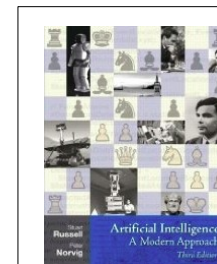# Uninformed Search

- ## Problem-solving agents
    - ### Single-State Problems
- ## Tree search algorithms
    - ### Breadth-First Search
    - ### Depth-First Search
    - ### Limited-Depth Search
   - ### Iterative Deepening
- ## Search with Partial Information

Many slides based on Russell & Norvig's slides
Artificial Intelligence:
A Modern Approach

Problem-Solving Agents

- Simple reflex agents
  - have a direct mapping from states to actions
  - typically too large to store
  - would take too long to learn
- Goal-Based agents
  - can consider future actions and the desirability of their outcomes (→ Planning)
- Problem-Solving Agents
  - special case of Goal-Based Agents
  - find sequences of actions that lead to desirable states
- Uninformed Problem-Solving Agents
  - do not have any information except the problem definition
- Informed Problem-Solving Agents
  - have knowledge where to look for solutions

Goal-Based Agent

- the agent knows what states are desirable
  - it will try to choose an action that leads to a desirable state
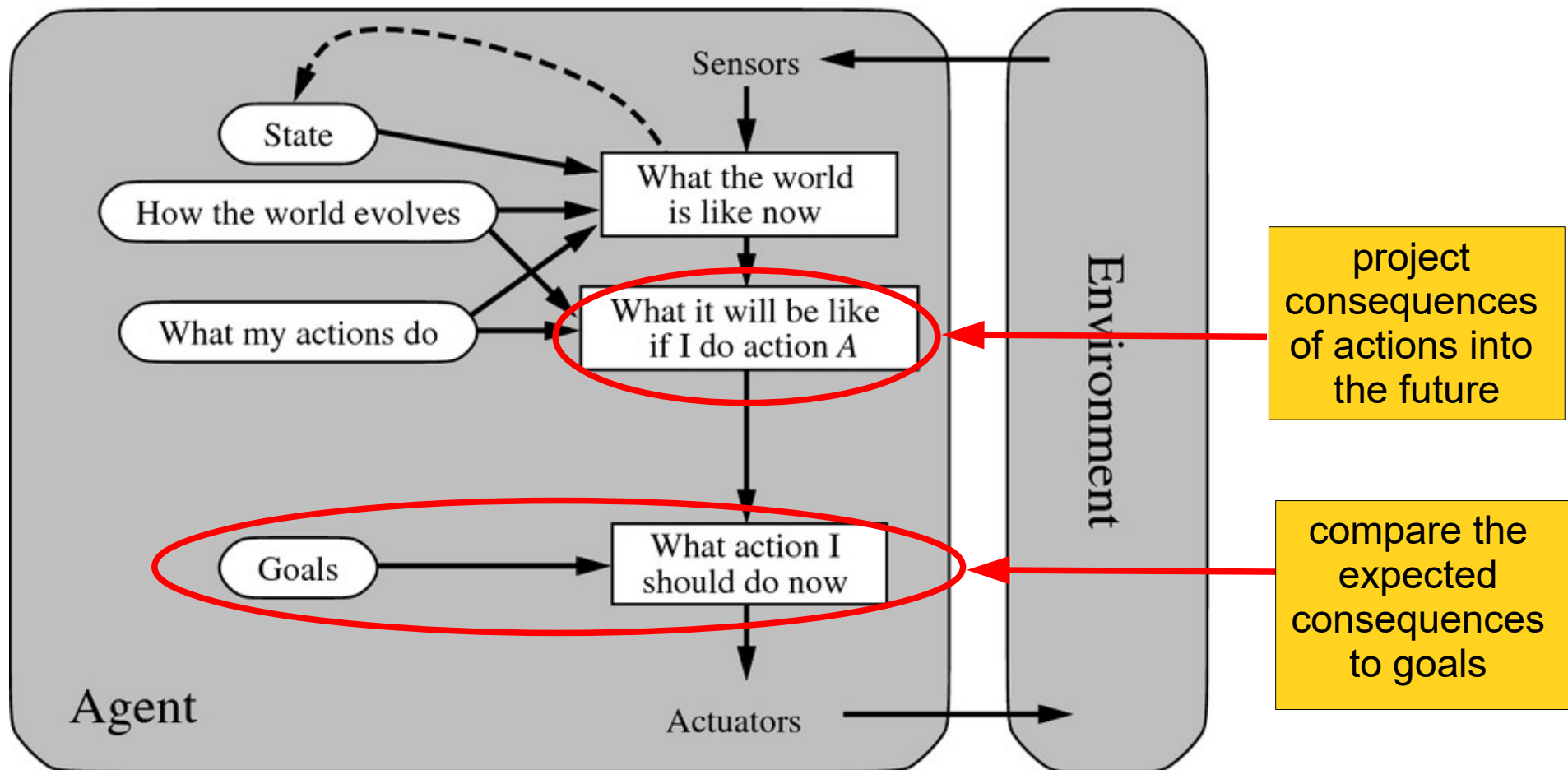
Formulate-Search-Execute Design

- Formulate:
  - Goal formulation:
    - A *goal* is a set of world states that the agents wants to be in (where the goal is achieved)
    - Goals help to organize behavior by limiting the objectives that the agent is trying to achieve
  - Problem formulation:
    - Process of which actions and states to consider, given a goal
- Search:
  - the process of finding the solution for a problem in the form of an action sequence

  > *an agent with several immediate options of unknown value can decide what to do by* **examining different possible sequences** *of actions that lead to states of known value, and then* **choosing the best**

- Execute:
  - perform the first action of the solution sequence

# Simple Problem-Solving Agent

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
    action ← RECOMMENDATION(seq, state)
    seq ← REMAINDER(seq, state)
    return action
```

Example: Navigate in Romania

- On holiday in Romania; currently in Arad.

- Flight leaves tomorrow from Bucharest

- Formulate goal:
  - be in Bucharest

- Formulate problem:
  - states: various cities
  - actions: drive between cities

- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Rimnicu Vilcea, Pitesti

- Assumption:
  - agent has a map of Romania, i.e., it can use this information to find out which of the three ways out of Arad is more likely to go to Bucharest

Example: Romania
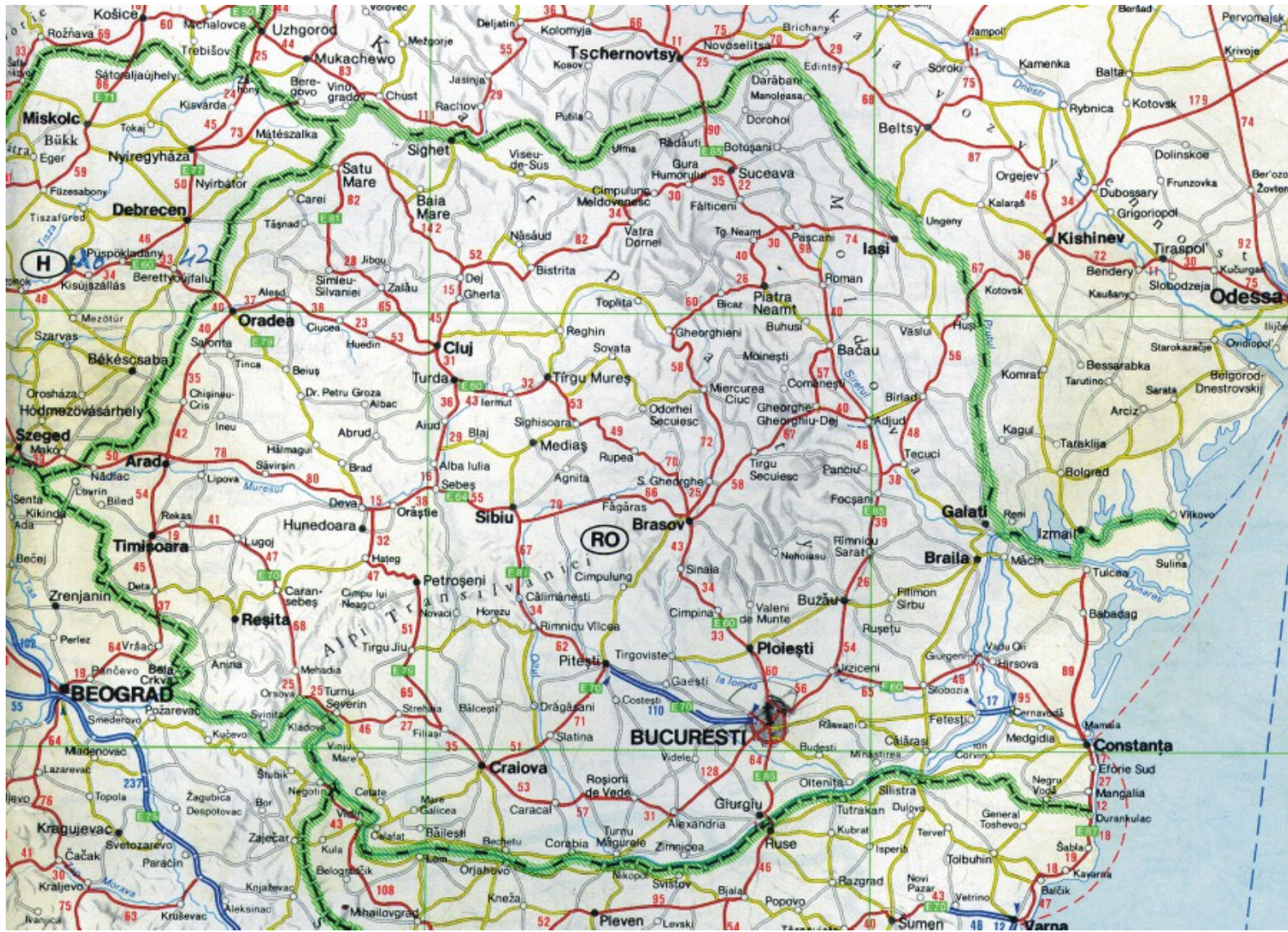
Single-state Problem Formulation

A problem is defined by four items:

- initial state
  - e.g., "at Arad"
- description of actions and their effects
  - typically as a successor function that maps a state s to a set $S(s)$ of action-state pairs
  - e.g., $S(\text{„at Arad“}) = \{<\text{„goto Zerind“}, \text{„at Zerind“}>, \dots \}$
- goal test, can be
  - explicit, e.g., $s = $ "at Bucharest"
  - implicit, e.g., $\text{Checkmate}(s)$, $\text{NoDirt}(s)$
- path cost (additive)
  - e.g., sum of distances, number of actions executed, etc.
