Learning

- Learning agents
- Inductive learning
  - Different Learning Scenarios
  - Evaluation
- Neural Networks
  - Perceptrons
  - Multilayer Perceptrons
- Reinforcement Learning
  - Temporal Differences
  - Q-Learning
  - SARSA

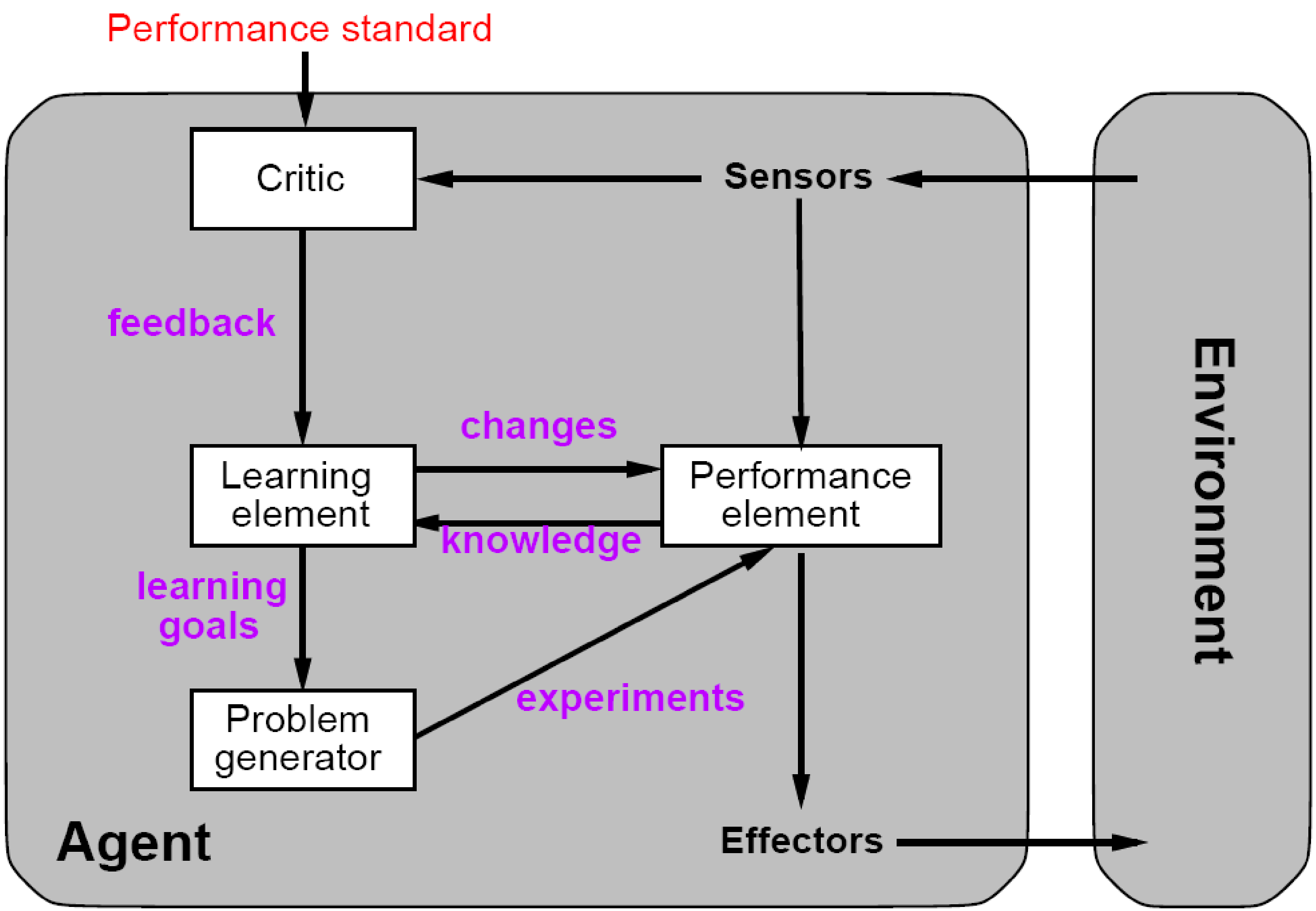
Material from Russell & Norvig, chapters 18.1, 18.2, 20.5 and 21

Slides based on Slides by Russell/Norvig, Ronald Williams, and Torsten Reil

Learning

- Learning is essential for unknown environments,
  - i.e., when designer lacks omniscience
- Learning is useful as a system construction method,
  - i.e., expose the agent to reality rather than trying to write it down
- Learning modifies the agent's decision mechanisms to improve performance

Learning Agents



Learning Element

- Design of a learning element is affected by
  - Which components of the performance element are to be learned
  - What feedback is available to learn these components
  - What representation is used for the components
- Type of feedback:
  - Supervised learning:
    - correct answers for each example
  - Unsupervised learning:
    - correct answers not given
  - Reinforcement learning:
    - occasional rewards for good actions

Different Learning Scenarios

Supervised Learning
- A teacher provides the value for the target function for all training examples (labeled examples)
- concept learning, classification, regression

Reinforcement Learning
- The teacher only provides feedback but not example values

Semi-supervised Learning
- Only a subset of the training examples are labeled

Unsupervised Learning
- There is no information except the training examples
- clustering, subgroup discovery, association rule discovery

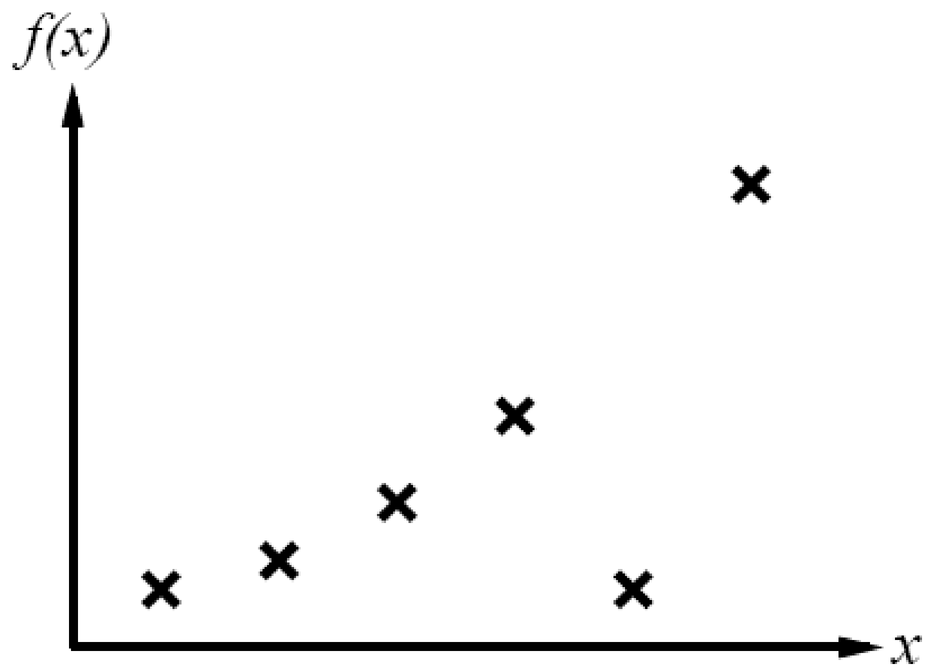
Inductive Learning

Simplest form: learn a function from examples

- f is the (unknown) target function
- An example is a pair $(x, f(x))$
- Problem: find a hypothesis h
  - given a training set of examples
  - such that $h \approx f$
  - on *all* examples
    - i.e. the hypothesis must generalize from the training examples
- This is a highly simplified model of real learning:
  - Ignores prior knowledge
  - Assumes examples are given

Inductive Learning Method

- Construct/adjust h to agree with f on training set
  - h is consistent if it agrees with f on all examples
- Example:
  - curve fitting



