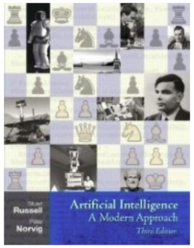
Uninformed Search

- Problem-solving agents
  - Single-State Problems
- Tree search algorithms
  - Breadth-First Search
  - Depth-First Search
  - Limited-Depth Search
  - Iterative Deepening
- Extensions
  - Graph search algorithms
  - Search with Partial Information



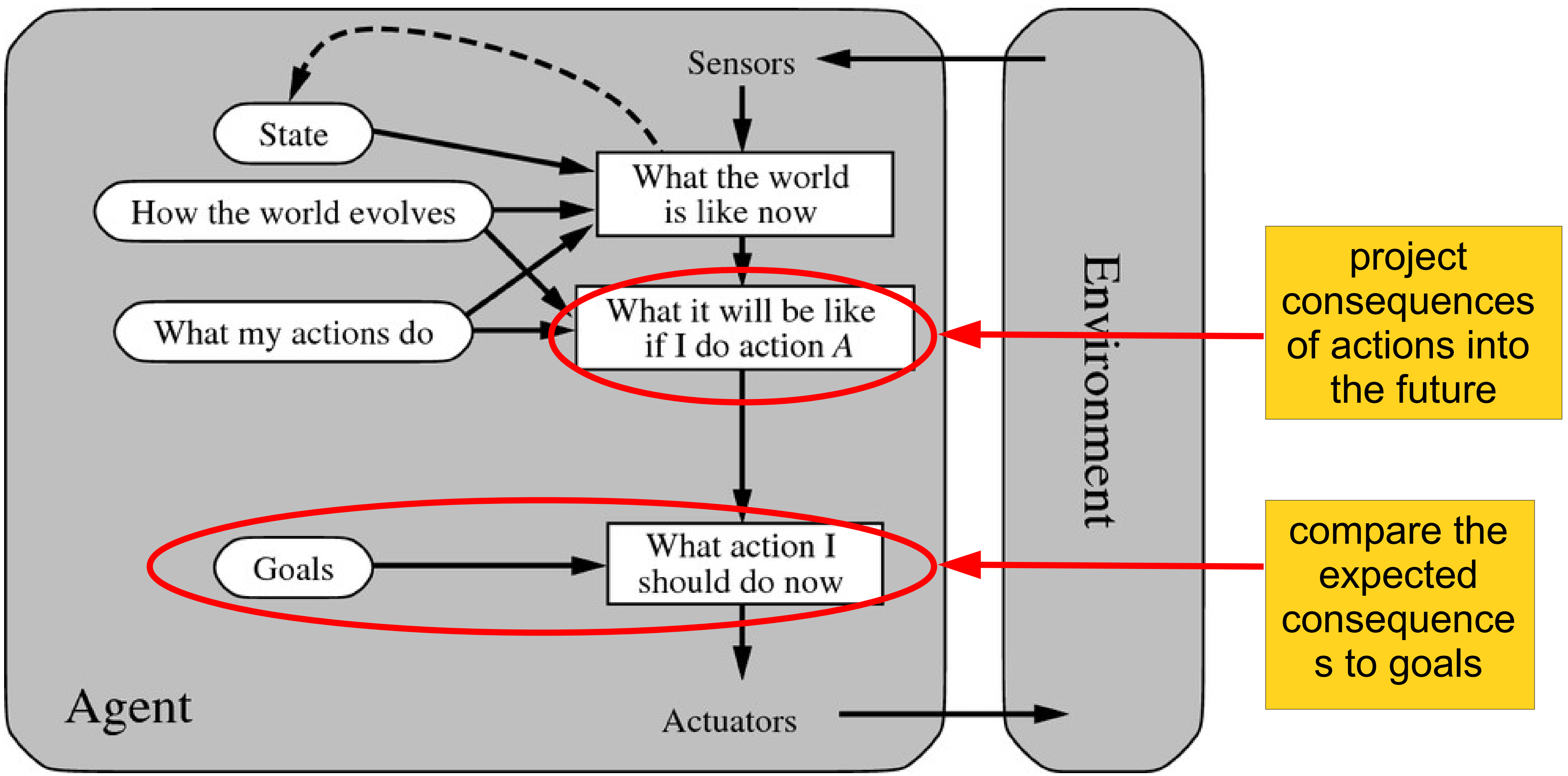
Many slides based on Russell & Norvig's slides *Artificial Intelligence: A Modern Approach*

Problem-Solving Agents

- Simple reflex agents
  - have a direct mapping from states to actions
  - typically too large to store
  - would take too long to learn
- Goal-Based agents
  - can consider future actions and the desirability of their outcomes
- Problem-Solving Agents
  - special case of Goal-Based Agents
  - find sequences of actions that lead to desirable states
- Uninformed Problem-Solving Agents
  - do not have any information except the problem definition
- Informed Problem-Solving Agents
  - have knowledge where to look for solutions

Goal-Based Agent

- the agent knows what states are desirable
  - it will try to choose an action that leads to a desirable state