  - $c(s_1, a, s_2)$ are the costs for one step (one action),
  - assumed to be ≥ 0

Single-State Problems

Yes

- 8-queens puzzle
- 8-puzzle
- Towers of Hanoi
- Cross-Word puzzles
- Sudoku
- Chess, Bridge, Scrabble puzzles
- Rubik's cube
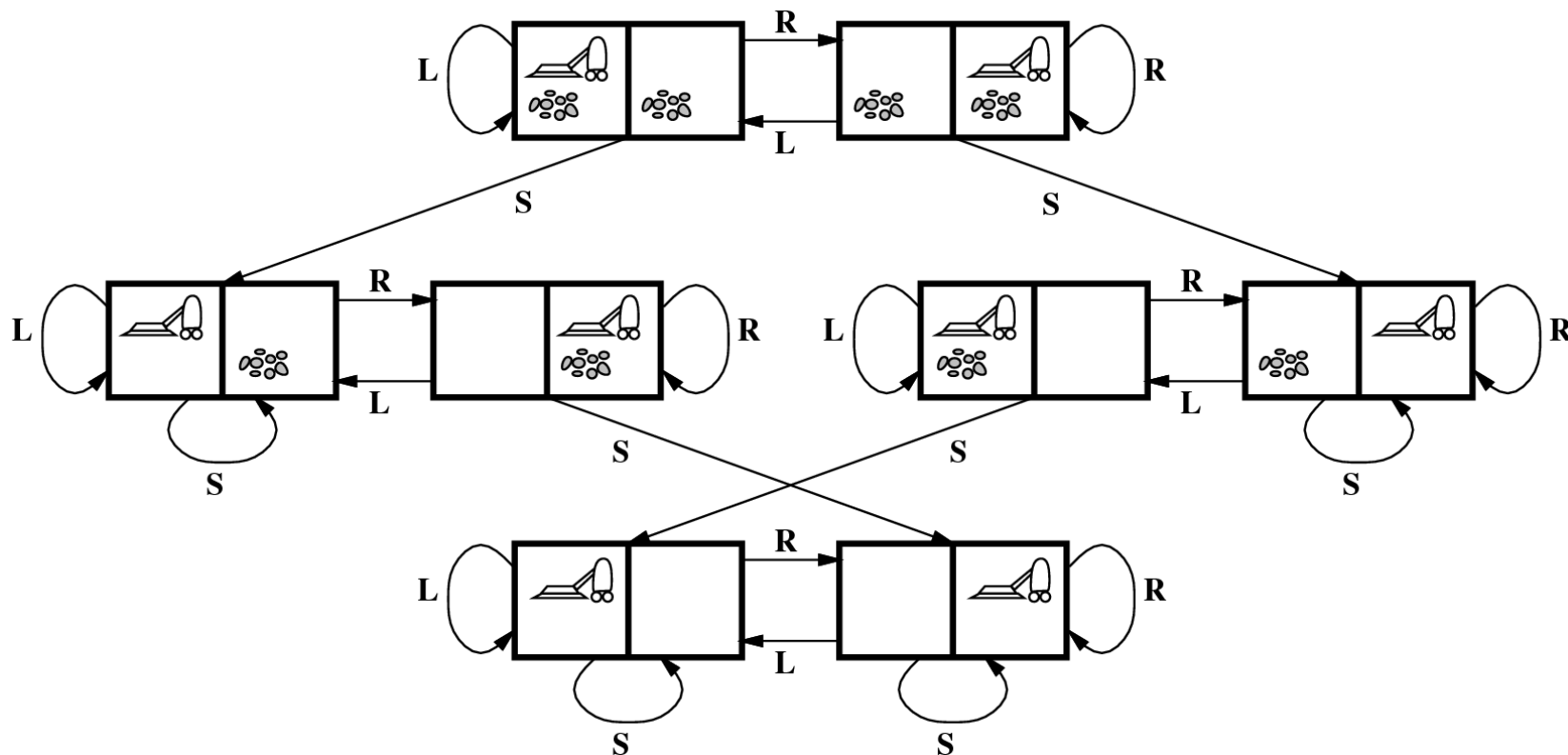- Sobokan
- Traveling Salesman Problem

No

- Tetris
  - dynamic not static
- Solitaire
  - only partially observable

State Space of a Problem

- ## State Space
    - the set of all states reachable from the initial state
    - implicitly defined by the initial state and the successor function

State Space of a Problem

- **State Space**
    - the set of all states reachable from the initial state
    - implicitly defined by the initial state and the successor function

- **Path**
    - a sequence of states connected by a sequence of actions

- **Solution**
    - a path that leads from the initial state to a goal state

- **Optimal Solution**
   - solution with the minimum path cost

Example: Romania

Selecting a State Space

Real world is absurdly complex

→ state space must be abstracted for problem solving

- (Abstract) state
  - corresponds to a set of real states
- (Abstract) action
  - corresponds to a complex combination of real actions
  - e.g., "go from Arad to Zerind" represents a complex set of possible routes, detours, rest stops, etc.
  - for guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"
  - each abstract action should be "easier" than the original problem
- (Abstract) solution
  - corresponds to a set of real paths that are solutions in the real world

Example: Romania – State Space
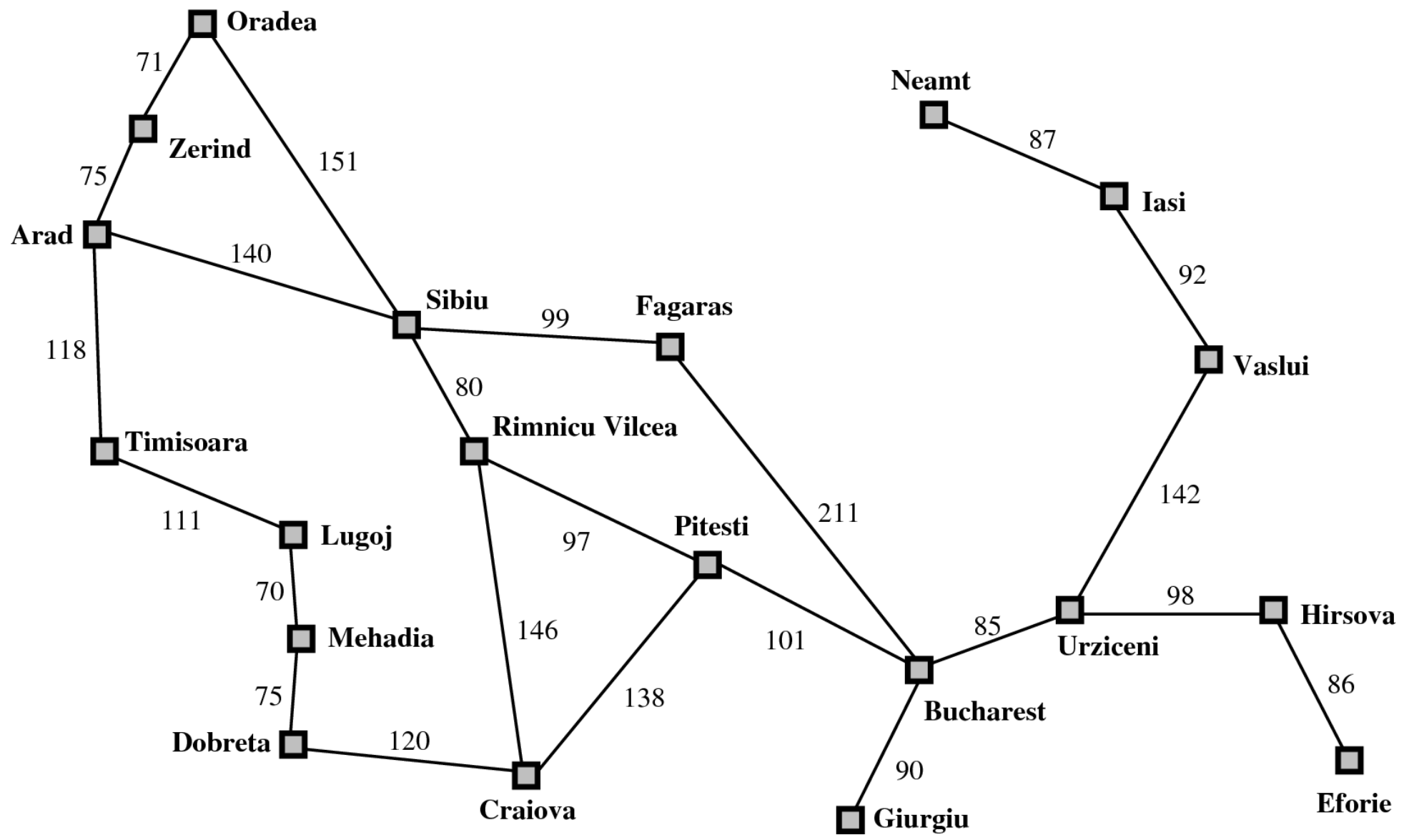
Example: The 8-puzzle

Start State   Goal State
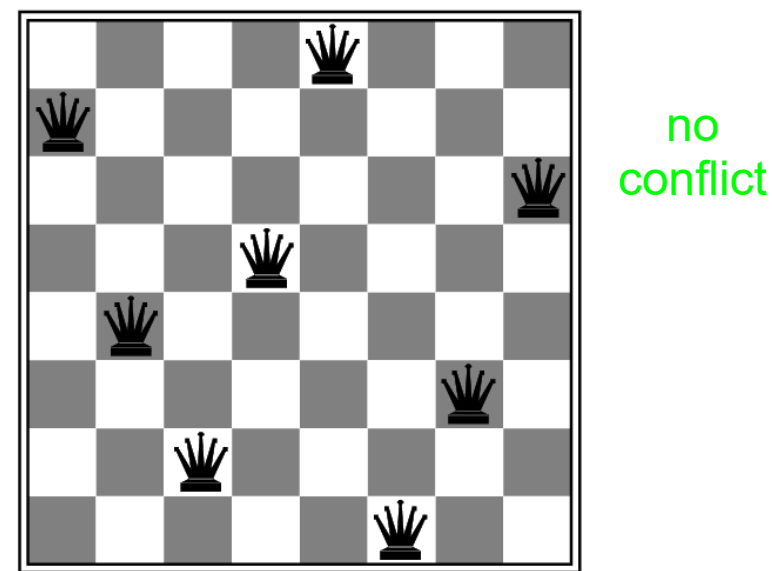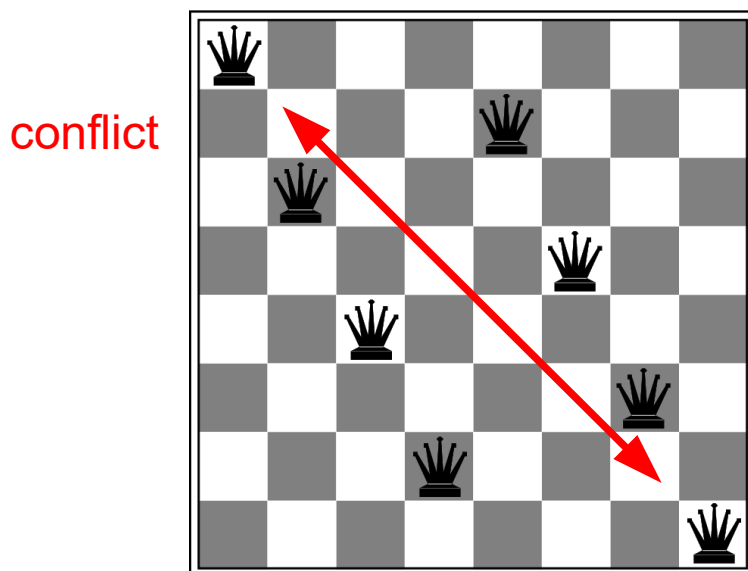
- **states?**
  - location of tiles
    - ignore intermediate positions during sliding
- **goal test?**
  - situation corresponds to goal state
- **path cost?**
  - number of steps in path (each step costs 1)

- **actions?**
  - move blank tile (left, right, up, down)
    - easier than having separate moves for each tile
    - ignore actions like unjamming slides if they get stuck

Example: The 8-Queens Problem