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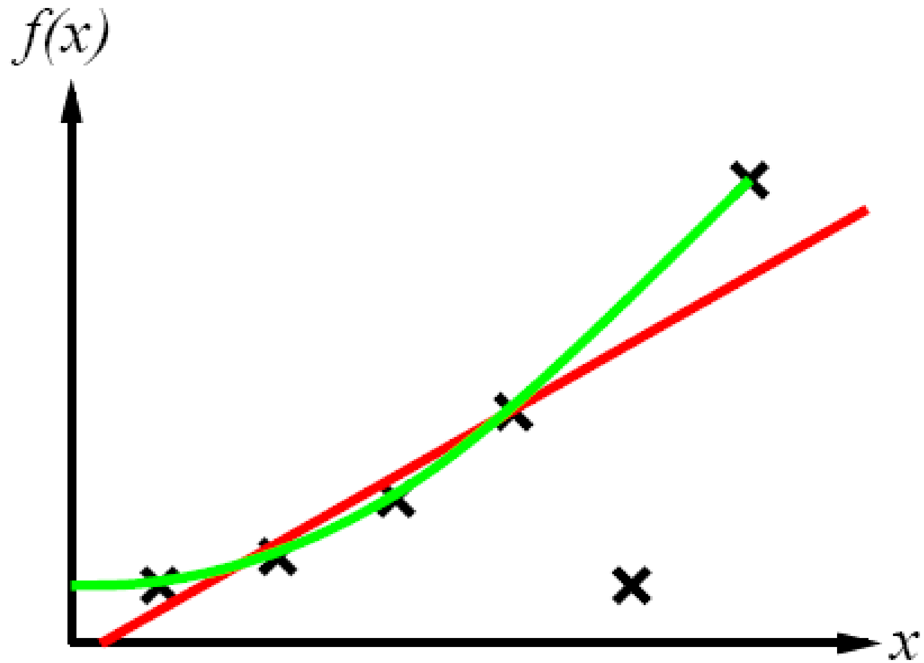

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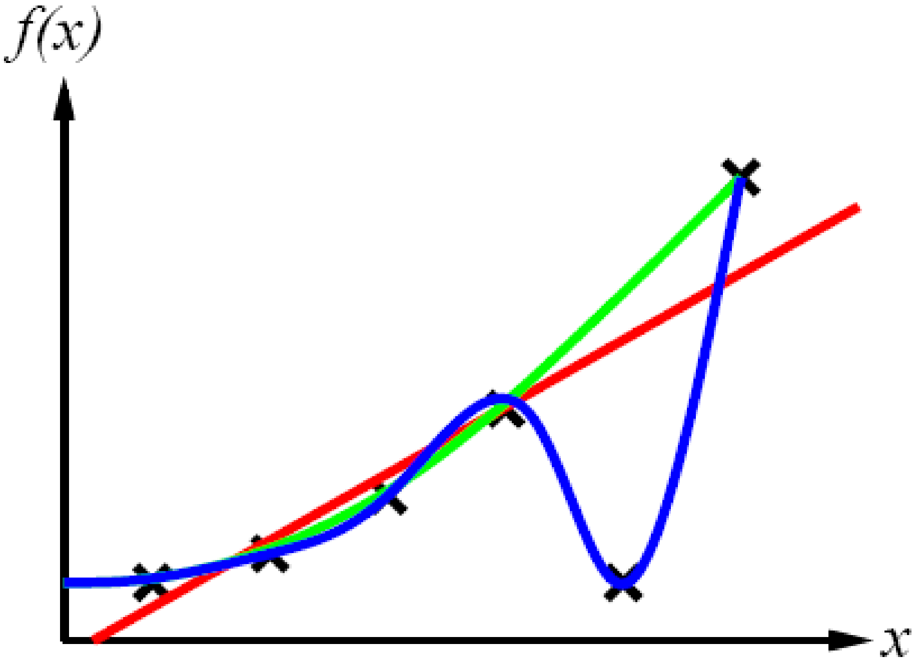

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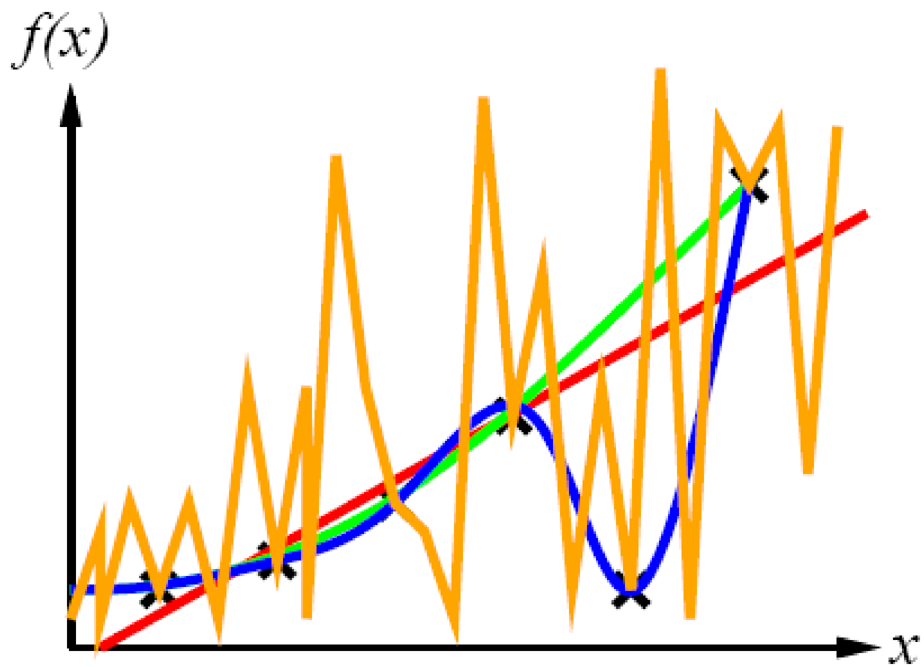
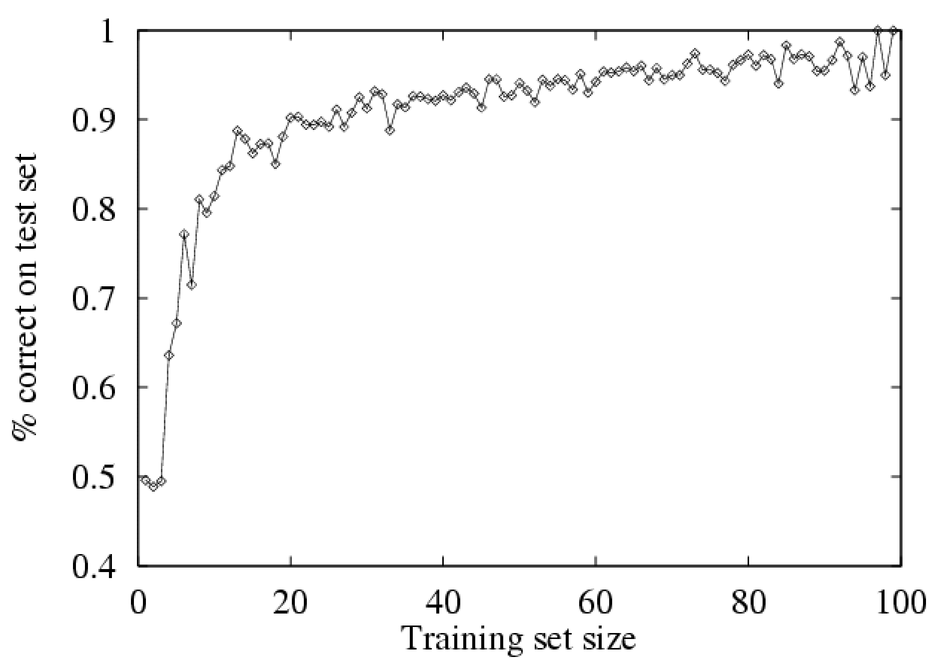


- Ockham's Razor
  - The best explanation is the simplest explanation that fits the data
- Overfitting Avoidance
  - maximize a combination of consistency and simplicity

Performance Measurement

- How do we know that $h \approx f$?
  - Use theorems of computational/statistical learning theory
  - Or try h on a new test set of examples where f is known (use same distribution over example space as training set)

Learning curve = % correct on test set over training set size



What are Neural Networks?

