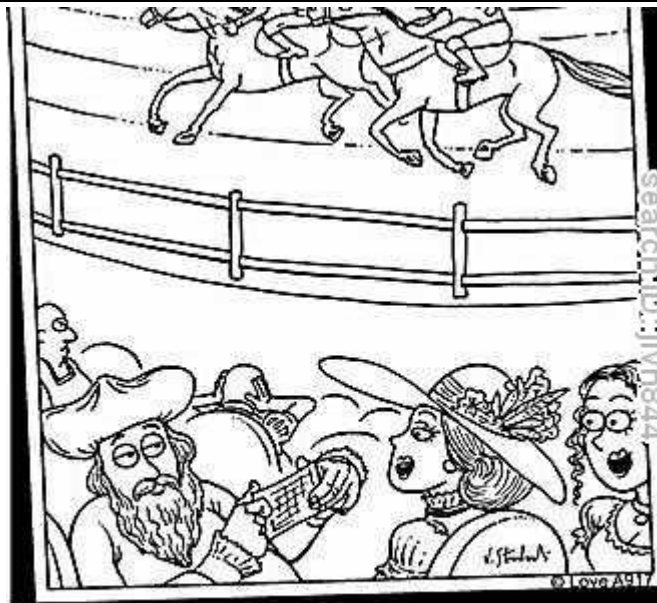


# Markets & Information



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## Networks, Crowds, and Markets Chapter 22



Before becoming the legend that he is today, Nostradamus first enjoyed a pretty good living at the tracks.



# Outline

- Introduction
- **Markets with Exogenous Events**
  - Horse Races, Bettings, and Beliefs
  - Wisdom of the Crowds
  - Prediction Markets & Stock Markets
- **Markets with Endogenous Events**
  - Market of Lemons
  - Asymmetric information
  - Signaling Quality
  - Quality Uncertainty On-Line

# Introduction

aka „Definitions“

## Exogenous Markets

Decisions do not affect the outcome of the contract.

- Prediction markets
- Horse Races

## Endogenous markets

Underlying events are affected by the decisions made (s. Braess's Paradox, Information cascades)

- Used car markets

# Horse Races, Bettings and Beliefs

## Definitions



- $w$  := Wealth distributed among bets
- $r$  := Fraction of  $w$  bet to A
- $(1-r)$  := Fraction of  $w$  bet to B
- Horse „A“
  - $a$  := Win probability (subjective)
  - $o_A$  := Odds for A
  - $rwo_A$  := Monetary payoff for A win
- Horse „B“
  - $b$  := Win probability (subjective)
  - $o_B$  := Odds for B
  - $(1-r)wo_B$  := Monetary payoff for B win

# Horse Races, Bettings and Beliefs

## Utility functions

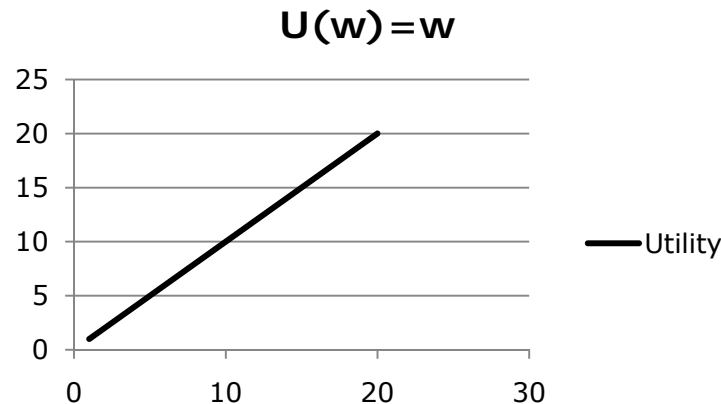
Utility Function  $U(w) \Leftrightarrow$  Attitude towards Risk

Simplest idea: Linear functions – i.e.  $U(w) = w$

More general  $U(w) = aw + \beta$

Idea: more wealth := more Utility, regardless of the actual absolute wealth

Realistic?



# Horse Races, Bettings and Beliefs

## Fair gamble-Test



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### *Fair gamble*

-> Expected value of wealth AFTER the gamble = current wealth

$$\frac{1}{2} \cdot (2w) + \frac{1}{2} \cdot 0 = w$$

Utility of accepting the gamble:  $\frac{1}{2}U(2w) + \frac{1}{2}U(0) = \frac{1}{2} \cdot (2w) + \frac{1}{2} \cdot 0 = w$