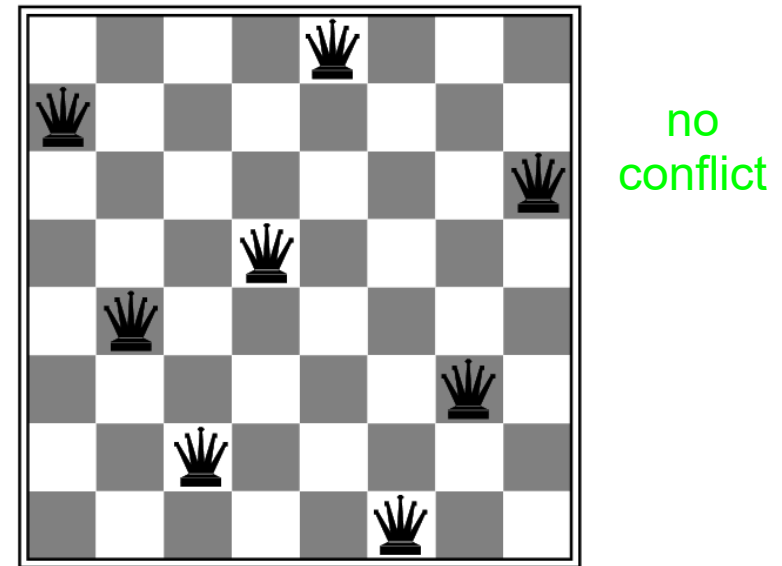
conflict

no conflict

- **states?**
  - any configuration of 8 queens on the board
- **goal test?**
  - no pair of queens can capture each other

- **actions?**
  - move one of the queens to another square
- **path cost?**
  - not of interest here

inefficient complete-state formulation
$\rightarrow 64 \cdot 63 \cdot ... \cdot 57 \approx 3 \cdot 10^{14}$ states

Example: The 8-Queens Problem

conflict

no conflict
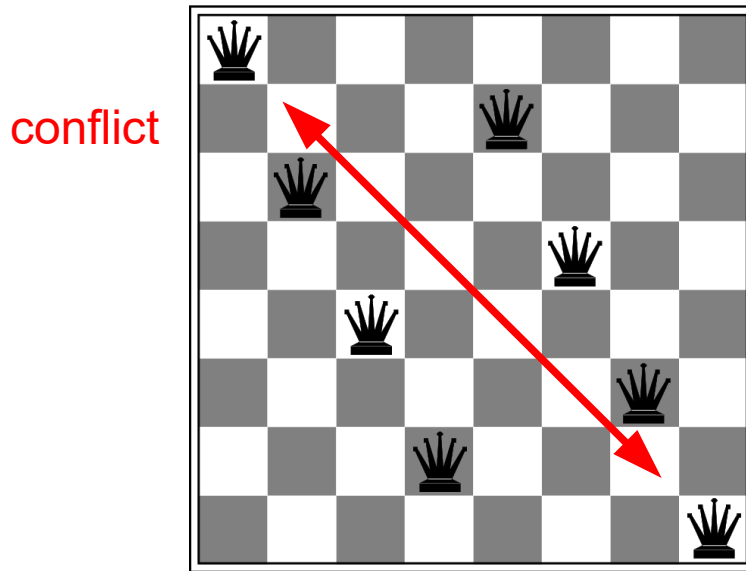
- **states?**
  - n non-attacking queens in the left n columns
- **goal test?**
  - no pair of queens can capture each other

- **actions?**
  - add queen in column $n+1$
  - without attacking the others
- **path cost?**
  - not of interest here

more efficient incremental formulation
→ only 2057 states

Tree Search Algorithms

- Treat the state-space graph as a tree
- Expanding a node
    - offline, simulated exploration of state space by generating successors of already-explored states (successor function)
- Search strategy
    - determines which node is expanded next
- General algorithm:

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

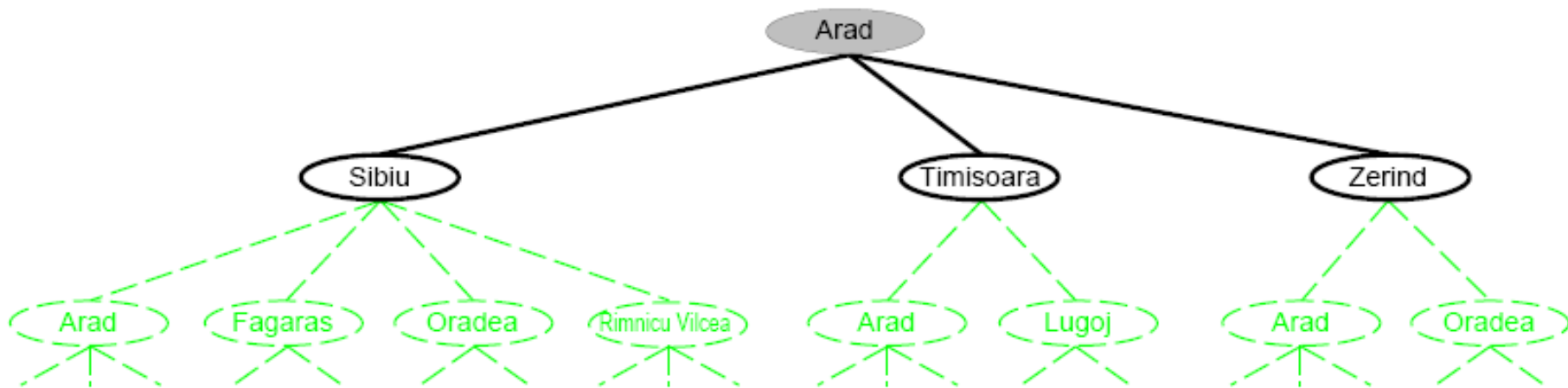# Tree Search Example

- Initial state: start with node Arad
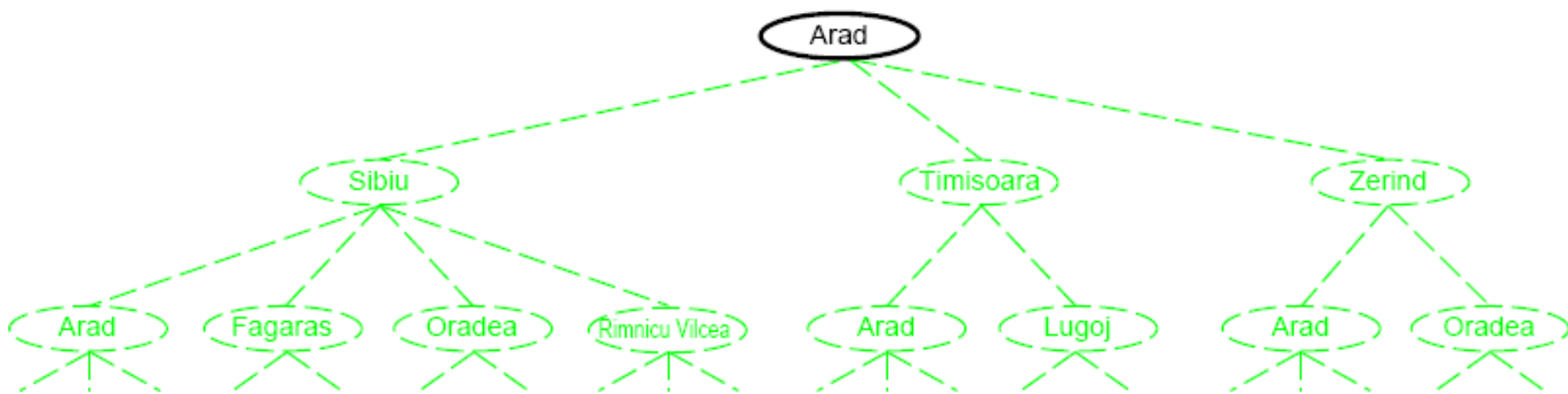
Tree Search Example

- Initial state: start with node Arad
- expand node Arad

Tree Search Example

- Initial state: start with node Arad
- expand node Arad
- expand node Sibiu

Tree Search Example

- Initial state: start with node Arad
- expand node Arad
- expand node Sibiu

depth of the
search tree

fringe of the
search tree
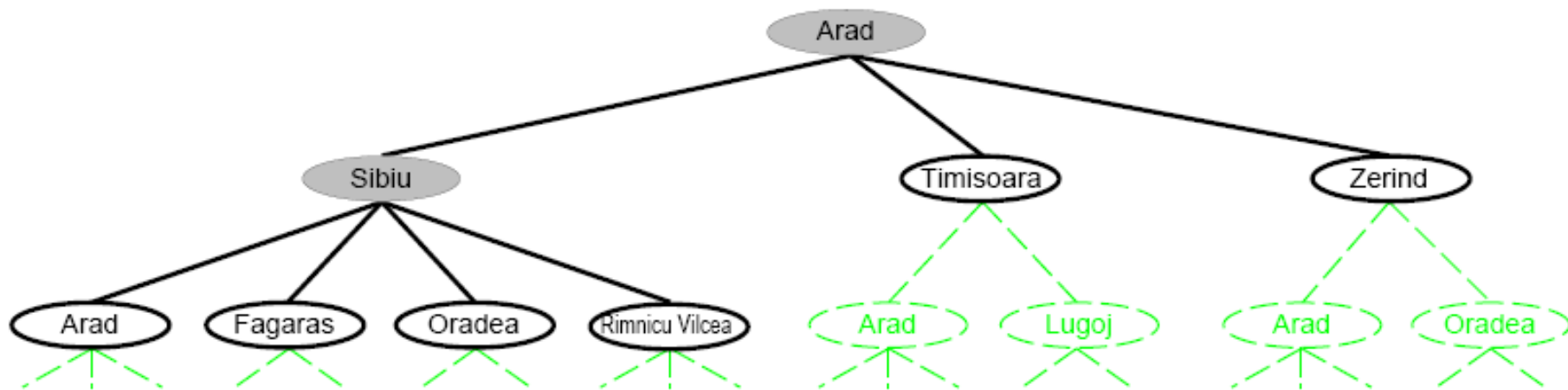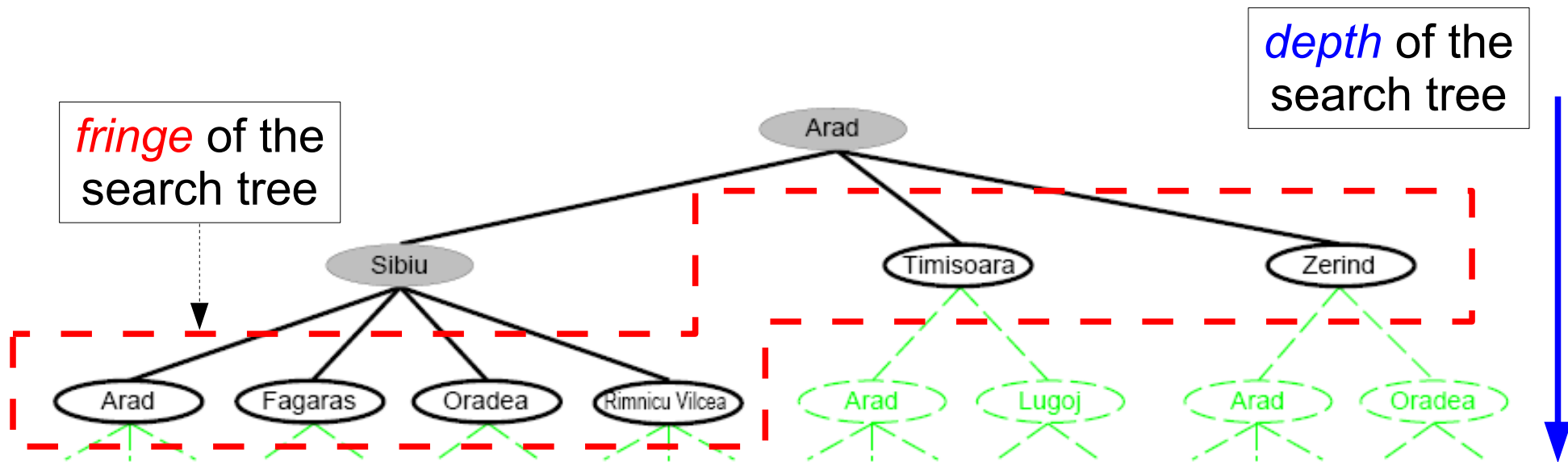
States vs. Nodes

- **State**
  - (representation of) a physical configuration
- **Node**
  - data structure constituting part of a search tree
  - includes
    - state
    - parent node
    - action
    - path cost *g(x)*
    - depth
- Expand
  - creates new nodes
  - fills in the various fields
  - uses the successor function to create the corresponding states
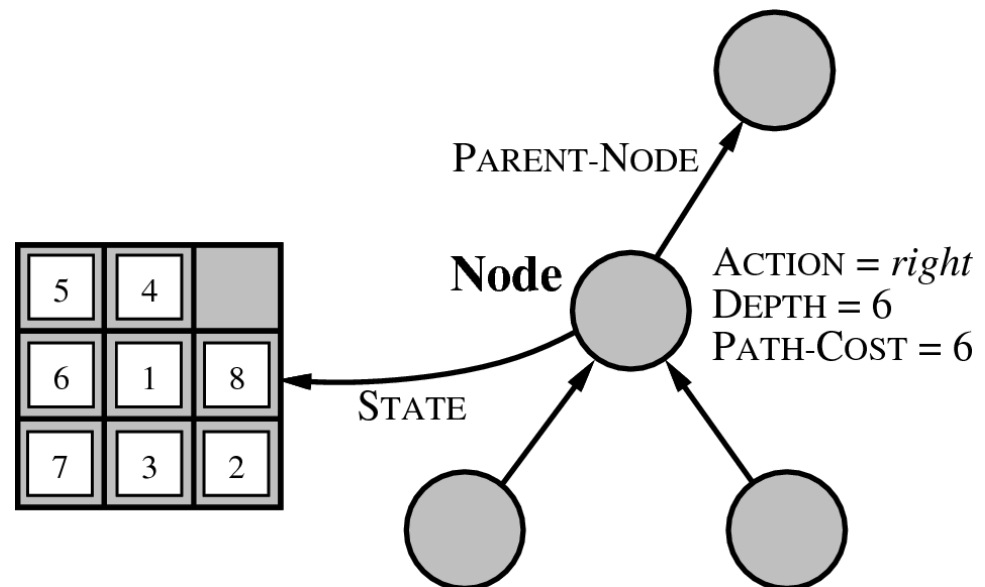
Implementation: General Tree Search

---

function TREE-SEARCH(*problem, fringe*) **returns** a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    loop do