- Models of the brain and nervous system
- Highly parallel
  - Process information much more like the brain than a serial computer
- Learning

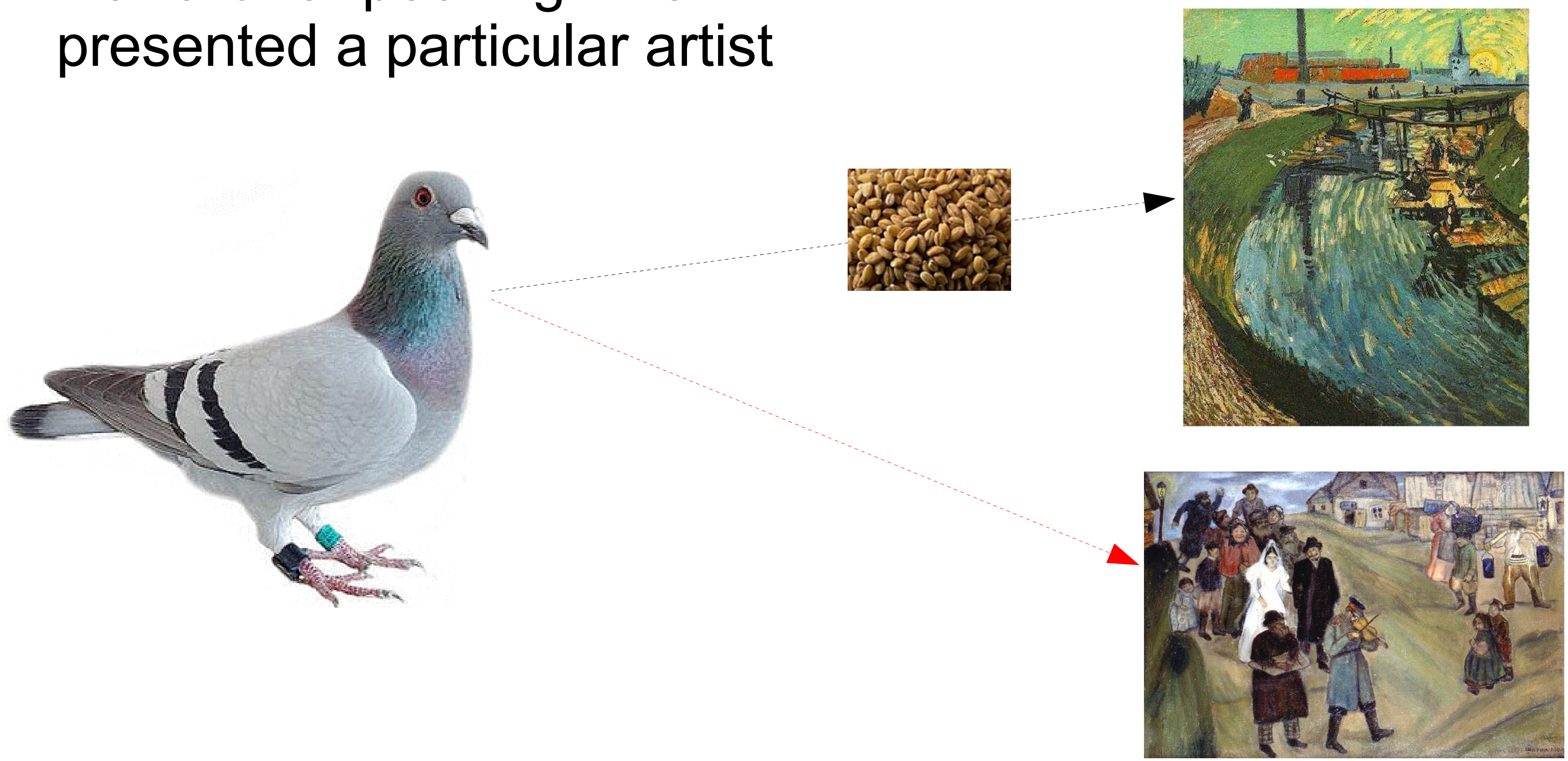
- Very simple principles
- Very complex behaviours

- Applications
  - As powerful problem solvers
  - As biological models

Pigeons as Art Experts

Famous experiment (Watanabe *et al.* 1995, 2001)

- Pigeon in Skinner box
- Present paintings of two different artists (e.g. Chagall / Van Gogh)
- Reward for pecking when presented a particular artist



Results

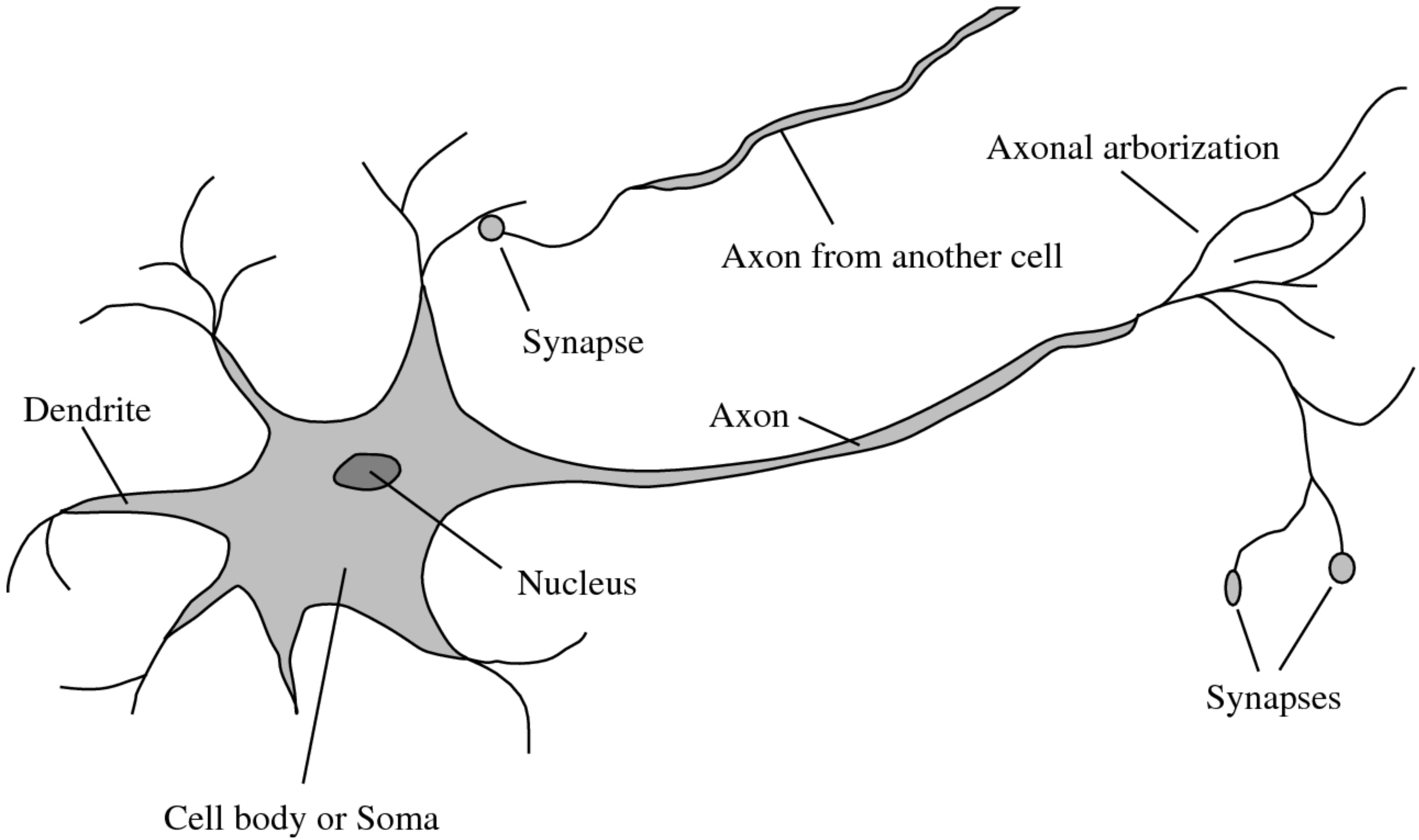
- Pigeons were able to discriminate between Van Gogh and Chagall with 95% accuracy
  - when presented with pictures they had been trained on

- Discrimination still 85% successful for previously unseen paintings of the artists

- Pigeons do not simply memorise the pictures
- They can extract and recognise patterns (the 'style')
- They generalise from the already seen to make predictions

