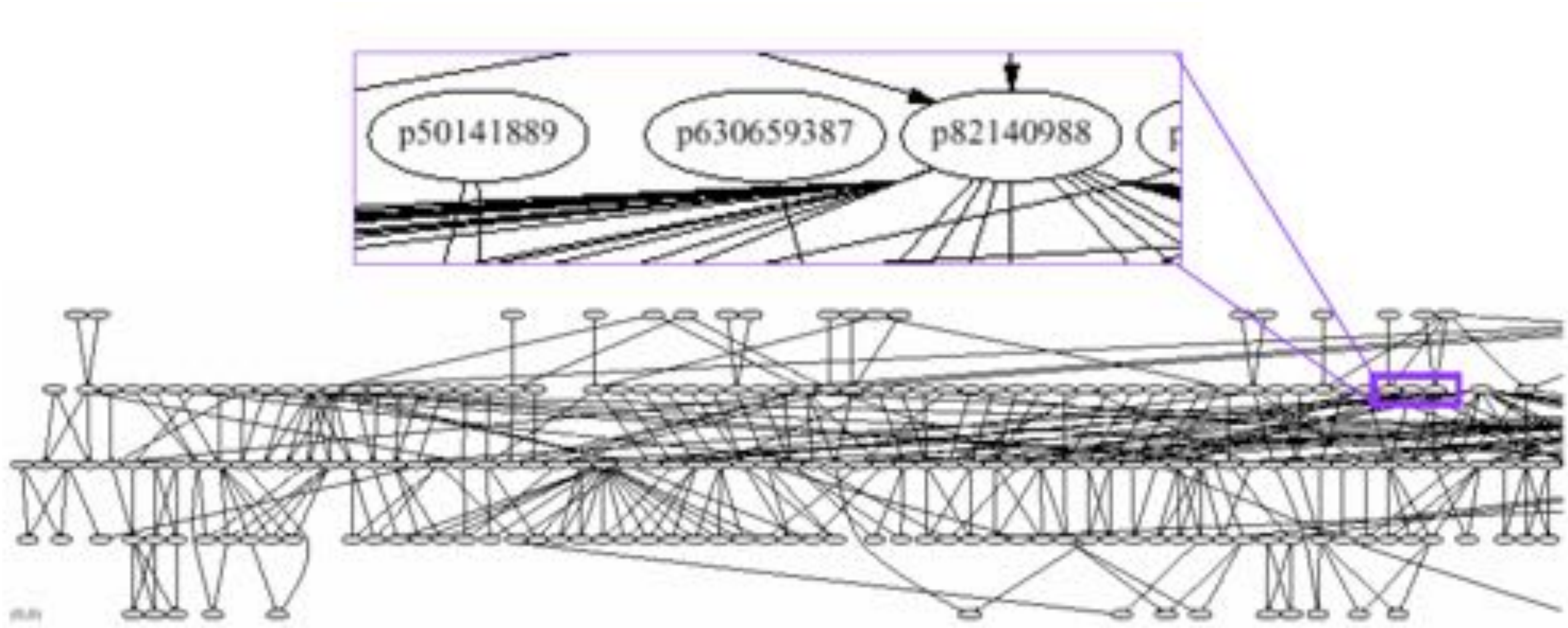
- Monitoring system for patients in intensive care



Slide by C. Boutilier and P. Poupart, 2003

Example: Pigs Network

- Determines pedigree of breeding pigs
- used to diagnose PSE disease
- half of the network structure shown here



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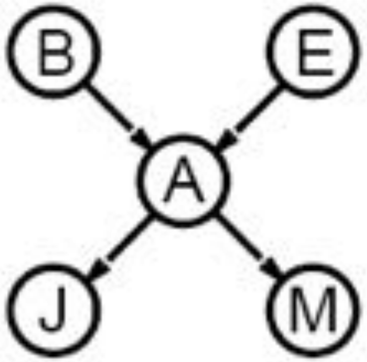
Compactness of a BN

A CPT for Boolean X_i with k Boolean parents has 2^k rows for the combinations of parent values



Each row requires one number p for $X_i = true$ (the number for $X_i = false$ is just $1 - p$)

If each variable has no more than k parents, the complete network requires $O(n \cdot 2^k)$ numbers