        if *fringe* is empty **then return** failure
        node ← REMOVE-FRONT(*fringe*)
        if GOAL-TEST(*problem*, STATE(*node*)) **then return** *node*
        fringe ← INSERTALL(EXPAND(*node, problem*), *fringe*)

---

function EXPAND(*node, problem*) **returns** a set of nodes
    successors ← the empty set
    for each *action, result* **in** SUCCESSOR-FN(*problem*, STATE[*node*]) **do**
        s ← a new NODE
        PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
        PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node, action, s*)
        DEPTH[*s*] ← DEPTH[*node*] + 1
        add *s* to *successors*
    return *successors*

Search Strategies

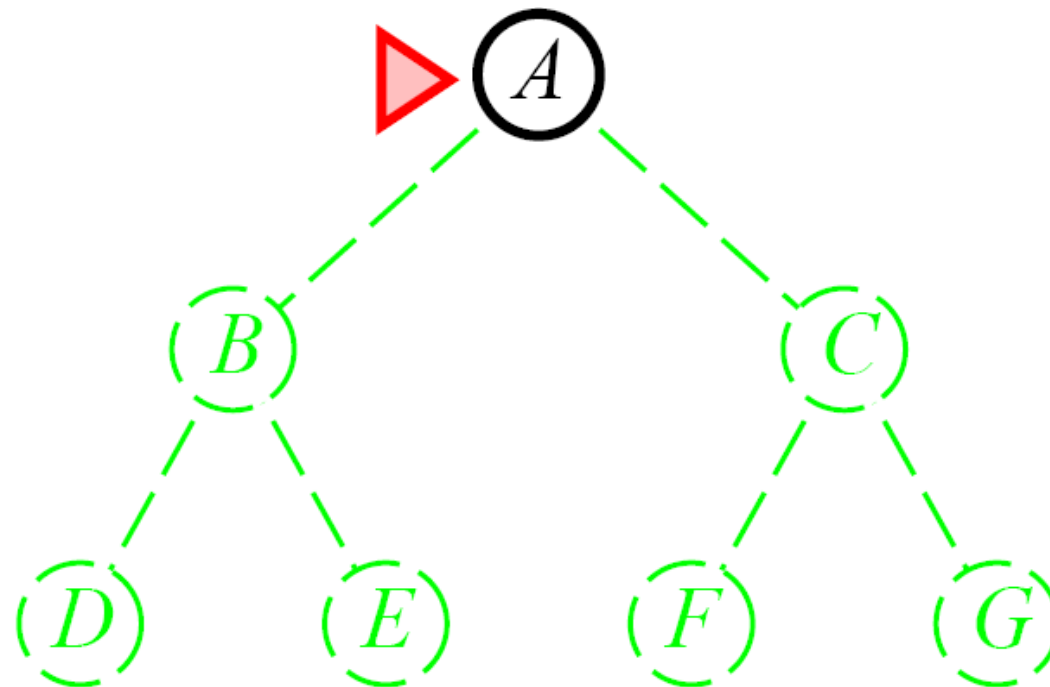- A search strategy is defined by picking the order of node expansion
  - implementation in a queue

- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

Search Strategies

- Uninformed (blind) search strategies use only the information available in the problem definition
    - Breadth-first search
    - Uniform-cost search
    - Depth-first search
    - Depth-limited search
    - Iterative deepening search

- Informed (heuristic) search strategies have knowledge that allows to guide the search to promising regions
    - Greedy Search
    - A* Best-First Search

Breadth-First Strategy

- Expand all neighbors of a node (breadth) before any of its successors is expanded (depth)

- Implemetation:
  - expand the shallowest unexpanded node
  - fringe is a FIFO queue (first-in-first-out, new nodes go to end of queue)

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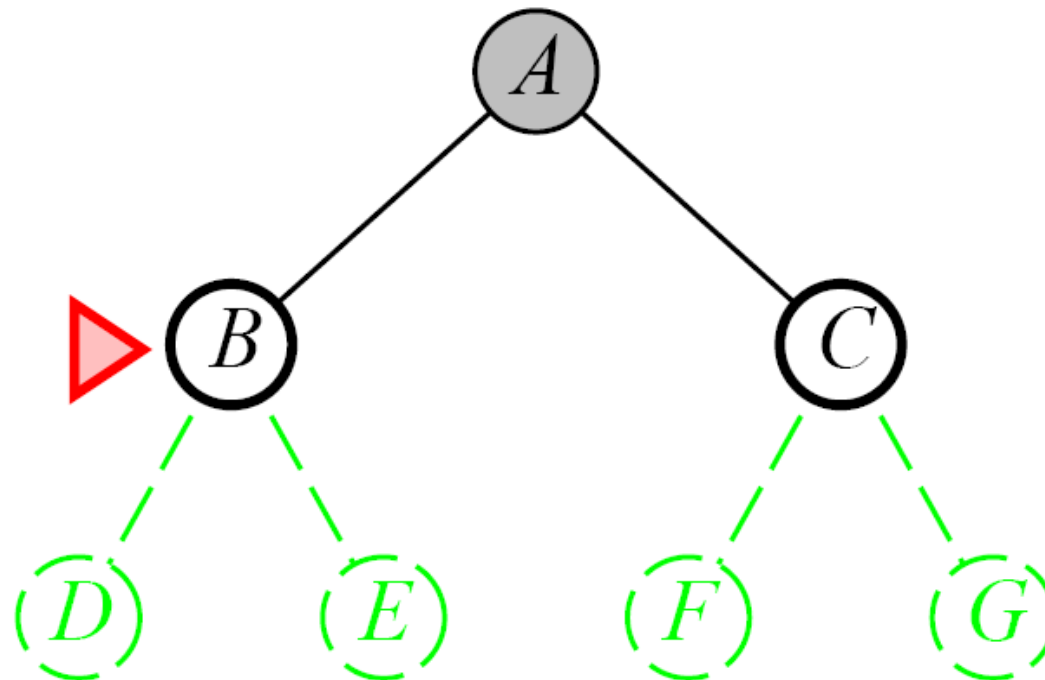 - fringe is a FIFO queue (first-in-first-out, new nodes go to end of queue)

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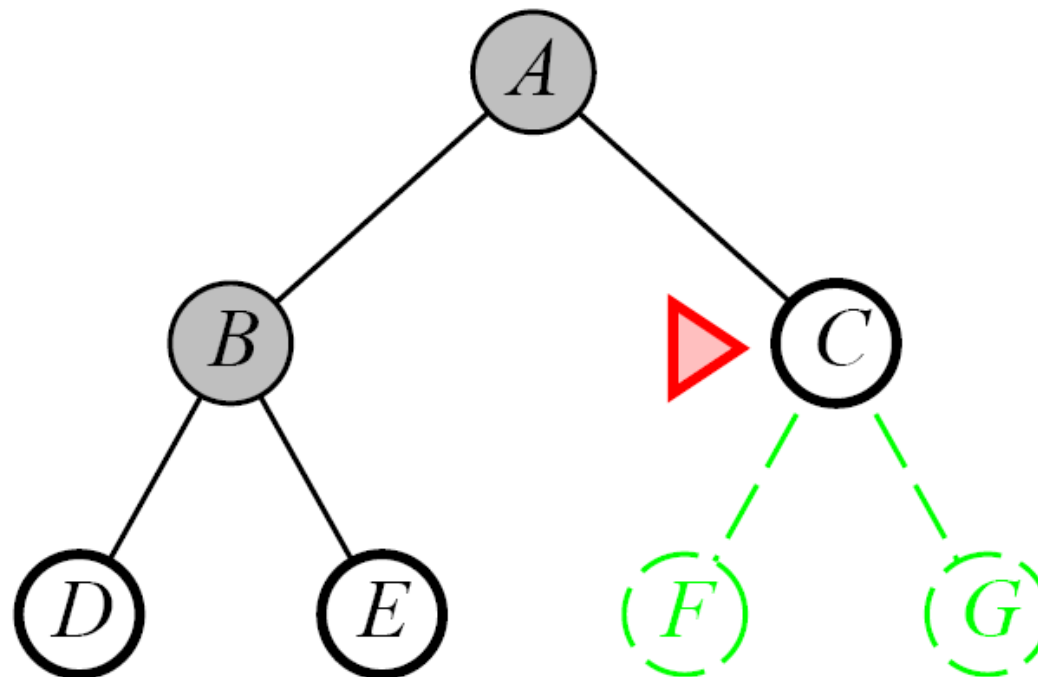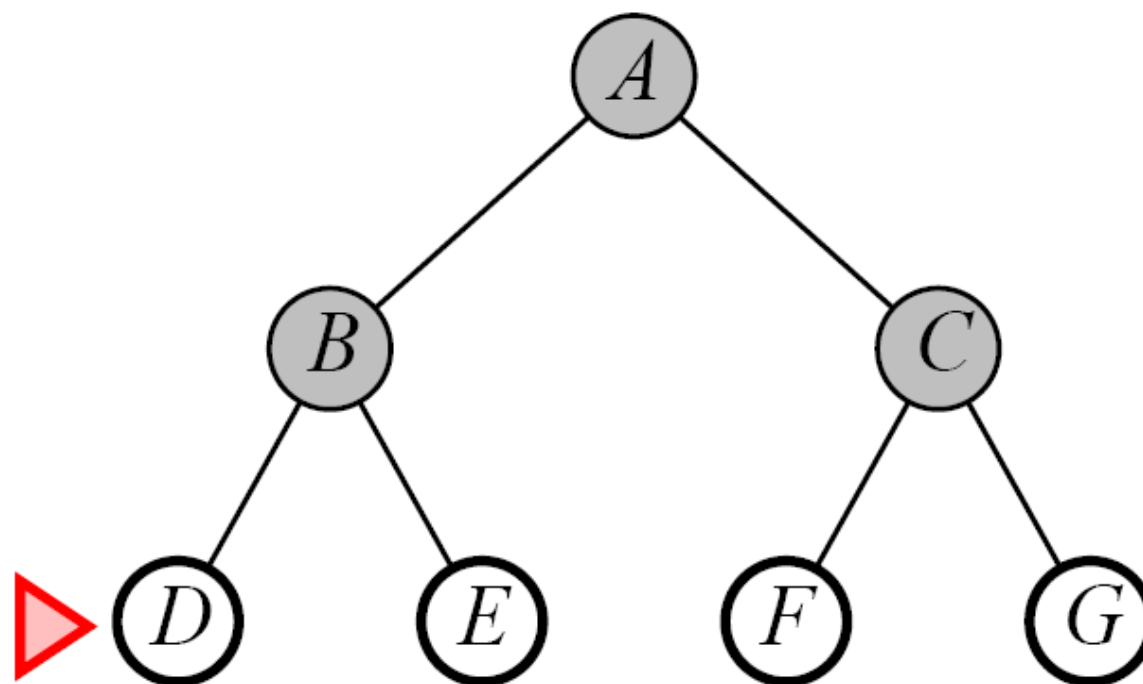 - fringe is a FIFO queue (first-in-first-out, new nodes go to end of queue)

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Properties of Breadth-First Search

- **Completeness**
  - Yes (if b is finite)
- **Time Complexity**
  - each depth has b times as many nodes as the previous
  - each node is expanded