- This is what neural networks (biological and artificial) are good at (unlike conventional computer)

A Biological Neuron

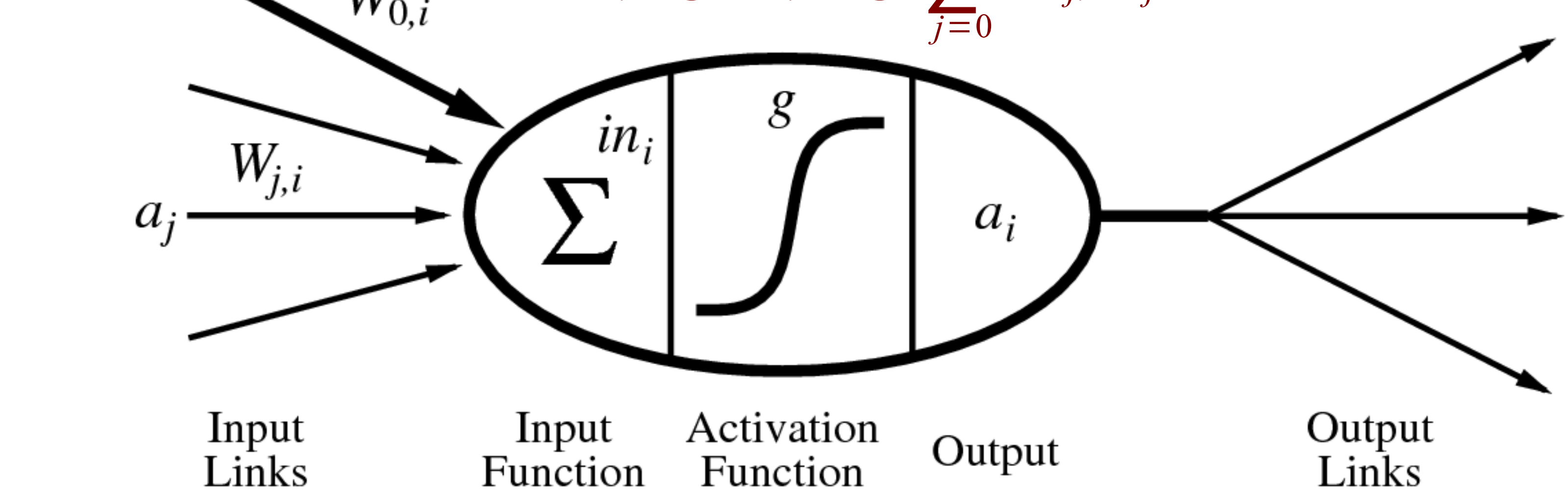


- Neurons are connected to each other via synapses
- If a neuron is activated, it spreads its activation to all connected neurons

An Artificial Neuron

(McCulloch-Pitts,1943)

$$a_i = g(\mathrm{in}_i) = g\left(\sum_{j=0}^{n} W_{ji} \cdot a_j\right)$$



- Neurons correspond to nodes or units
- A link from unit j to unit i propagates activation a_j from j to i
- The weight $W_{j,i}$ of the link determines the strength and sign of the connection
- The total input activation is the sum of the input activations
- The output activation is determined by the activiation function g

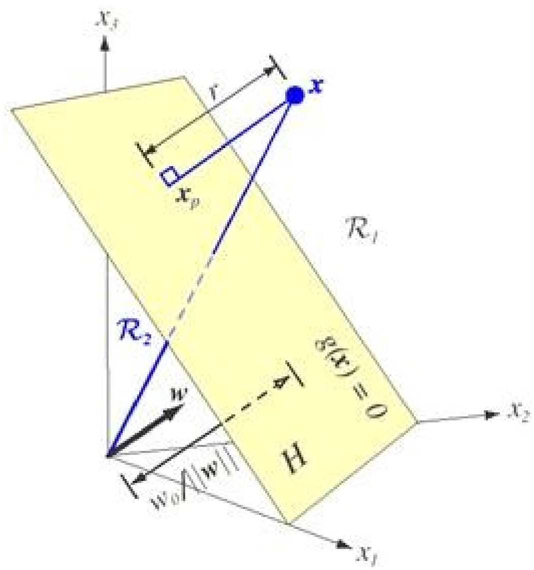
Perceptron

(Rosenblatt 1957, 1960)

- A single node
  - connecting n input signals a_j with one output signal a
  - typically signals are -1 or $+1$
- Activation function
  - A simple threshold function:

$$a = \begin{cases} -1 & \text{if } \sum_{j=0}^{n} W_j \cdot a_j \leq 0 \\ 1 & \text{if } \sum_{j=0}^{n} W_j \cdot a_j > 0 \end{cases}$$

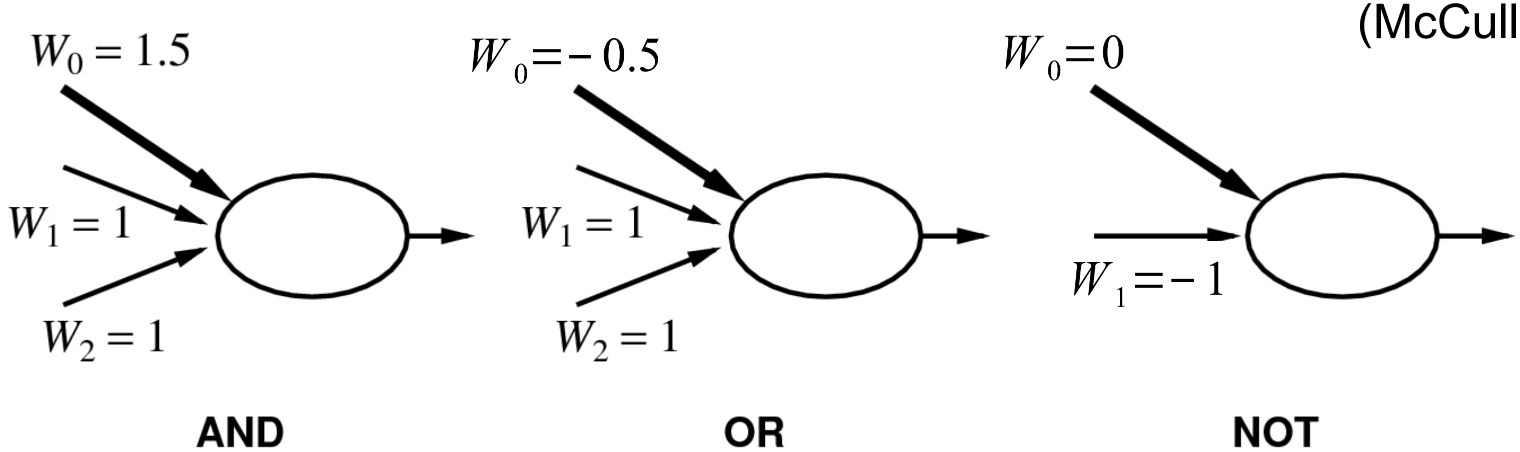
- Thus it implements a linear separator
  - i.e., a hyperplane that divides n-dimensional space into a region with output -1 and a region with output 1



Perceptrons and Boolean Fucntions

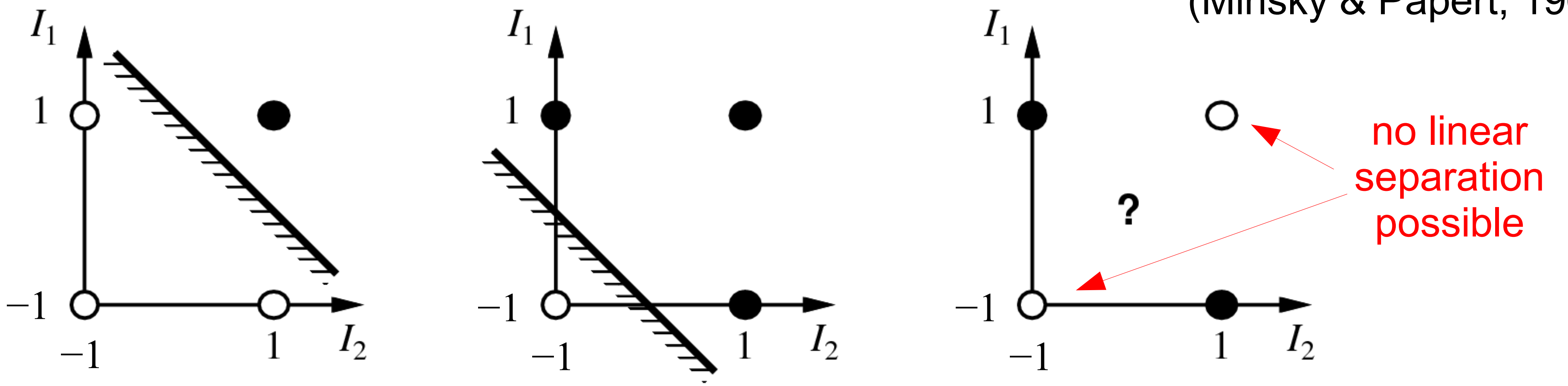
- a Perceptron can implement all elementary logical functions