I.e., grows linearly with n, vs. $O(2^n)$ for the full joint distribution

For burglary net, $1 + 1 + 4 + 2 + 2 = 10$ numbers (vs. $2^5 - 1 = 31$)

Compact Conditional Distributions

CPT grows exponentially with number of parents
CPT becomes infinite with continuous-valued parent or child

Solution: canonical distributions that are defined compactly

Deterministic nodes are the simplest case:
$X = f(Parents(X))$ for some function f

E.g., Boolean functions
$NorthAmerican \Leftrightarrow Canadian \vee US \vee Mexican$

E.g., numerical relationships among continuous variables

$$\frac{\partial Level}{\partial t} = \text{inflow} + \text{precipitation - outflow - evaporation}$$

Compact Conditional Distributions Independent Causes

Noisy-OR distributions model multiple noninteracting causes

1) Parents $U_1 \ldots U_k$ include all causes (can add leak node)
2) Independent failure probability q_i for each cause alone

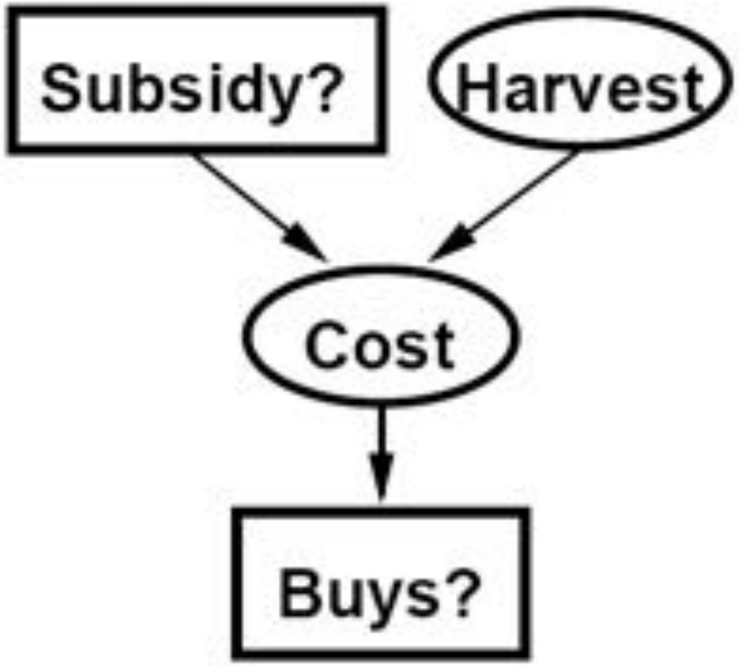
$$\Rightarrow P(X|U_1 \ldots U_j, \neg U_{j+1} \ldots \neg U_k) = 1 - \prod_{i=1}^{j} q_i$$

$Cold$	Flu	$Malaria$	$P(Fever)$	$P(\neg Fever)$
F	F	F	**0.0**	1.0
F	F	T	0.9	**0.1**
F	T	F	0.8	**0.2**
F	T	T	0.98	$0.02 = 0.2 \times 0.1$
T	F	F	0.4	**0.6**
T	F	T	0.94	$0.06 = 0.6 \times 0.1$
T	T	F	0.88	$0.12 = 0.6 \times 0.2$
T	T	T	0.988	$0.012 = 0.6 \times 0.2 \times 0.1$

Number of parameters **linear** in number of parents

Hybrid Networks

Discrete (*Subsidy?* and *Buys?*); continuous (*Harvest* and *Cost*)



Option 1: discretization—possibly large errors, large CPTs
Option 2: finitely parameterized canonical families

1) Continuous variable, discrete+continuous parents (e.g., *Cost*)
2) Discrete variable, continuous parents (e.g., *Buys?*)

Continuous Conditional Distributions

Need one conditional density function for child variable given continuous parents, for each possible assignment to discrete parents