  - except the goal node in level d
    - worst case: goal is last node in this level

  $$\Rightarrow 1 + b + b^2 + b^3 + ... + b^d + (b^{(d+1)} - b) = O(b^{d+1})$$

  > If goal-property is checked right after generation instead before expansion, it reduces to $O(b^d)$

- **Space Complexity**
  - every node must remain in memory
    - it is either a fringe node or an ancestor of a fringe node
    - in the end, the goal will be in the fringe, and its ancestors will be needed for the solution path

  $$\Rightarrow O(b^{d+1})/O(b^d)$$

- **Optimality**
  - Yes, for uniform costs (e.g., if cost = 1 per step)

Combinatorial Explosion

- ## Breadth-first search
  - branching factor $b = 10$, 1,000,000 nodes/sec, 1000 bytes/node

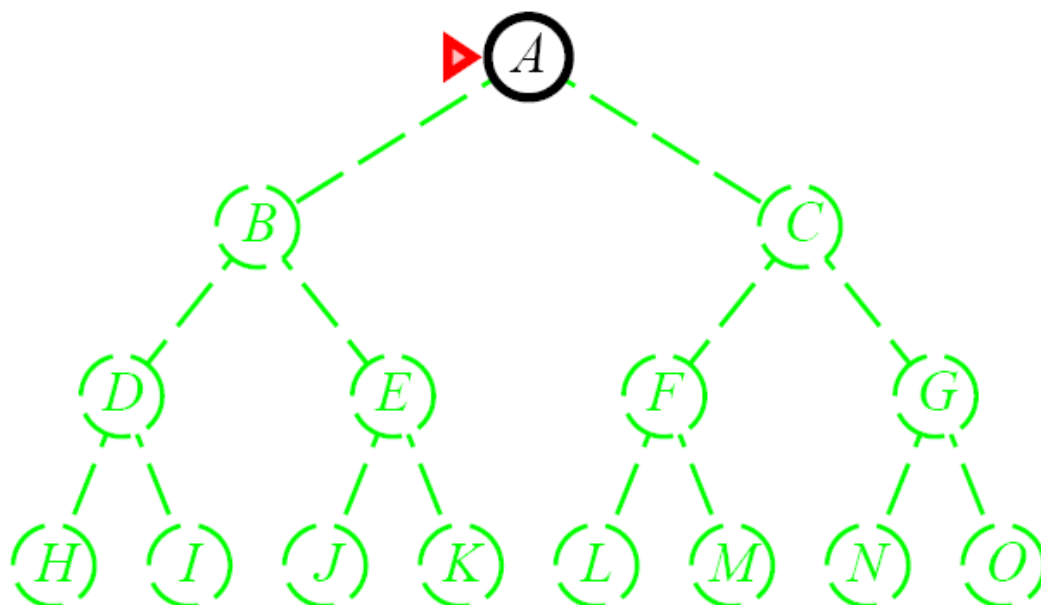| Depth	Nodes	Time	Memory
2	110	.11 msecs	107 kB
4	11 110	11 msecs	10.6 MB
6	10^6	1.1 secs	1 GB
8	10^8	2 minutes	103 GB
10	10^{10}	3 hours	10 TB
12	10^{12}	13 days	1 PetaBytes
14	10^{14}	3.5 years	99 PetaBytes
16	10^{16}	350 years	10 ExaBytes

- ## Space is the bigger problem
  - Storing 10 TB every 3 hours is a challenge...
- ## But Time is also quite challenging.

Uniform-Cost Search

- Breadth-first search can be generalized to cost functions
  - each node now has associated costs
  - costs accumulate over path
  - instead of expanding the shallowest path,
    expand the least-cost unexpanded node
  - breadth-first is special case where all costs are equal
- Implementation
  - fringe = queue ordered by path cost
- Completeness
  - yes, if each step has a positive cost (cost ≥ ε)
  - otherwise infinite loops are possible
- Space and Time complexity $b^{1+O\left(\lfloor C^*/\epsilon \rfloor\right)}$
  - number of nodes with costs < costs of optimal solution C^*
- Optimality
  - Yes – nodes expanded in increasing order of path costs

Depth-First Strategy

- Expand all successors of a node (depth) before any of its neighbors is expanded (breadth)

- Implemetation:

  - expand the deepest unexpanded node