Utility of passing:  $U(w) = w$

⇒ Investor is indifferent between accepting and passing the gamble

⇒ ... indifferent whether to possess \$2,000,000 or end up at \$0

Again: Realistic? 😊



# Horse Races, Bettings and Beliefs

## Fair gamble-Test



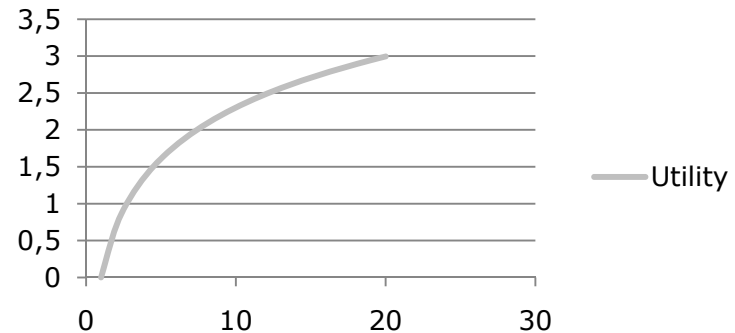
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Better Idea: Utility function with decreasing growth

$$U(w) = w^{1/2}$$

$$U(w) = \ln(w)$$

$$U(w) = \ln(w)$$



Again: *Fair gamble*-test

$$\text{Utility of accepting: } \frac{1}{2}U(2w) + \frac{1}{2}U(w \rightarrow 0) = \frac{1}{2}\ln(2w) + \frac{1}{2}\lim_{x \rightarrow 0} \ln(x) = -\infty$$

$$\text{Utility of passing: } U(w) = \ln(w)$$



# Horse Races, Bettings and Beliefs

## Optimal Strategy

Strategy := Distribution of wealth between two bets (A and B)

Remember: An *optimal strategy* is such, that maximizes the player's utility.

$$a \ln(rw_{O_A}) + (1-a) \ln((1-r)w_{O_B})$$
$$= a \ln(r) + (1-a) \ln(1-r) + a \ln(w_{O_A}) + (1-a) \ln(w_{O_B})$$

$$\Rightarrow a \ln(r) + (1-a) \ln(1-r) = \text{MAX}$$

$$\Rightarrow \frac{a}{r} - \frac{1-a}{1-r} = 0$$

$$\Rightarrow a = r$$

**Optimal strategy is to bet your beliefs**



# Wisdom of Crowds

## Aggregate Beliefs

- Where are the odds gone?

⇒ Determinated by the aggregate Beliefs

⇒ The race-track wants to break even in either case

Total amount bet on A:  $\sum_{n=1..N} a_n w_n$

Total amount bet on B:  $\sum_{n=1..N} b_n w_n$

Total payoff if A wins:  $o_A \sum_{n=1..N} a_n w_n$

Total payoff if B wins:  $o_B \sum_{n=1..N} b_n w_n$

# Wisdom of Crowds

## Aggregate Beliefs

$$w = \sum_{n=1..N} w_n$$

In order to break even : payoffs = bets

$$o_A \sum_{n=1..N} a_n w_n = w$$

$$\sum_{n=1..N} \frac{a_n w_n}{w} = o_A^{-1}$$

$$f_n = \frac{w_n}{w}$$

$$\sum_{n=1..N} a_n f_n = o_A^{-1}$$

$$o_B \sum_{n=1..N} b_n w_n = w$$

$$\sum_{n=1..N} \frac{b_n w_n}{w} = o_B^{-1}$$

$$f_n = \frac{w_n}{w}$$

$$\sum_{n=1..N} b_n f_n = o_B^{-1}$$

$p_A = o_A^{-1} \equiv$  "price of a dollar" for the event that A wins (state price for "A wins")

$p_B = o_B^{-1} \equiv$  "price of a dollar" for the event that B wins (state price for "B wins")

# Wisdom of Crowds

## Interpretation

- 1) If  $a_n = a$  for all  $n \Rightarrow p_A = a$   
 $\Rightarrow$  If everyone agrees in the probability  $a$ , it reflects exactly the market price
- 2) State prices are weighted averages of the bettors' beliefs
- 3) More wealth = more influence to the market
- 4) For a crowd, which is big enough:

**The aggregated weighted beliefs of the crowd converges to the true probability of A to win**

=

**„Wisdom of Crowds“**

- Limitations:
- 1) opinions are independent (symmetric information distribution)
  - 2) wealth shares are equal

# Prediction & Stock Markets

## ... of horses and markets



### Prediction markets

Example: Trading a \$1 contract on win of elections of the U.S. presidential elections

Difference to horse bets:

- „Bettors“ trade their contracts with each other, rather than with an institutional „race-track“
- Interpretation of the state prices: The price of one-dollar return in the event specified by the contract
- Wisdom of the Crowds proven experimentally



# Prediction & Stock Markets

## ... of horses and markets

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### Stock markets

Difference to horse bets:

- No clear mutual state prices
- No „odds“ in real sense: the payoffs are determined by the value of the stock in each state