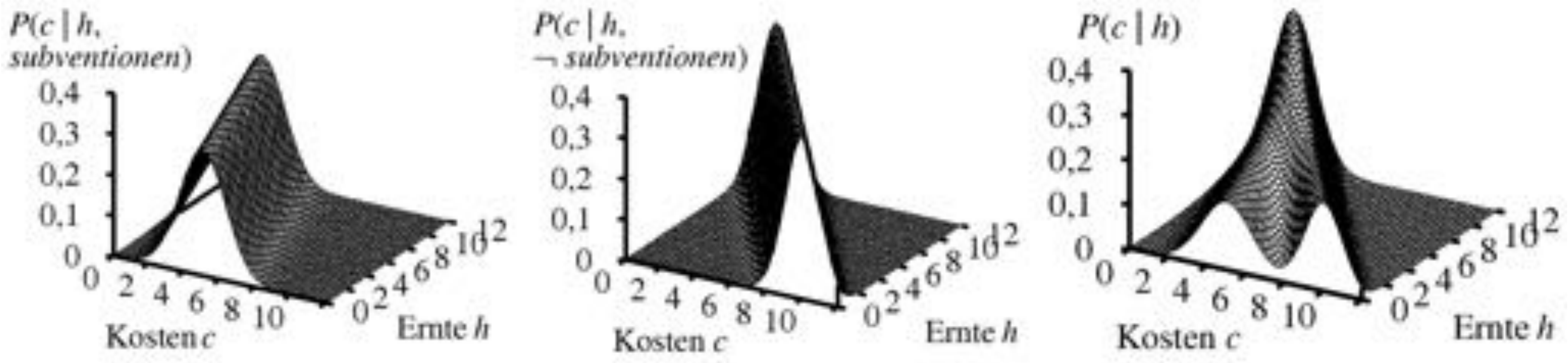
Most common is the linear Gaussian model, e.g.,:

$$
\begin{aligned}
& P(Cost = c \mid Harvest = h, Subsidy? = true) \\
&= N(a_t h + b_t, \sigma_t)(c) \\
&= \frac{1}{\sigma_t \sqrt{2\pi}} exp\left(-\frac{1}{2} \left(\frac{c - (a_t h + b_t)}{\sigma_t} \right)^2 \right)
\end{aligned}
$$

Mean $Cost$ varies linearly with $Harvest$, variance is fixed

Linear variation is unreasonable over the full range
but works OK if the **likely** range of $Harvest$ is narrow

Continuous Conditional Distributions



$\mathbf{P}(Cost \mid Harvest, subsidy)$ $\quad$ $\mathbf{P}(Cost \mid Harvest, \neg subsidy)$ $\quad$ $\mathbf{P}(Cost \mid Harvest)$

$$= \boldsymbol{P}(Cost \mid Harvest, subsidy) + \boldsymbol{P}(Cost \mid Harvest, \neg subsidy)$$

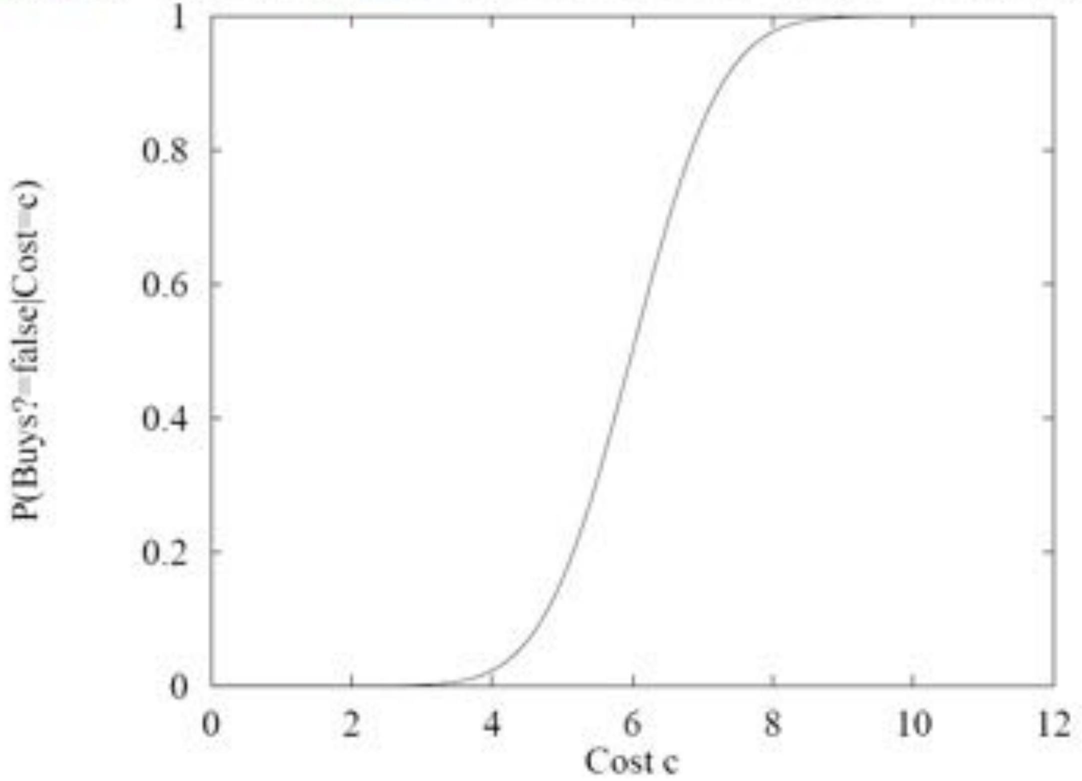
All-continuous network with LG distributions