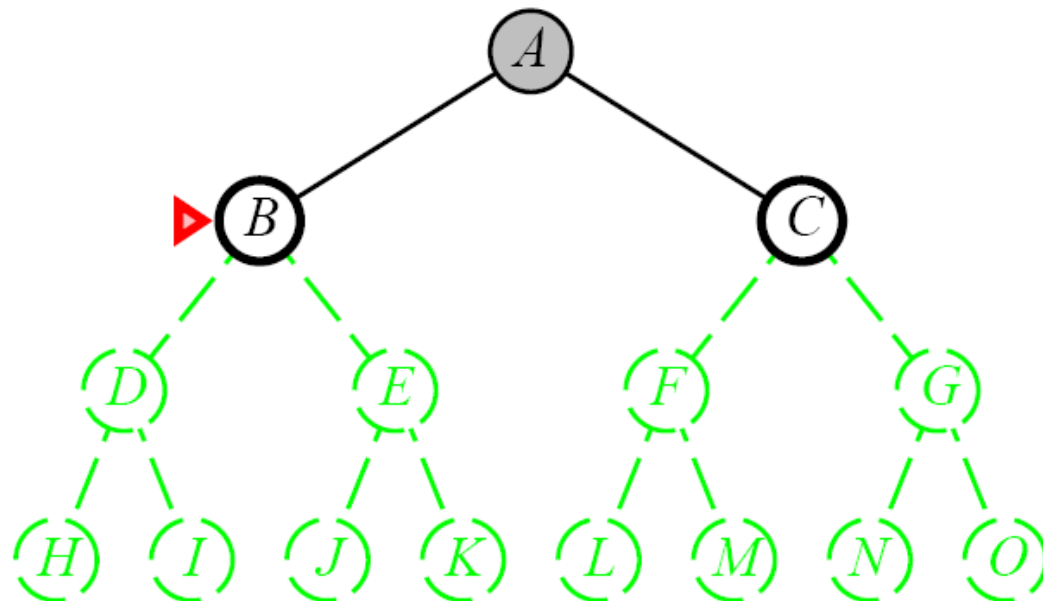 - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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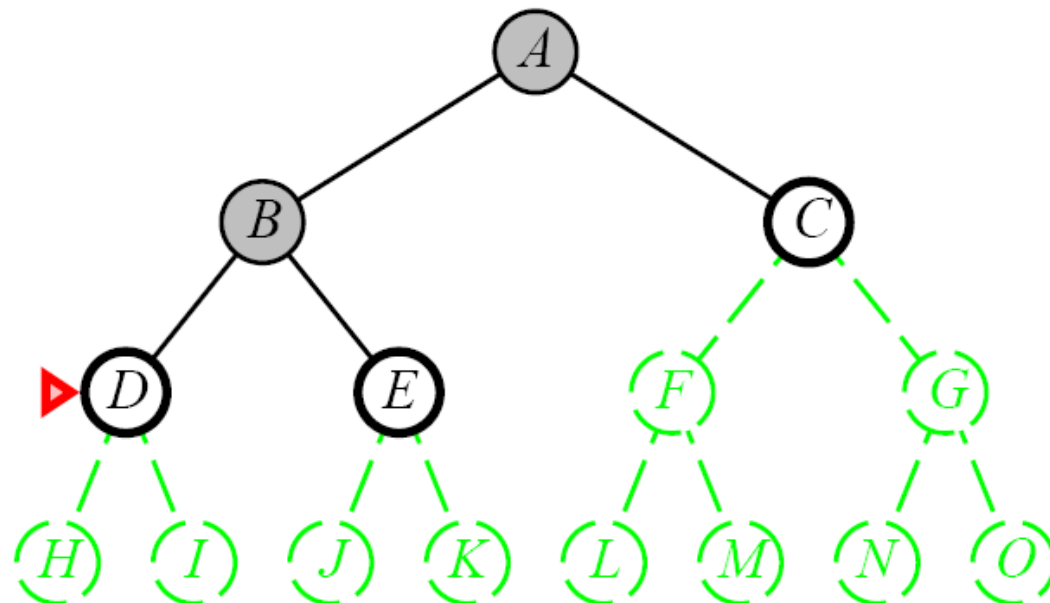 - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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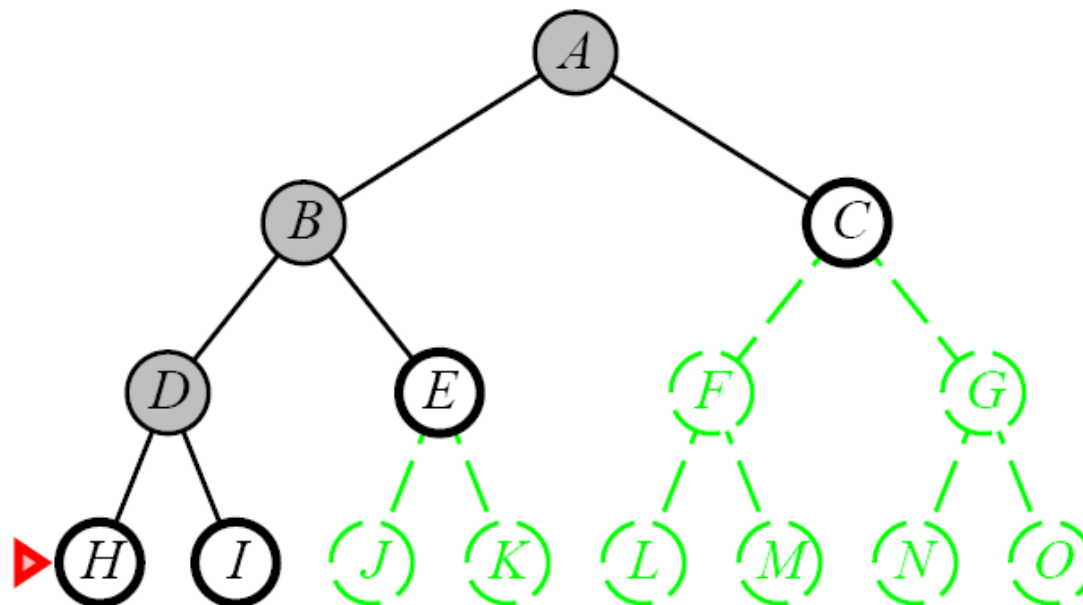   - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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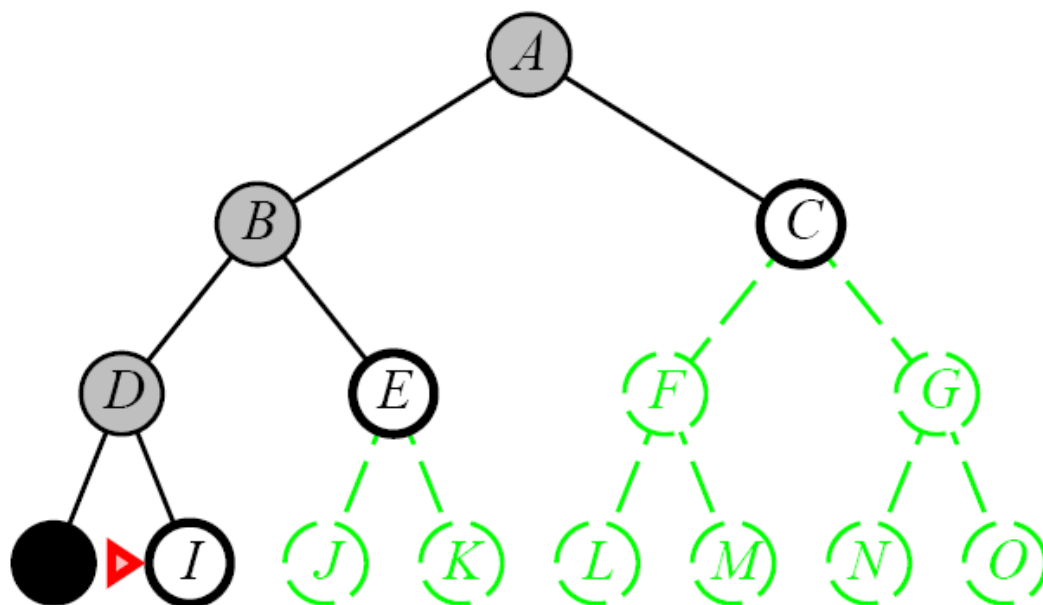   - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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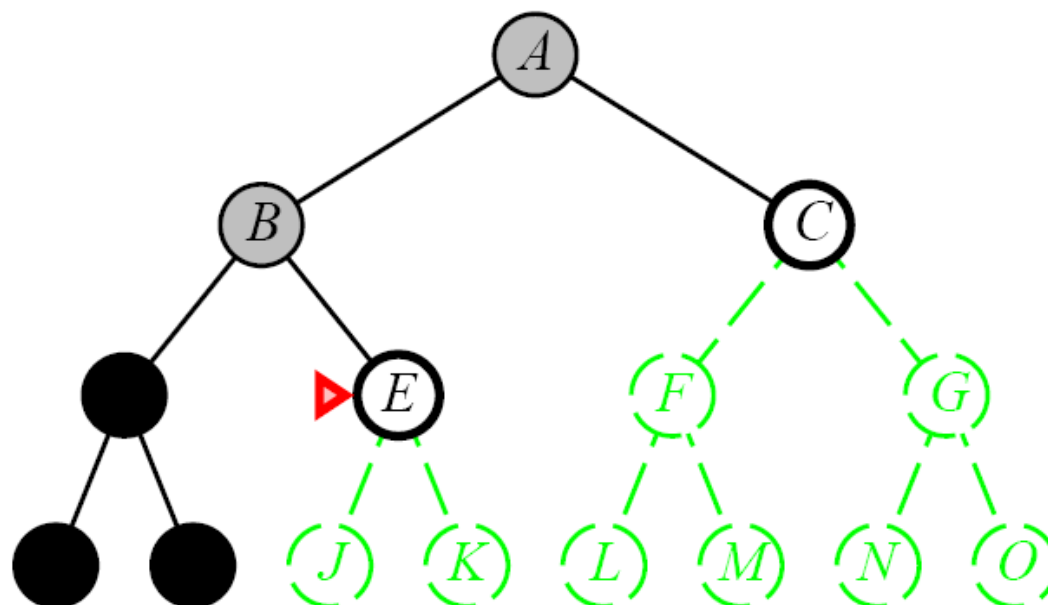 - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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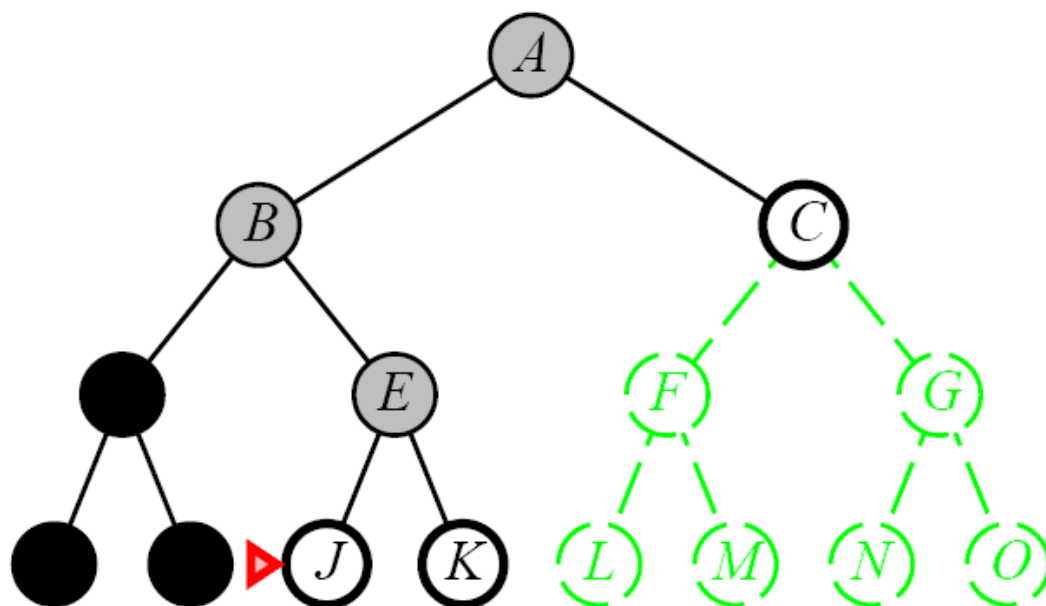   - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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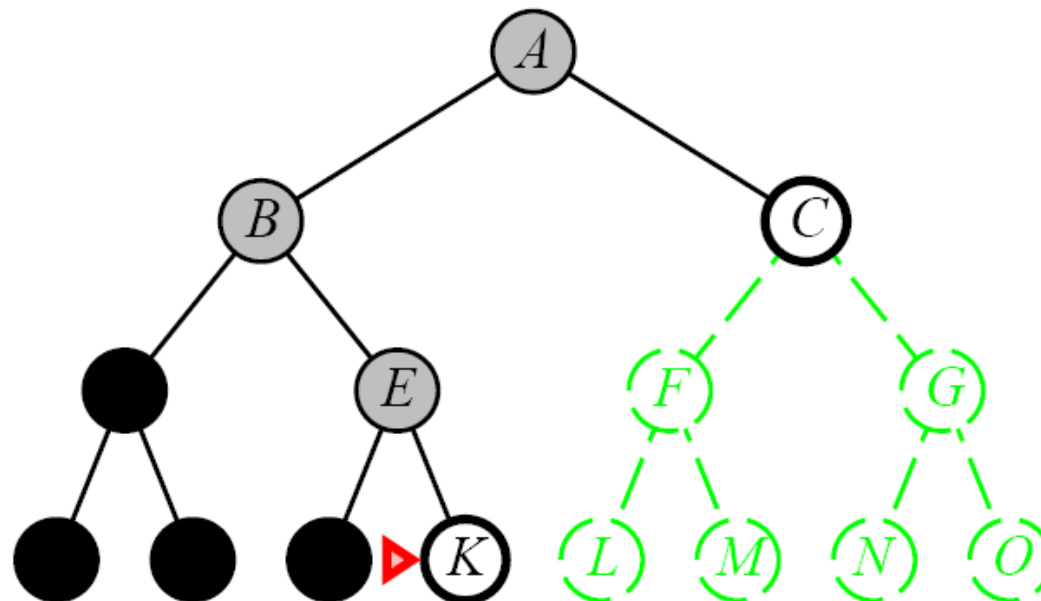 - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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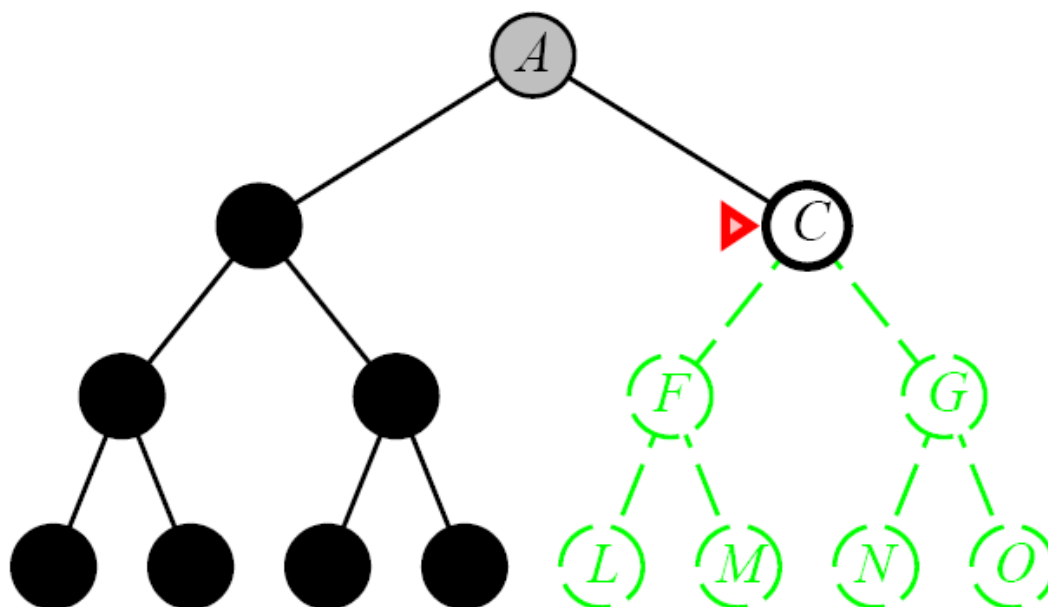 - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

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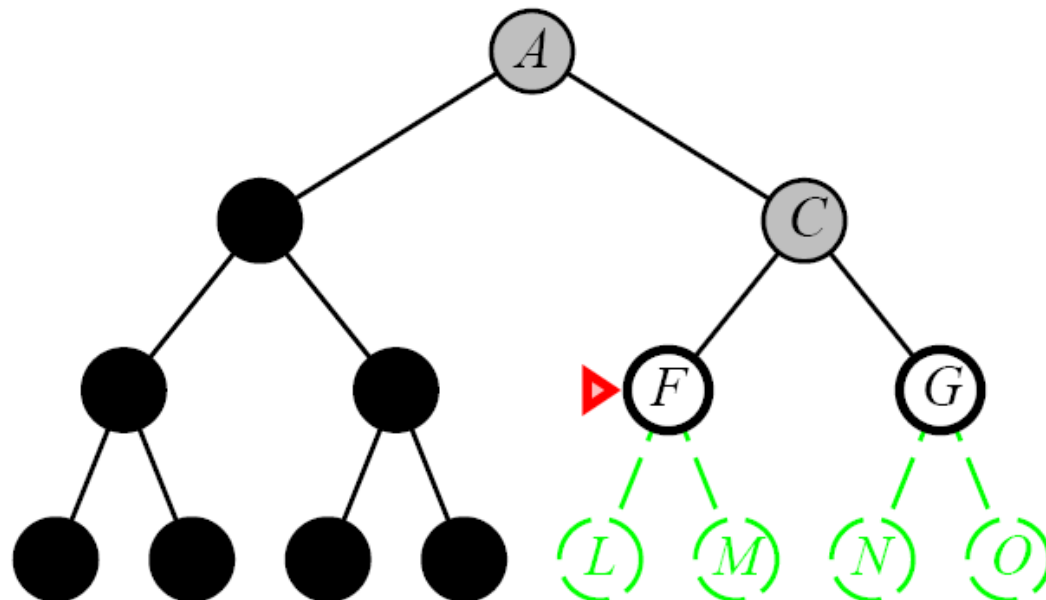 - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

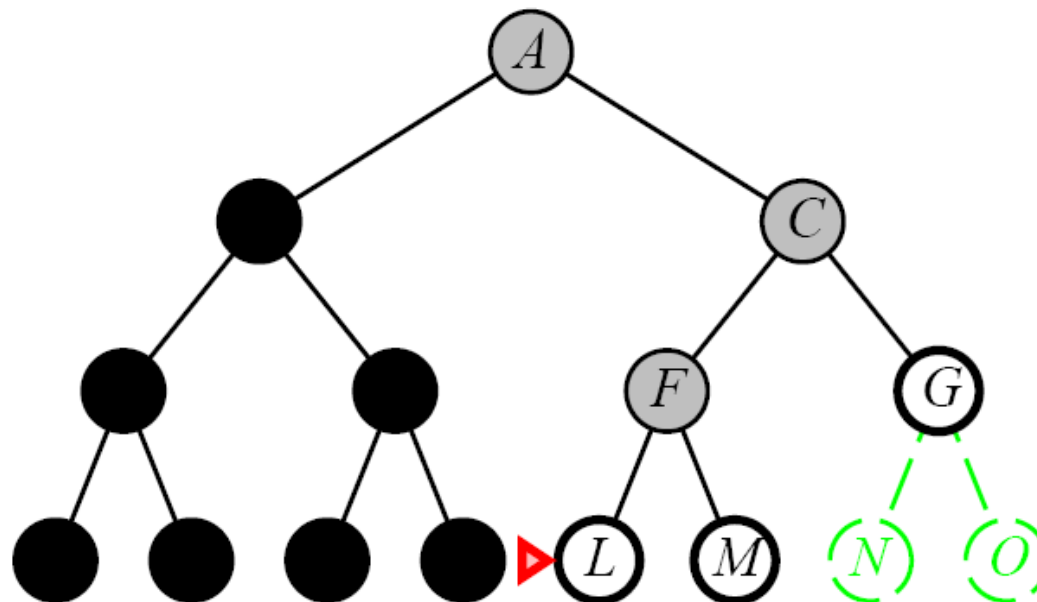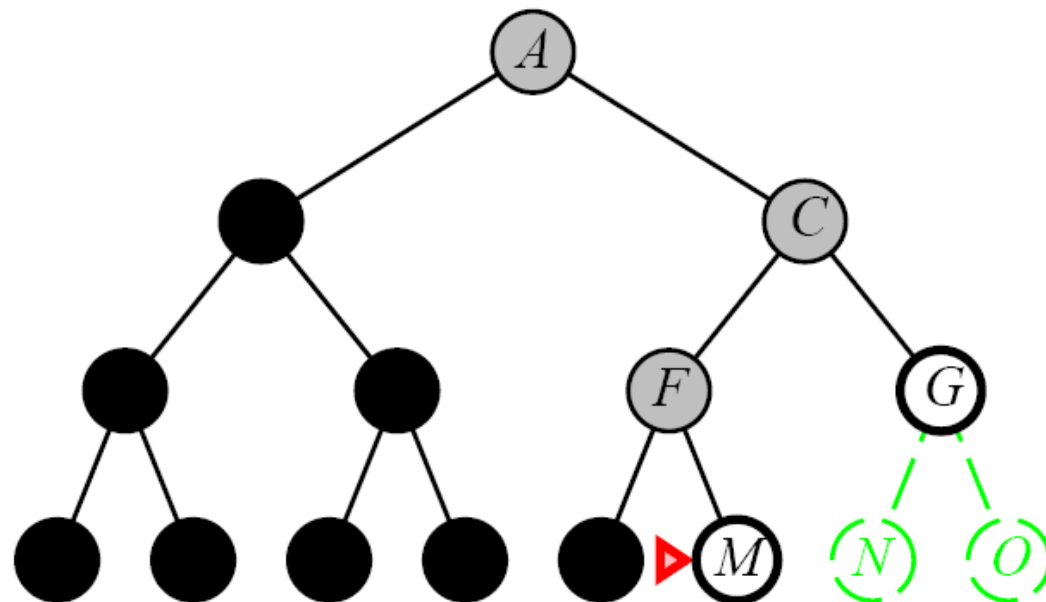# Depth-First Strategy

- Expand all successors of a node (depth) before any of its neighbors is expanded (breadth)

- Implemetation:

  - expand the deepest unexpanded node
  - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)

Properties of Depth-First Search

- **Completeness**
  - No, fails in infinite-depth search spaces and spaces with loops
  - complete in finite spaces if modified so that repeated states are avoided
- **Time Complexity**
  - has to explore each branch until maximum depth $m \Rightarrow O(b^m)$
  - terrible if $m > d$ (depth of goal node)
  - but may be faster than breadth-first if solutions are dense
- **Space Complexity**
  - only nodes in current path and their unexpanded siblings need to be stored
  - $\Rightarrow$ only linear complexity $O(m \cdot b)$
- **Optimality**
  - No, longer (more expensive) solutions may be found before shorter (cheaper) ones

Backtracking Search

Even more space-efficient variant

- does not store all expanded nodes, but only the current path

$$\Rightarrow O(m)$$