(McCullogh & Pitts, 1943)



- more complex functions like XOR cannot be modeled

(Minsky & Papert, 1969)



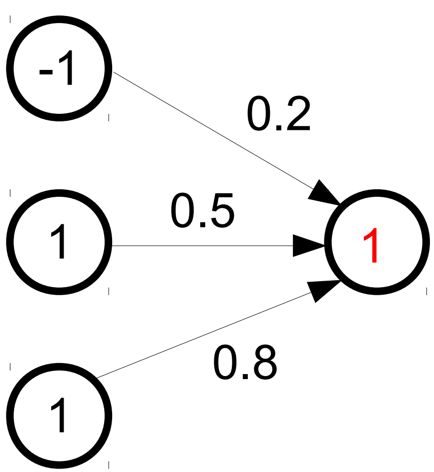
(a) I_1 **and** I_2 (b) I_1 **or** I_2 (c) I_1 **xor** I_2

Perceptron Learning

- Perceptron Learning Rule for Supervised Learning

$$W_j \leftarrow W_j + \underbrace{\alpha}_{\text{learning rate}} \cdot \underbrace{(f(\mathbf{x}) - h(\mathbf{x}))}_{\text{error}} \cdot x_j$$

- Example:



Computation of output signal $h(x)$

$$\text{in}(x) = -1 \cdot 0.2 + 1 \cdot 0.5 + 1 \cdot 0.8 = 1.1$$

$$h(x) = 1 \text{ because } \text{in}(x) > 0 \text{ (activation function)}$$

Assume target value $f(x) = -1$ (and $\alpha = 0.5$)

$$W_0 \leftarrow 0.2 + 0.5 \cdot (-1 - 1) \cdot -1 = 0.2 + 1 = 1.2$$

$$W_1 \leftarrow 0.5 + 0.5 \cdot (-1 - 1) \cdot 1 = 0.5 - 1 = -0.5$$

$$W_2 \leftarrow 0.8 + 0.5 \cdot (-1 - 1) \cdot 1 = 0.8 - 1 = -0.2$$

Measuring the Error of a Network

- The error for one training example $\mathbf{x}$ can be measured by the squared error
  - the squared difference of the output value $h(\mathbf{x})$ and the desired target value $f(\mathbf{x})$

$$E(\mathbf{x}) = \frac{1}{2} Err^2 = \frac{1}{2}(f(\mathbf{x}) - h(\mathbf{x}))^2 = \frac{1}{2}\left(f(\mathbf{x}) - g\left(\sum_{j=0}^{n} W_j \cdot x_j\right)\right)^2$$

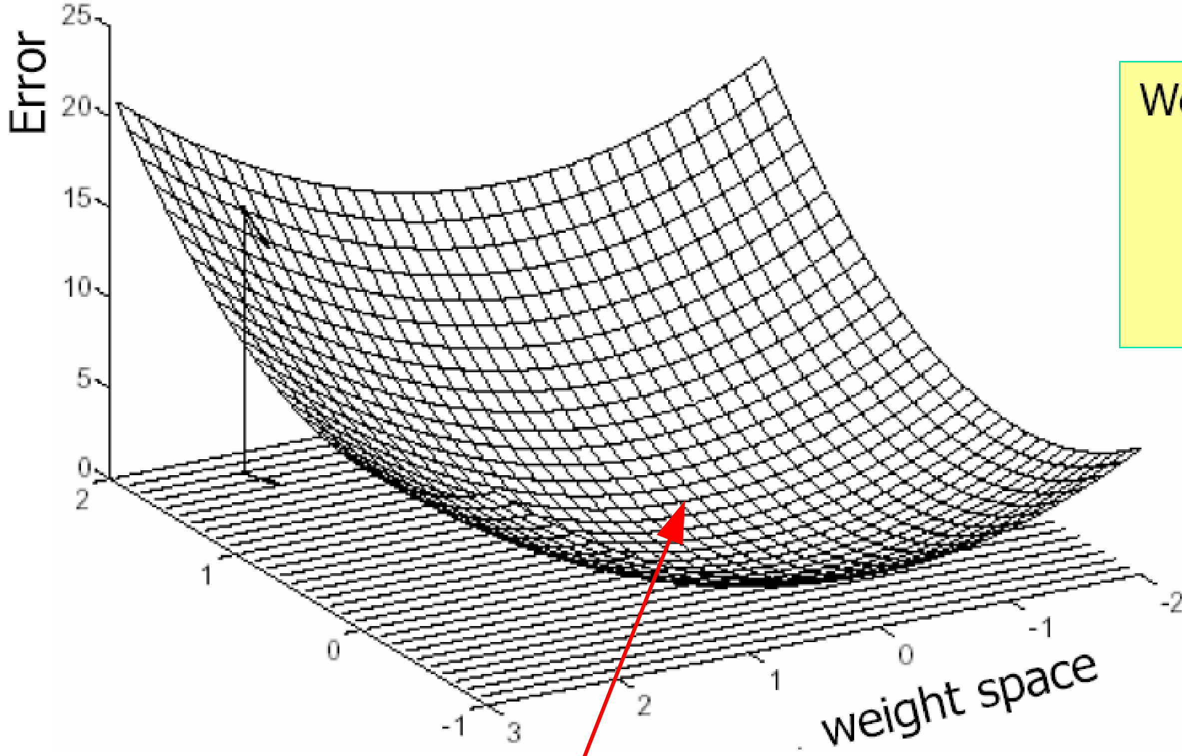
- For evaluating the performance of a network, we can try the network on a set of datapoints and average the value

  (= sum of squared errors)

$$E(Network) = \sum_{i=1}^{N} E(\mathbf{x}_i)$$

Error Landscape

- The error function for one training example may be considered as a function in a multi-dimensional weight space



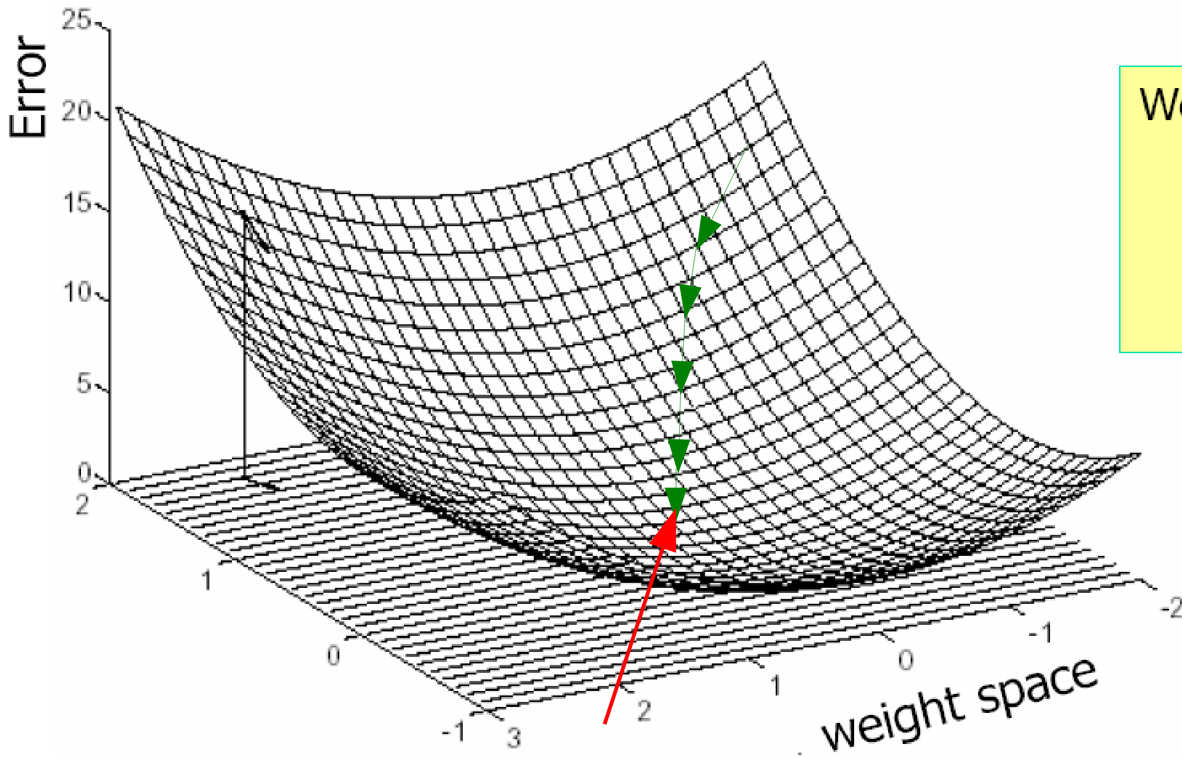
Weight space is N-dimensional, where N is the total number of weights in the network