$\Rightarrow$ full joint distribution is a multivariate Gaussian

Discrete+continuous LG network is a conditional Gaussian network i.e., a multivariate Gaussian over all continuous variables for each combination of discrete variable values

Discrete Variables with Continuous Parents

Probability of $Buys?$ given $Cost$ should be a "soft" threshold:



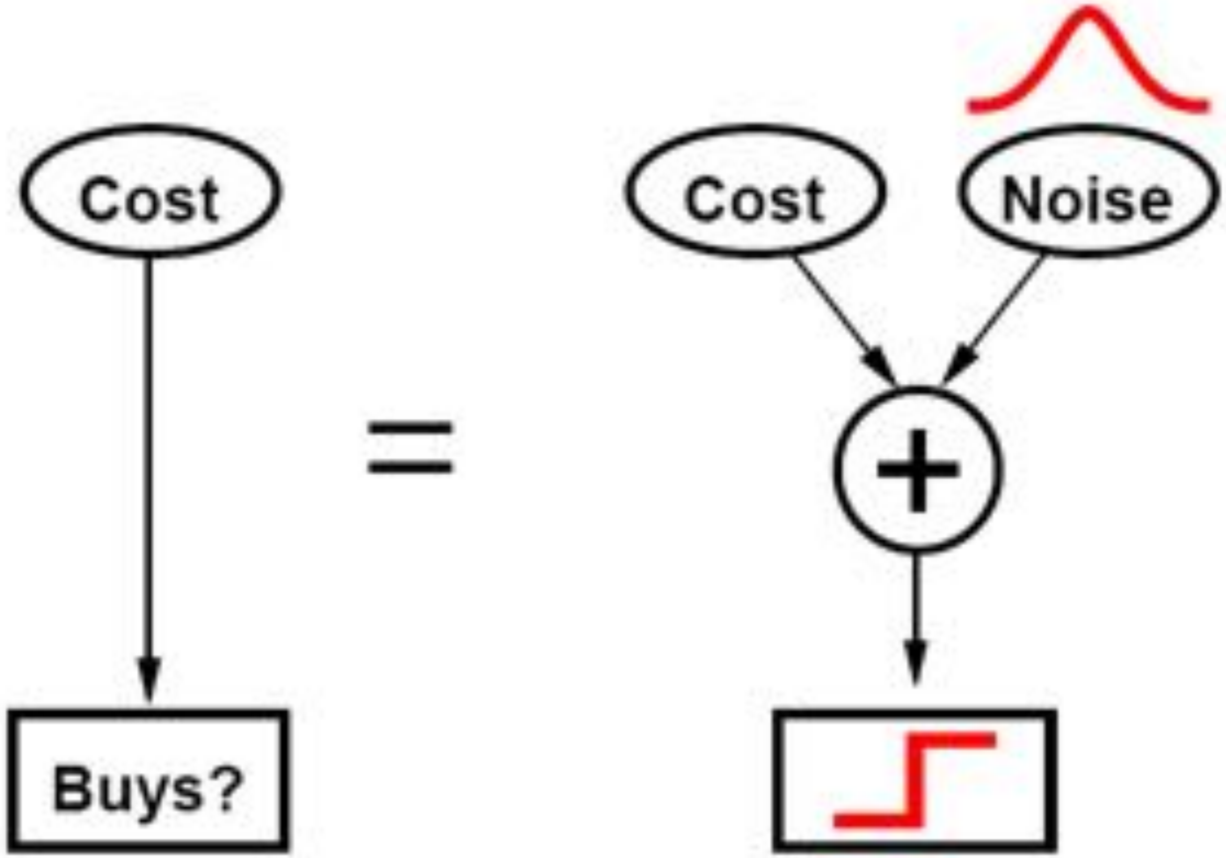
Probit distribution uses integral of Gaussian:

$\Phi(x) = \int_{-\infty}^{x} N(0,1)(x)dx$

$P(Buys? = true \mid Cost = c) = \Phi((-c + \mu)/\sigma)$

Why Probit?