  - if no further expansion is possible, go back to the predecessor
  - each node is able to generate the *next* successor

- only needs to store and modify one state

  - actions can do and undo changes on this one state

Depth-limited Search

- depth-first search is provided with a depth limit l
  - nodes with depths $d > l$ are not considered → incomplete
  - if $d < l$ it is not optimal (like depth-first search)
  - time complexity $O(b^l)$, space complexity $O(bl)$

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST(problem, STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative Deepening Search

- ## Main problem with depth-limited search is setting of l

- ## Simple solution:
  - ### try all possible $l = 0, 1, 2, 3, ...$

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

  - ### costs are dominated by the last iteration, thus the overhead is marginal
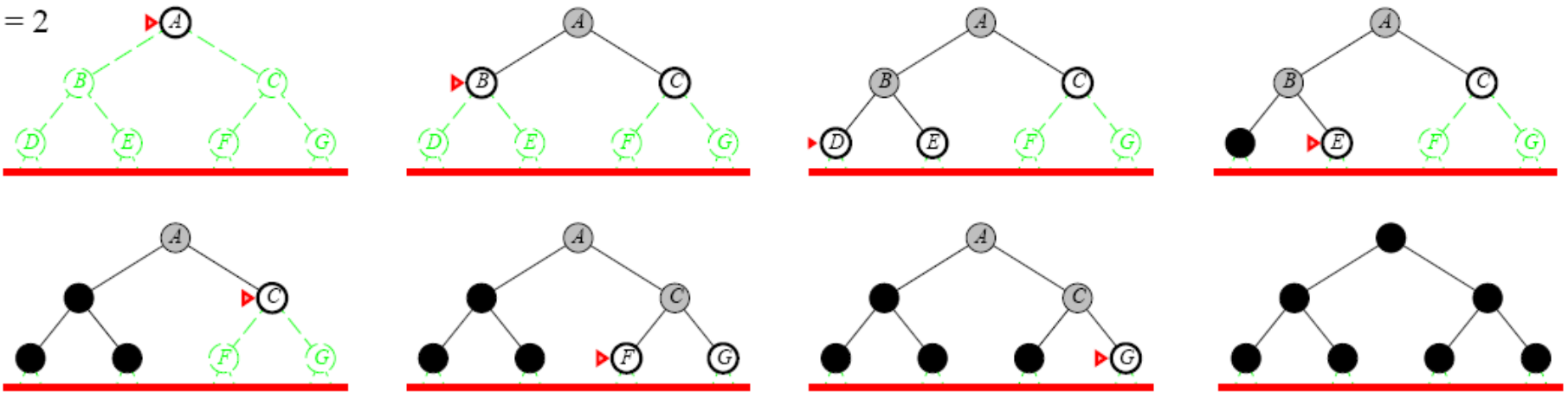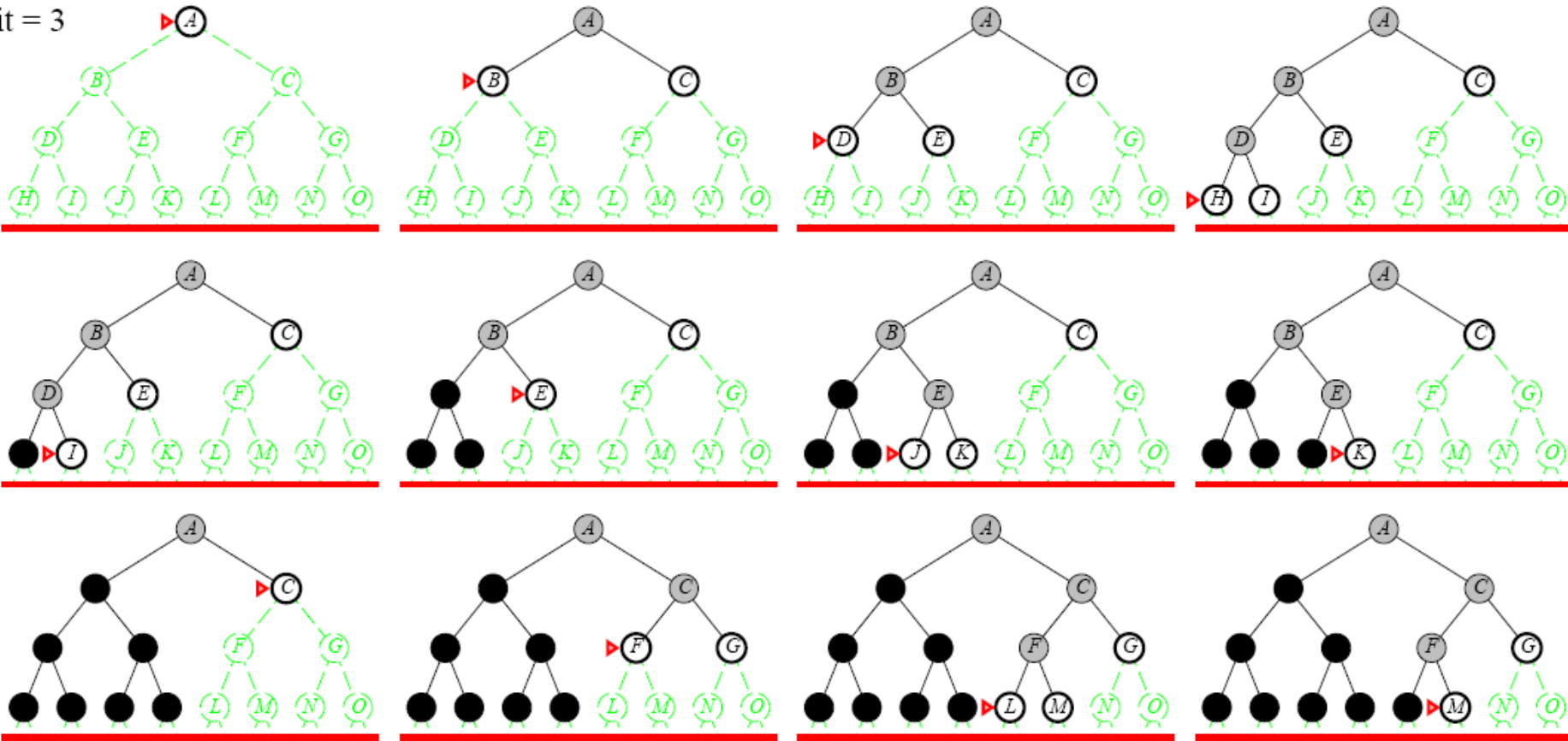
Iterative Deepening Search

Iterative Deepening Search

Properties of
Iterative Deepening Search

- **Completeness**
  - Yes (no infinite paths)
- **Time Complexity**
  - first level has to be searched d times
  - last level has to be searched once

$$\Rightarrow d \cdot b + (d-1)b^2 + ... + 1 \cdot b^d = \sum_{i=1}^{d}(d-i+1) \cdot b^i$$

- **Space Complexity**

  $\Rightarrow$ only linear complexity $O(bd)$
- **Optimality**
  - Yes, the solution is found at the minimum depth

$\Rightarrow$ combines advantages of depth-first and breadth-first search

Comparison of Time Complexities

Worst-case (goal is in right-most node at level d)

- Depth-Limited Search

$$N_{DLS} = b + b^2 + ... + b^d = \sum_{i=1}^{d} b^i$$

- Iterative Deepening

$$N_{IDS} = d \cdot b + (d-1) b^2 + ... + 1 \cdot b^d = \sum_{i=1}^{d} (d-i+1) \cdot b^i$$
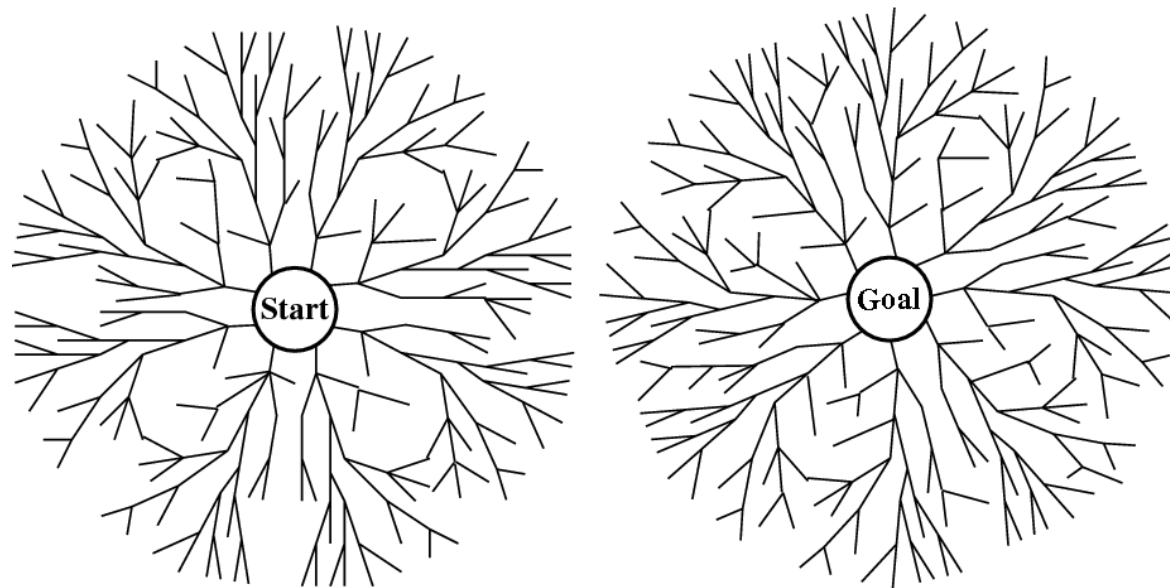
Example: $b = 10$, $d = 5$

$$N_{DLS} = 10 + 100 + 1000 + 10,000 + 100,000 = 111,110$$

$$N_{IDS} = 50 + 400 + 3000 + 20,000 + 100,000 = 123,450$$

Overhead of IDS only ca. 10%

Bidirectional Search

- Perform two searches simultaneously
    - forward starting with initial state
    - backward starting with goal state

  check whether generated node is in fringe of the other search