$$E(W)=\frac{1}{2}\left(f(\mathbf{x})-g\left(\sum_{j=0}^{n} W_j \cdot x_j\right)\right)^2$$

- The best weight setting for one example is where the error measure for this example is minimal

Error Minimization via Gradient Descent

- In order to find the point with the minimal error:
  - go downhill in the direction where it is steepest



Weight space is N-dimensional, where N is the total number of weights in the network

$$E(W)=\frac{1}{2}\left(f(\mathbf{x})-g\left(\sum_{j=0}^{n} W_j \cdot x_j\right)\right)^2$$

  - … but make small steps, or you might shoot over the target

Error Minimization

- It is easy to derive a perceptron training algorithm that minimizes the squared error

$$E=\frac{1}{2}Err^2=\frac{1}{2}(f(\mathbf{x})-h(\mathbf{x}))^2=\frac{1}{2}\left(f(\mathbf{x})-g(\sum_{j=0}^{n}W_j\cdot x_j)\right)^2$$

- Change weights into the direction of the steepest descent of the error function

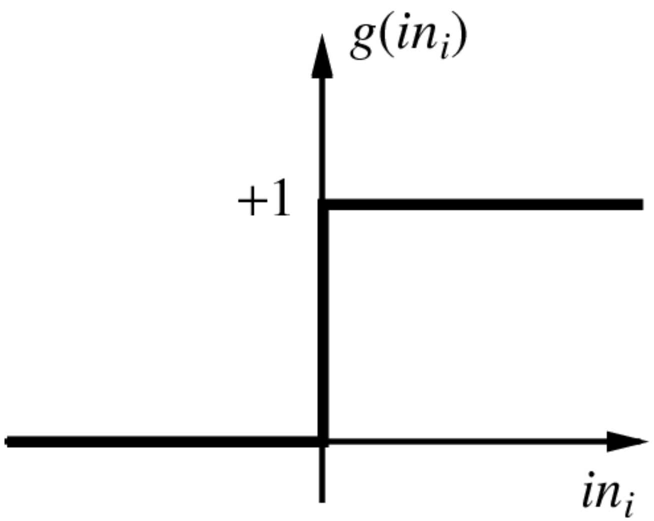
$$\frac{\partial E}{\partial W_j}=Err\cdot\frac{\partial Err}{\partial W_j}=Err\cdot\frac{\partial}{\partial W_j}\left(f(\mathbf{x})-g(\sum_{k=0}^{n}W_k\cdot x_k)\right)=-Err\cdot g'(\mathrm{in})\cdot x_j$$

  - To compute this, we need a continuous and differentiable activation function g!
- Weight update with learning rate α: $W_j\leftarrow W_j+\alpha\cdot Err\cdot g'(\mathrm{in})\cdot x_j$
  - positive error → increase network output
    - increase weights of nodes with positive input
    - decrease weights of nodes with negative input

Threshold Activation Function

- The regular threshold activation function is problematic
  - $g'(x) = 0$, therefore $\frac{\partial E}{\partial W_{j,i}} = -Err \cdot g'(\mathrm{in}_i) \cdot x_j = 0$

    $\rightarrow$ no weight changes!

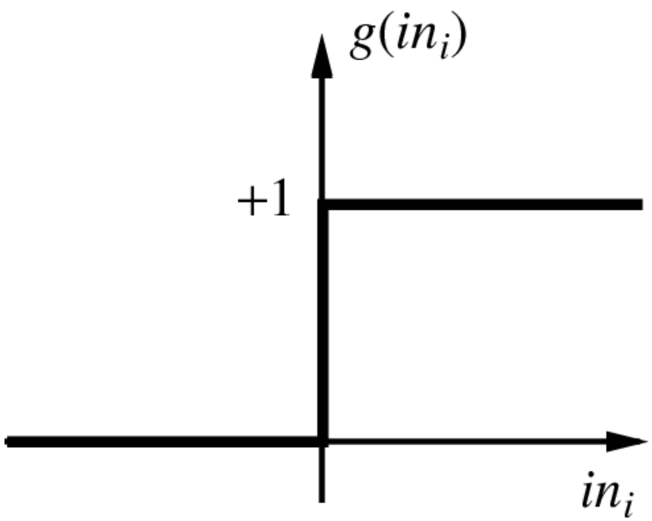


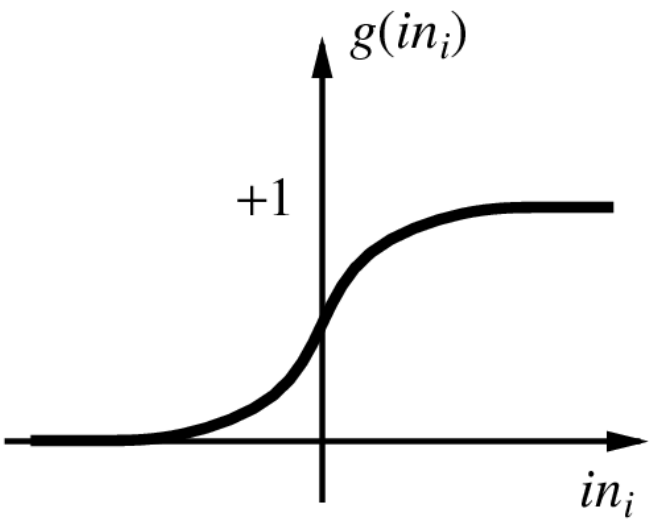
$$g(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

$$g'(x) = 0$$

Sigmoid Activation Function

- A commonly used activation function is the sigmoid function
  - similar to the threshold function
  - easy to differentiate
  - non-linear