1. It's sort of the right shape
2. Can view as hard threshold whose location is subject to noise

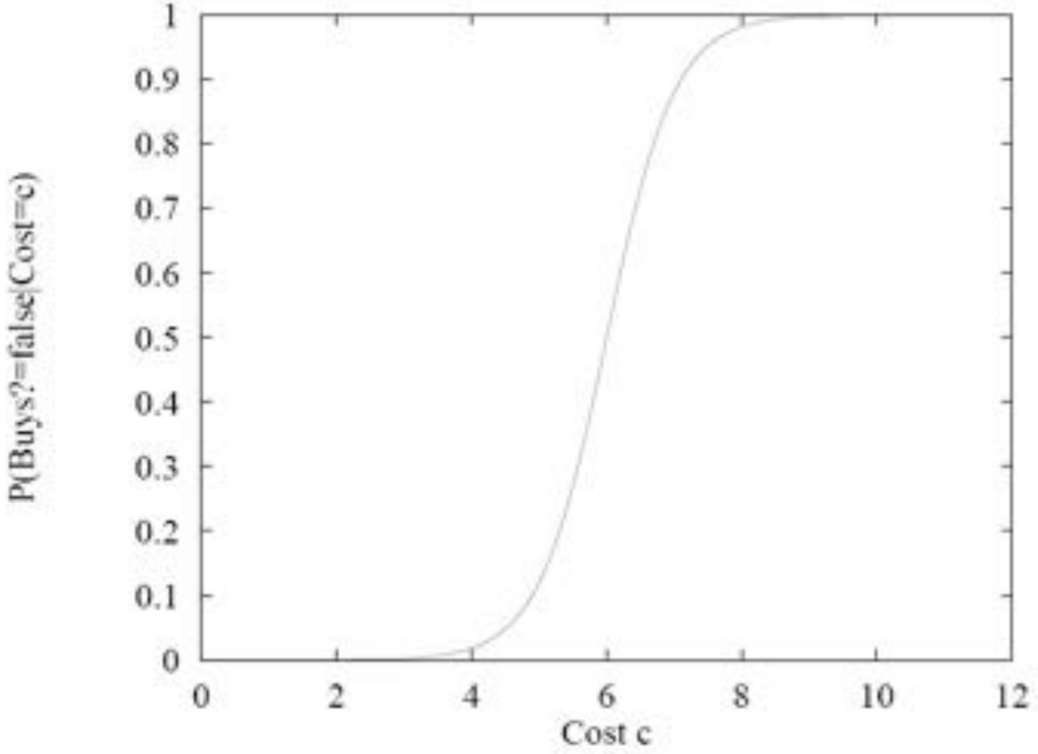


Discrete Variables with Continuous Parents

Sigmoid (or logit) distribution also used in neural networks:

$$P(Buys? = true \mid Cost = c) = \frac{1}{1 + exp(-2\frac{-c+\mu}{\sigma})}$$

Sigmoid has similar shape to probit but much longer tails:



Real-World Applications of BN

- Industrial
- Processor Fault Diagnosis - by Intel
- Auxiliary Turbine Diagnosis - GEMS by GE
- Diagnosis of space shuttle propulsion systems - VISTA by NASA/Rockwell
- Situation assessment for nuclear power plant – NRC

- Military
- Automatic Target Recognition - MITRE
- Autonomous control of unmanned underwater vehicle - Lockheed Martin
- Assessment of Intent

Real-World Applications of BN

- Medical Diagnosis (also at TUDA)
- Internal Medicine
- Pathology diagnosis - Intellipath by Chapman & Hall
- Breast Cancer Manager with Intellipath

- NLP, CV, Cognitive Science, Plant Phenotpying (also at TUDA)

- Commercial
- Financial Market Analysis
- Information Retrieval
- Software troubleshooting and advice - Windows 95 & Office 97
- Pregnancy and Child Care – Microsoft
- Software debugging - American Airlines' SABRE online reservation system