# Prediction & Stock Markets

## ... of horses and markets

### Very simple example

Two companies, two states:

State 1 ( $s_1$ ): „Company 1 does well, Company 2 does not“

- Company 1 is worth \$1
- Company 2 is worth \$0

State 2 ( $s_2$ ): „Company 2 does well, Company 1 does not“

- Company 1 is worth \$0
- Company 2 is worth \$1

=> Same as horse races: stocks are equivalent to contracts,  
prices are market probabilities

# Prediction & Stock Markets

## ... of horses and markets

### Advanced example

Two companies, two states:

State 1 ( $s_1$ ): „Company 1 does well, Company 2 **not quite well**“

- Company 1 is worth **\$2**
- Company 2 is worth **\$1**

State 2 ( $s_2$ ): „Company 2 does well, Company 1 **not quite well**“

- Company 1 is worth **\$1**
- Company 2 is worth **\$2**

# Prediction & Stock Markets

## ... of horses and markets

### Advanced example

$p_x$ : := state price for state  $x$

$v_y$ : := value of a share of company  $y$

Price of a share for Company 1 = value of the future worth of the company.

$$v_1 = 2p_1 + 1p_2$$

$$v_2 = 1p_1 + 2p_2$$

⇔

$$p_1 = \frac{2v_1 - v_2}{3}$$

$$p_2 = \frac{2v_2 - v_1}{3}$$

Problem: We need enough information to calculate prices so that the equitation systems yields a solution => The set of stocks has to be „rich enough“



# Markets with Endogenous Events



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**Endogenous** := The outcome of the event depends on the actions the individuals take

Examples:

- Joining a social network
- Market of used cars

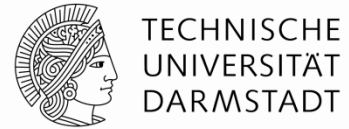
Common theme:

***Self-fulfilling expectations***



# Markets with Endogenous Events

## Self-fulfilling expectations



### Example: Social network joining

Everyone expects the payoff of joining a social network to be low

⇒ Barely anyone joins the network

⇒ The payoff is low indeed

### Example: Market for used cars

Everyone expects to be only low-quality cars on the market

⇒ No potential buyer is willing to pay a price of a good car

⇒ No potential seller of a good car is willing to sell for a low price

⇒ Only low-quality cars will be on the market



# Markets with Endogenous Events

## Asymmetric information

### Example: Market for used cars

An important difference in this case: Asymmetric information distribution among (prospective) buyers and sellers

A seller knows more about the quality of the car than the prospective buyer



# Markets with Endogenous Events

## The Market of Lemons



### Example: Market for used cars

Simplification: There are only good and bad cars on the market

$v_S^+$  := seller's value of a good car = 10

$v_S^-$  := seller's value of a bad car = 4

$v_B^+$  := buyer's value of a good car = 12

$v_B^-$  := buyer's value of a bad car = 6

$p^+$  := price of a good car

$p^-$  := price of a bad car

$g$  := fraction of good cars

$h$  := fraction of good cars for sale



# Markets with Endogenous Events

## The Market of Lemons



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### Example: Market for used cars

In a market with symmetric information:

$$v_S^+ < p^+ < v_B^+$$

$$v_S^- < p^- < v_B^-$$

In a market with asymmetric information:

$$v_B^* = 12h + 6(1-h) = 6 + 6h$$



# Markets with Endogenous Events

## The Market of Lemons



### Example: Market for used cars

Self-fulfilling-expectations equilibria:

For  $h=g$ :

$p^*$  := a price at which all sellers would offer their cars for sell

$$p^* = 6+6g \geq 10$$

$$\Rightarrow g \geq 2/3$$

$g = 2/3$  : Critical point of an equilibrium in this value-setting



# Markets with Endogenous Events

## The Market of Lemons



### Advanced Example: Market for used cars with lemons

- There are good cars, bad cars and *lemons*
- There are more buyers than sellers
- Good cars:  $v_S^+ = 10, v_B^+ = 12$
- Bad cars:  $v_S^- = 4, v_B^- = 6$
- Lemons:  $v_S^0 = v_B^0 = 0$

In a market with symmetric information:

All cars are sold at \$12 or \$6, buyers and sellers are indifferent to buy a lemon



# Markets with Endogenous Events

## The Market of Lemons



### Advanced Example: Market for used cars with lemons

In a market with asymmetric information:

Three candidates for equilibrium:

- a) All cars are offered for sale
- b) Only bad cars and lemons are offered for sale
- c) Only lemons are offered for sale





# Markets with Endogenous Events

## The Market of Lemons



### Advanced Example: Market for used cars with lemons

a) All cars are offered for sale

$$p_B = \frac{12 + 6 + 0}{3} = 6$$

- ⇒ Price below sellers of good cars expectations
- ⇒ No good cars are offered for sale
- ⇒ No equilibrium