$$g(x)=\begin{cases}0 & \text{if } x\leq 0\\ 1 & \text{if } x>0\end{cases}$$

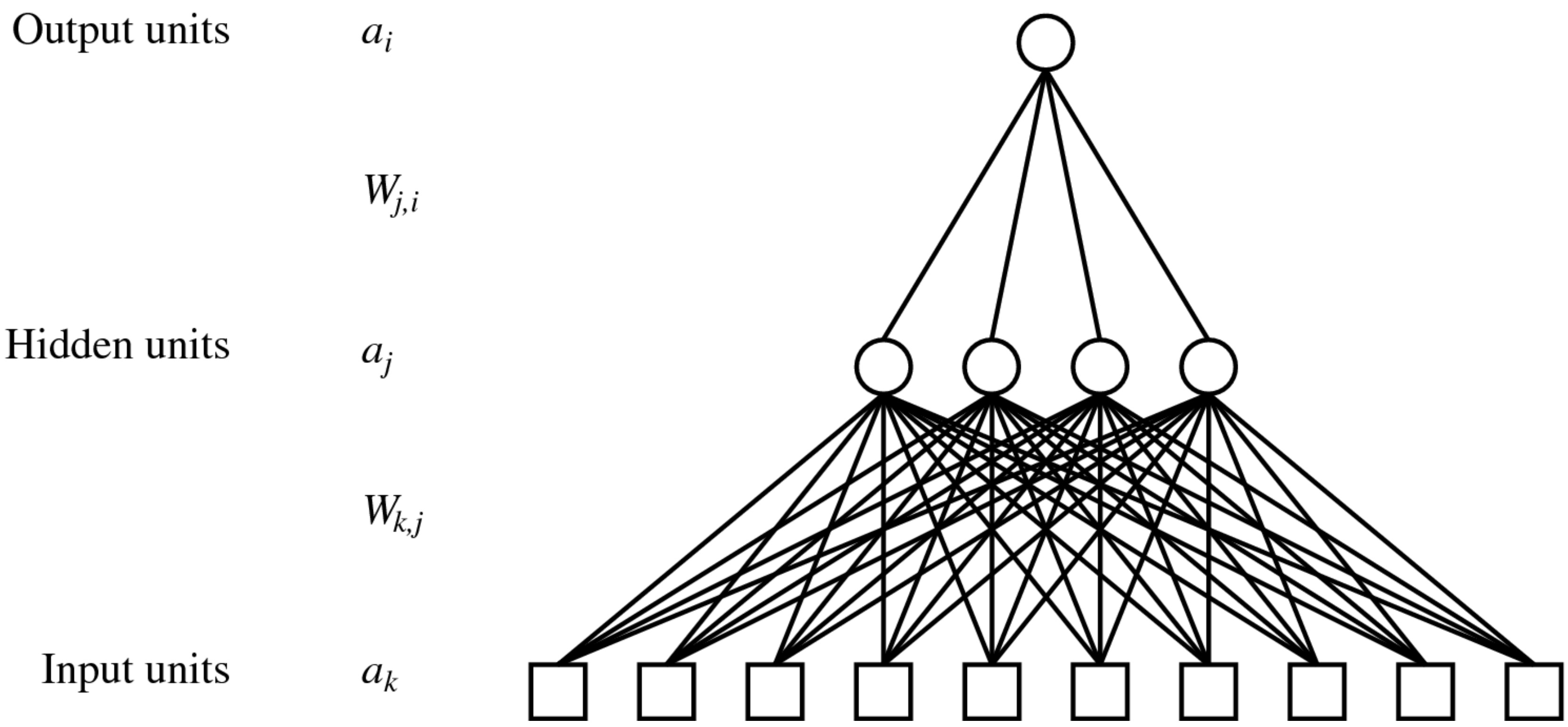
$$g'(x)=0$$

$$g(x)=\frac{1}{1+e^{-x}}$$

$$g'(x)=g(x)\left(1-g(x)\right)$$

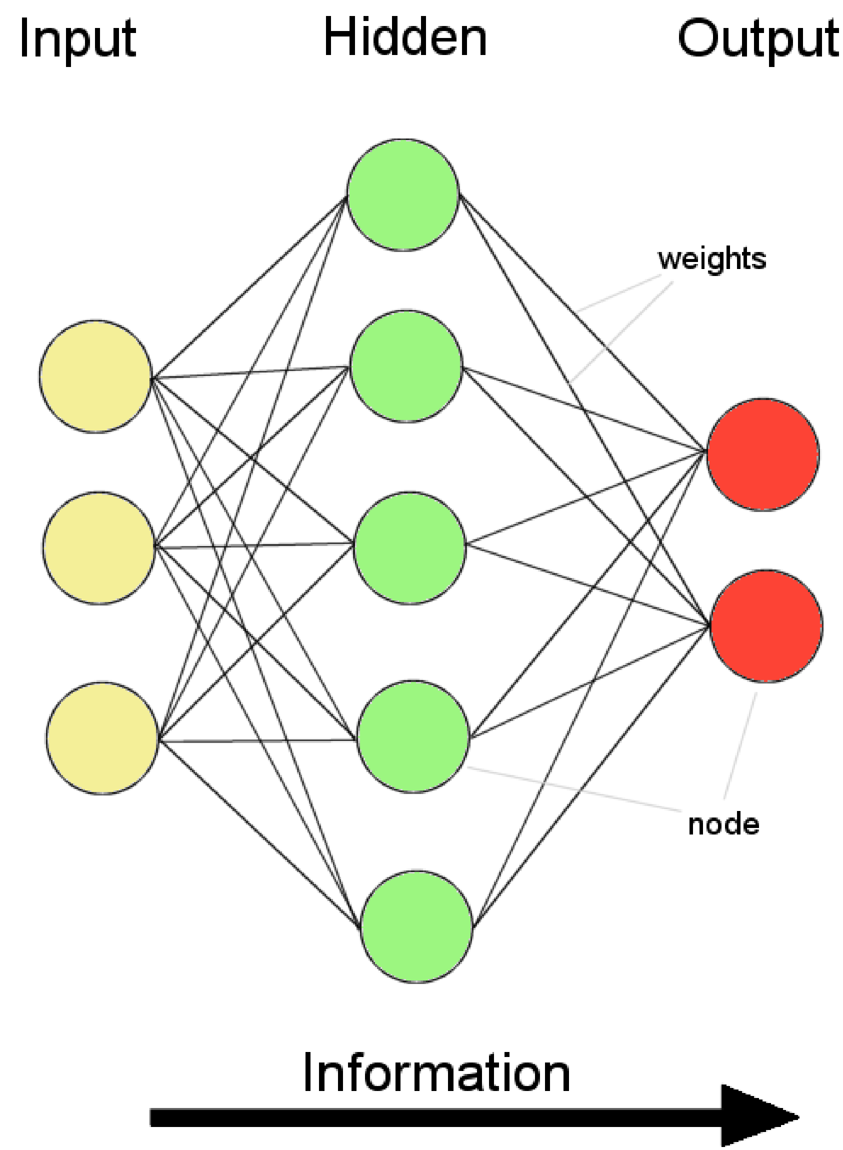
Multilayer Perceptrons

- Perceptrons may have multiple output nodes
  - may be viewed as multiple parallel perceptrons
- The output nodes may be combined with another perceptron
  - which may also have multiple output nodes
- The size of this hidden layer is determined manually



Multilayer Perceptrons

- Information flow is unidirectional
  - Data is presented to *Input layer*
  - Passed on to *Hidden Layer*
  - Passed on to *Output layer*
- Information is distributed
- Information processing is parallel



Expressiveness of MLPs

- Every continuous function can be modeled with three layers
  - i.e., with one hidden layer
- Every function can be modeled with four layers
  - i.e., with two hidden layers

Backpropagation Learning

- The output nodes are trained like a normal perceptron

$$W_{ji} \leftarrow W_{ji} + \alpha \cdot Err_i \cdot g'(\mathrm{in}_i) \cdot x_j = W_{ji} + \alpha \cdot \Delta_i \cdot x_j$$