- Properties
    - reduction in complexity $(b^{d/2} + b^{d/2} \ll b^d)$
    - only possible if actions can be reversed
    - search paths may not meet for depth-first bidirectional search

Summary of Algorithms

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

- Variety of uninformed search strategies

- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Assumptions about the Environment

- **static**
  - we do not pay attention to possible changes in the environment
- **observable**
  - we can at least observe our initial state
- **discrete**
  - possible actions can be enumerated
- **deterministic**
  - the expected outcome of an action is always identical to the true outcome
  - once we have a plan, we can execute it „with eyes closed"

$\rightarrow$ easiest possible scenario

Problems with Partial Information

- **Single-State Problem**

  | *deterministic, fully observable* |

  - agent knows exactly which state it will be in
  - solution is a sequence

- **Conformant Problem** (sensorless problem)

  | *non-observable* |

  - agent may have no idea where it is
  - solution (if any) is a sequence

- **Contingency Problem**

  | *nondeterministic and/or partially observable* |

  - percepts provide new information about current state
  - solution is a contingent plan (tree) or a policy
  - search and execution often interleaved
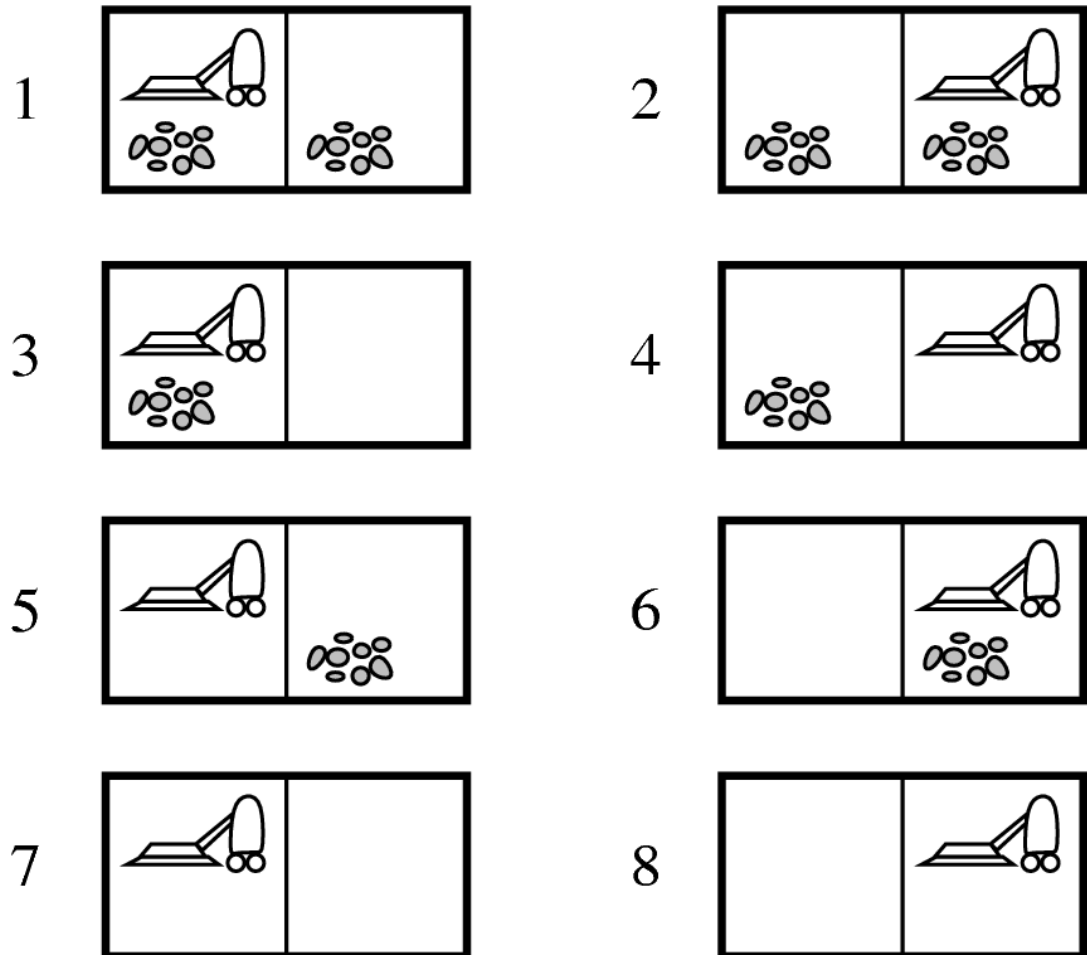
- **Exploration Problem**

  | *state-space is not known* | $\rightarrow$ Reinforcement Learning

Example: Vacuum World

- **Single-state Problem**
  - start in #5
  - goal
    - no dirt
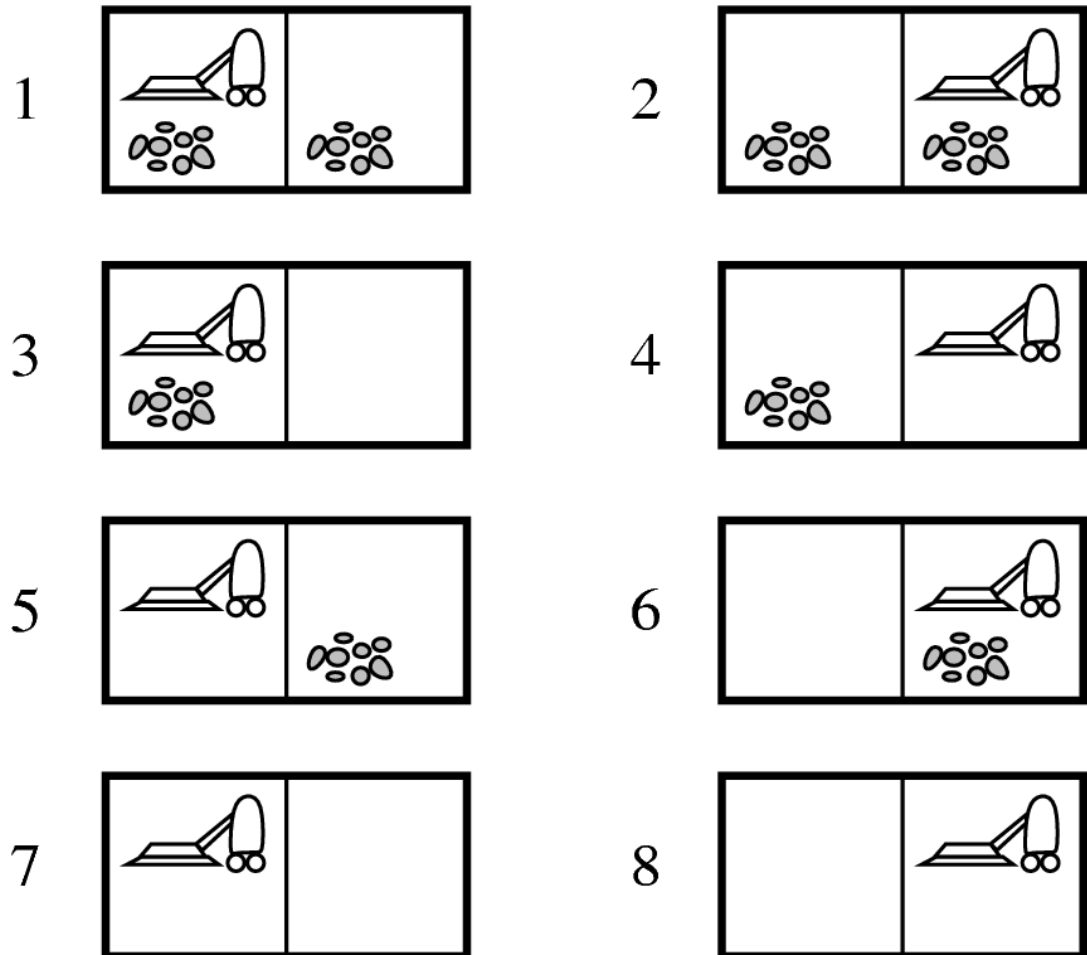- **Solution**
  - [*Right, Suck*]

Example: Vacuum World

- **Conformant Problem**
  - start in any state
    (we can't sense)
    - $start \leftarrow \{1,2,3,4,5,6,7,8\}$
  - actions
    - e.g., *Right*
      goes to $\{2,4,6,8\}$
  - goal
    - no dirt
- **Solution**
  - [*Right, Suck, Left, Suck*]

Example: Vacuum World

- **Contingency Problem**
  - start in #5
  - indeterministic actions
    - *Suck* can dirty a clean carpet
  - sensing
    - dirt at current location?
  - goal
    - no dirt
- **Solution**
  - [*Right,* **if** *dirt* **then** *Suck*]