# Markets with Endogenous Events

## The Market of Lemons



### Advanced Example: Market for used cars with lemons

b) Only bad cars and lemons are offered for sale

$$p_B = \frac{6+0}{2} = 3$$

- ⇒ Price below sellers of bad cars expectations
- ⇒ Only lemons are offered for sale
- ⇒ No equilibrium



# Markets with Endogenous Events

## The Market of Lemons



### Advanced Example: Market for used cars with lemons

c) Only lemons are offered for sale

$$p_B = 0$$

⇒ An equilibrium, but worthless

### Conclusion

a) leads to b), which leads to c)

⇒ Complete market failure



# Markets with Endogenous Events

## The Market of Lemons - Summary



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### Characteristics of the lemon market:

- (i) Items of different qualities
- (ii) For each quality, the buyer's value is at least as high as seller's value => market succeeds in symmetric information
- (iii) Asymmetric information: Only one side of transaction actually knows about the quality
- (iv) Following from (iii): all items are priced with the same value, items priced below their seller's value won't be put for sale



# Asymmetric information in markets

## Labor market

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### The Labor Market

- (i) Different qualities of workers: productive and unproductive
- (ii) Employers hire employees at wages appropriate to their quality, if they could determine it
- (iii) Asymmetric information: Employees know more about their qualities than the employers
- (iv) Following from (iii): wages can't directly depend on employee's quality => a uniform wage is offered to all potential workers, workers with higher wage expectations will leave the market into self-employment



# Asymmetric information in markets

## Labor market



### The Labor Market

#### Assumptions:

- Productive workers generate a revenue of \$80.000, but may choose to produce an alternate income of \$55.000 by being self-employed
- Unproductive workers' generated revenue is \$40.000, their alternate income through self-employment is \$25.000

In *symmetric information*, a company would pay \$55.000 to \$80.000 to productive employees and \$25.000 - \$40.000 to unproductive ones



# Asymmetric information in markets

## Labor market – asymmetric information



### The Labor Market

#### Assumptions:

- Productive workers generate a revenue of \$80.000, but may choose to produce an alternate income of \$55.000 by being self-employed
- Unproductive workers' generated revenue is \$40.000, their alternate income through self-employment is \$25.000

In *asymmetric information*, a company has to offer a wage of  $w$  based upon its expectations to the distribution of the two types of job-seekers on the market



# Asymmetric information in markets

## Labor market – asymmetric information



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In *asymmetric information* market:

Assumption:  $\frac{1}{2}$  of the workers are productive,  $\frac{1}{2}$  are not

Expectation: All workers are on the market

$$w = \frac{\$40.000 + \$80.000}{2} = \$60.000 \quad \Rightarrow \text{All employees get hired}$$

Expectation: Only unproductive workers are on the market

$$w = \$40.000 \quad \Rightarrow \text{Only unproductive employees get hired}$$

Both are self-fulfilling-expectation equilibria, so far so good





# Asymmetric information in markets

## Labor market – asymmetric information



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In *asymmetric information* market:

Assumption:  $\frac{1}{4}$  of the workers are productive,  $\frac{3}{4}$  are not

Is there an equilibrium?

Expectation: All workers are on the market

$$w = \frac{3}{4} \cdot \$40.000 + \frac{1}{4} \cdot \$80.000 = \$50.000 \quad \Rightarrow \text{Only unproductive employees get hired}$$

$\Rightarrow$  There is no self-fulfilling-expectation equilibrium, productive workers are driven out of the market by the high frequency of the unproductive ones



# Signaling Quality

Signal := An approach to break the information asymmetry

The trader being in information advantage „signals“ a bit of his information to the other trader.

Signals are more expensive to traders of „bad“ goods

- Used-cars market:

- TÜV-Certificates
- Test drives
- Guarantees

- Labor market:

- Education certificates as prove of qualification
- Education certificates as prove of effort-willingness

# Quality Uncertainty On-Line

Many Web-Sites for online commerce are motivated by considerations of asymmetric information and signaling

## ⇒ Reputation Systems (eBay)

- An eBay-bidder is uncertain about seller's fairness
- If the risk to „buy a lemon“ is too high, the bidders won't bid a price high enough for the sellers of a good items, the market would fail

## Solution: Reputation as signal for quality

„Expensive“ to obtain, but much cheaper for good seller than for a bad one.

## ⇒ Ad Quality in Keywords-Based Advertising

- An ad „looking good“, but pointing to an uninteresting landing page would generate a high click-rate in the short term, but might decrease the general click-rate in the long term.

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# Thank you!



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# Questions?