  - Δ_i is the error term of output node i times the derivation of its inputs

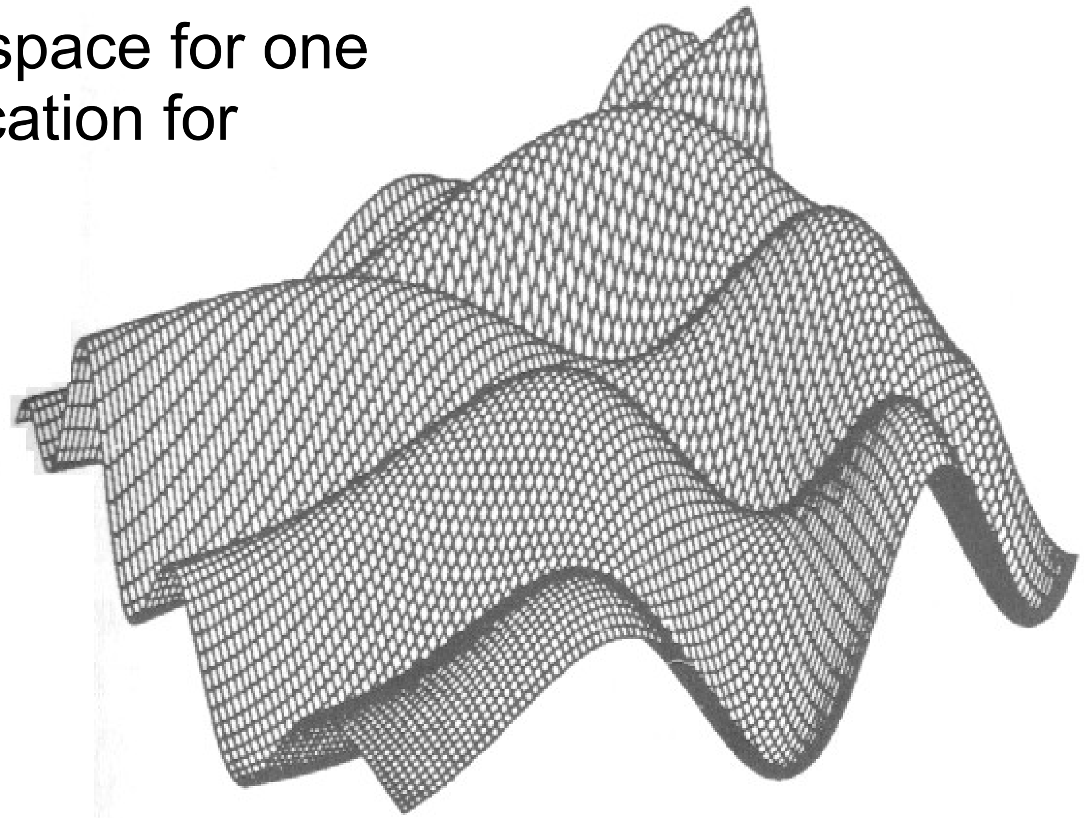
- the error term Δ_i of the output layers is propagated back to the hidden layer

$$\Delta_j = \left(\sum_i W_{ji} \cdot \Delta_i \right) \cdot g'(\mathrm{in}_j) \qquad W_{kj} \leftarrow W_{kj} + \alpha \cdot \Delta_j \cdot x_k$$

  - the training signal of hidden layer node j is the weighted sum of the errors of the output nodes

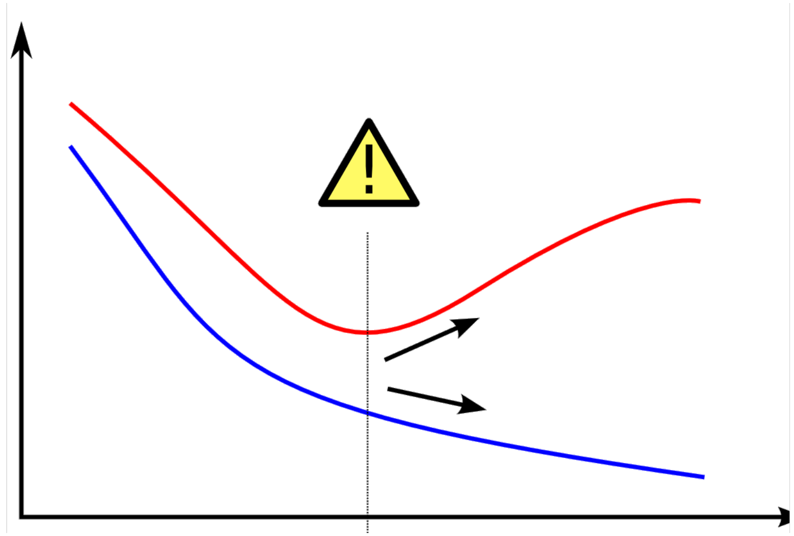
Minimizing the Network Error

- The error landscape for the entire network may be thought of as the sum of the error functions of all examples
  - will yield many local minima → hard to find global minimum
- Minimizing the error for one training example may destroy what has been learned for other examples
  - a good location in weight space for one example may be a bad location for another examples
- Training procedure:
  - try all examples in turn
  - make small adjustments for each example
  - repeat until convergence
- One Epoch = One iteration through all examples



Overfitting

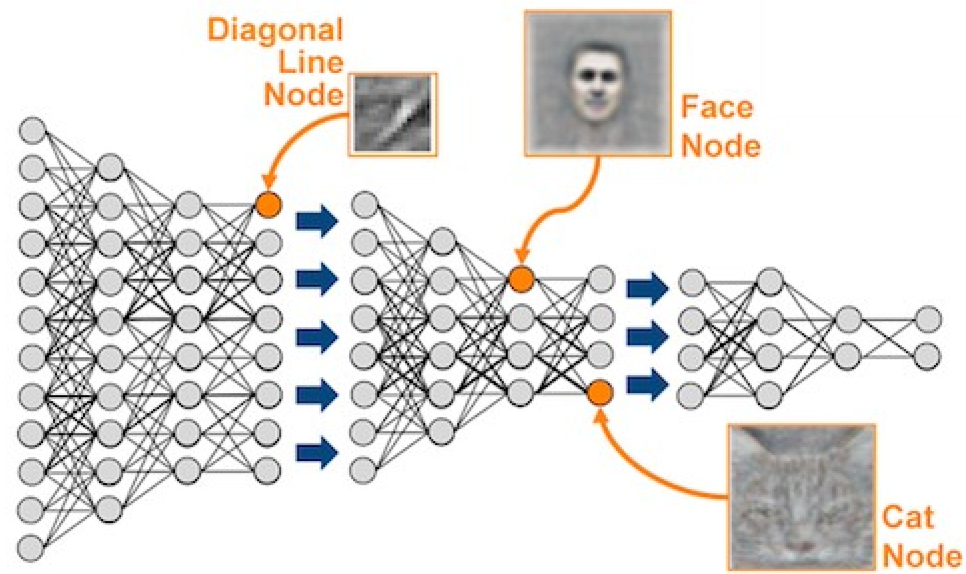
- Training Set Error continues to decrease with increasing number of training examples / number of epochs
  - an epoch is a complete pass through all training examples
- Test Set Error will start to increase because of overfitting



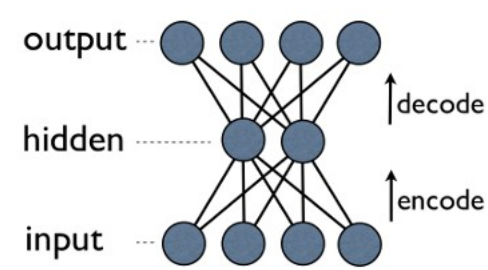
- Simple training protocol:
  - keep a separate validation set to watch the performance
    - validation set is different from training and test sets!
  - stop training if error on validation set gets down

Deep Learning

- In the last years, great success has been observed with training „deep“ neural networks
  - Deep networks are networks with multiple layers
- Successes in particular in image classification
  - Idea is that layers sequentially extract information from image
    - 1st layer → edges,
    - 2nd layer → corners, etc…

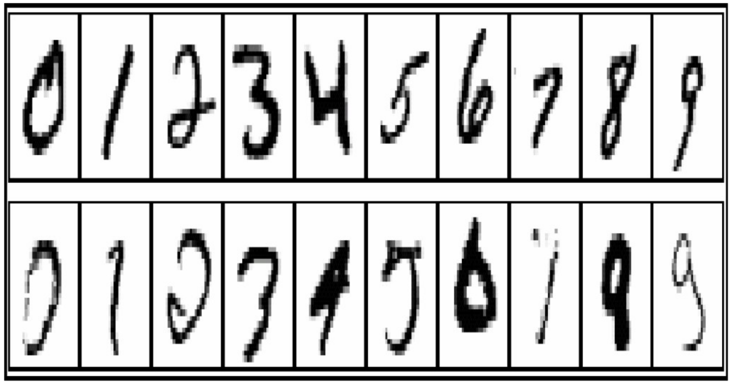


- Key ingredients:
  - A lot of training data are needed and available (big data)
  - Fast processing and a few new tricks made fast training for big data possible
  - Unsupervised pre-training of layers
    - **Autoencoder** use the previous layer as input and output for the next layer



Wide Variety of Applications

- Speech Recognition
- Autonomous Driving
- Handwritten Digit Recognition
- Credit Approval
- Backgammon
- etc.



- Good for problems where the final output depends on combinations of many input features
  - rule learning is better when only a few features are relevant
- Bad if explicit representations of the learned concept are needed
  - takes some effort to interpret the concepts that form in the hidden layers