Formulate-Search-Execute Design

- Formulate:
  - Goal formulation:
    - A *goal* is a set of world states that the agents wants to be in (where the goal is achieved)
    - Goals help to organize behavior by limiting the objectives that the agent is trying to achieve
  - Problem formulation:
    - Process of which actions and states to consider, given a goal
- Search:
  - the process of finding the solution for a problem in the form of an action sequence

> *an agent with several immediate options of unknown value can decide what to do by* ***examining different possible sequences*** *of actions that lead to states of known value, and then* ***choosing the best***

- Execute:
  - perform the first action of the solution sequence

Simple Problem-Solving Agent

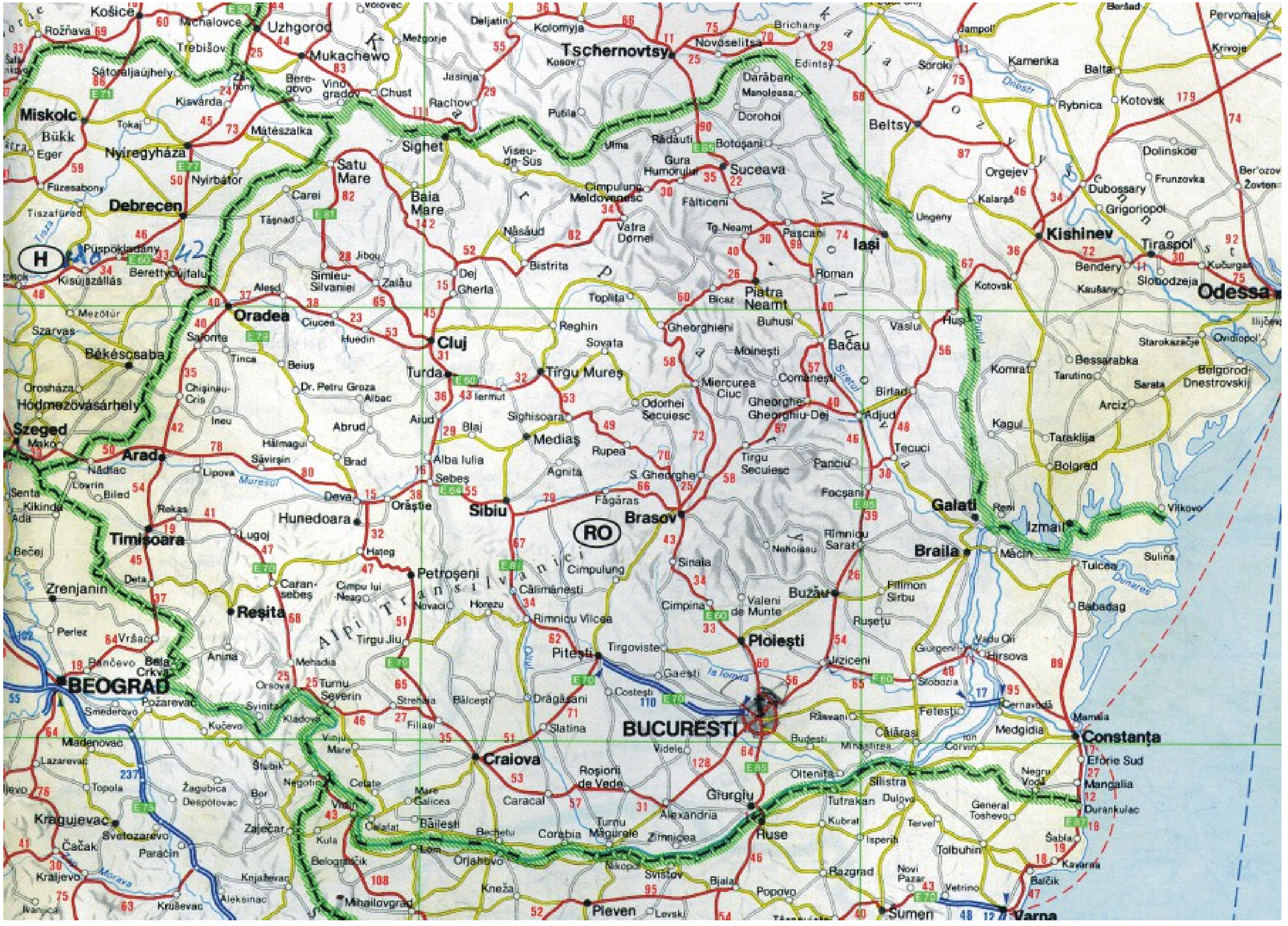
```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
    action ← RECOMMENDATION(seq, state)
    seq ← REMAINDER(seq, state)
    return action
```

Example: Navigate in Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Rimnicu Vilcea, Pitesti
- Assumption:
  - agent has a map of Romania, i.e., it can use this information to find out which of the three ways out of Arad is more likely to go to Bucharest

Example: Romania



Single-state Problem Formulation

A problem is defined by four items:

- initial state
  - e.g., "at Arad"
- description of actions and their effects
  - typically as a successor function that maps a state s to a set $S(s)$ of action-state pairs
  - e.g., S(„at Arad“) = {<„goto Zerind“, „at Zerind“>, … }
- goal test, can be
  - explicit, e.g., s = "at Bucharest"
  - implicit, e.g., Checkmate(s), NoDirt(s)
- path cost (additive)
  - e.g., sum of distances, number of actions executed, etc.
  - $c(s_1, a, s_2)$ are the costs for one step (one action),
  - assumed to be ≥ 0

Single-State Problems

Yes

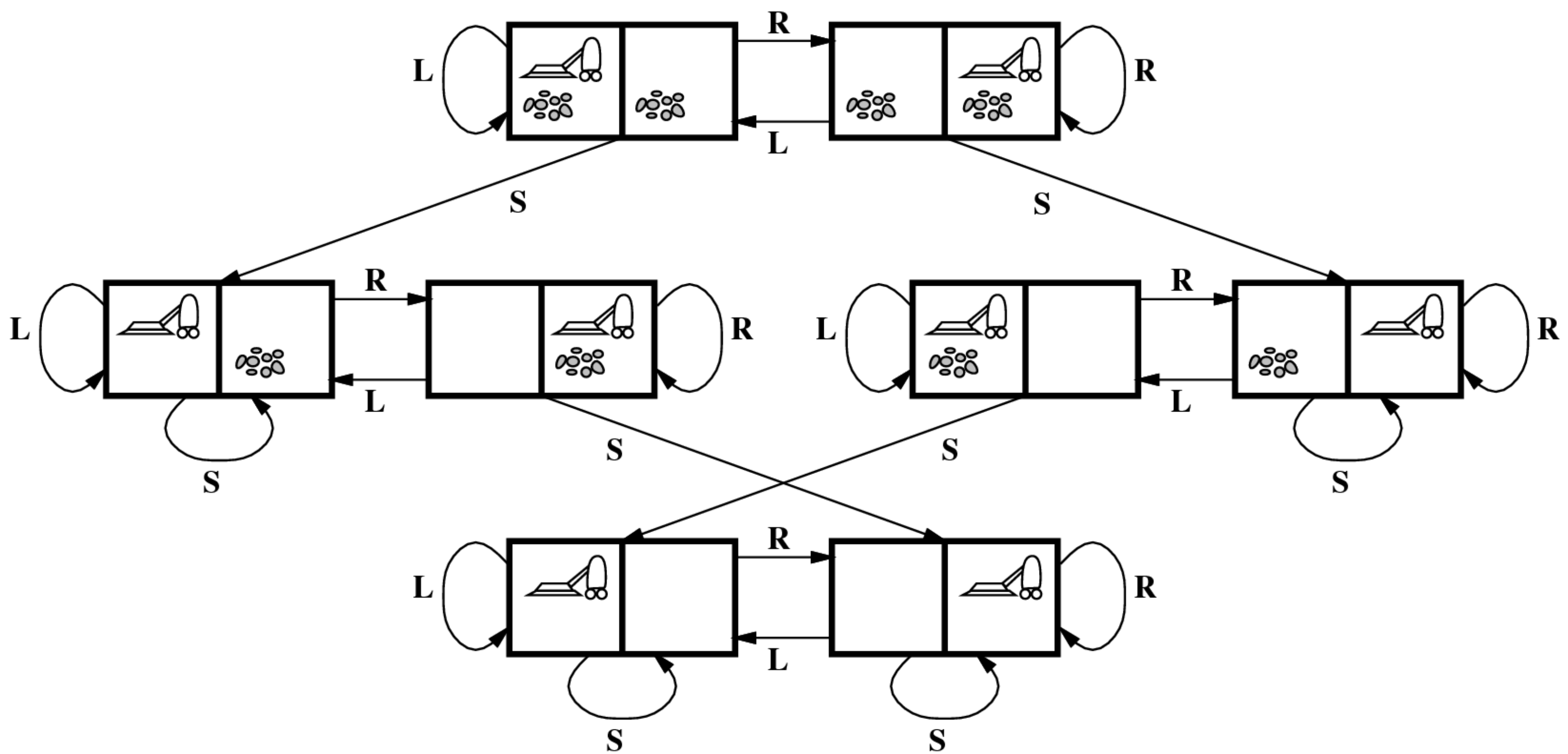
- 8-queens puzzle
- 8-puzzle
- Towers of Hanoi
- Cross-Word puzzles
- Sudoku
- Chess, Bridge, Scrabble puzzles
- Rubik's cube
- Sobokan
- Traveling Salesman Problem

No

- Tetris
  - dynamic not static
- Solitaire
  - only partially observable

State Space of a Problem

- State Space
  - the set of all states reachable from the initial state
  - implicitly defined by the initial state and the successor function



State Space of a Problem

- State Space
  - the set of all states reachable from the initial state
  - implicitly defined by the initial state and the successor function
- Path
  - a sequence of states connected by a sequence of actions
- Solution
  - a path that leads from the initial state to a goal state
- Optimal Solution
  - solution with the minimum path cost

Example: Romania

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arad, romania ⇄ bucharest, romania [Get Directions]

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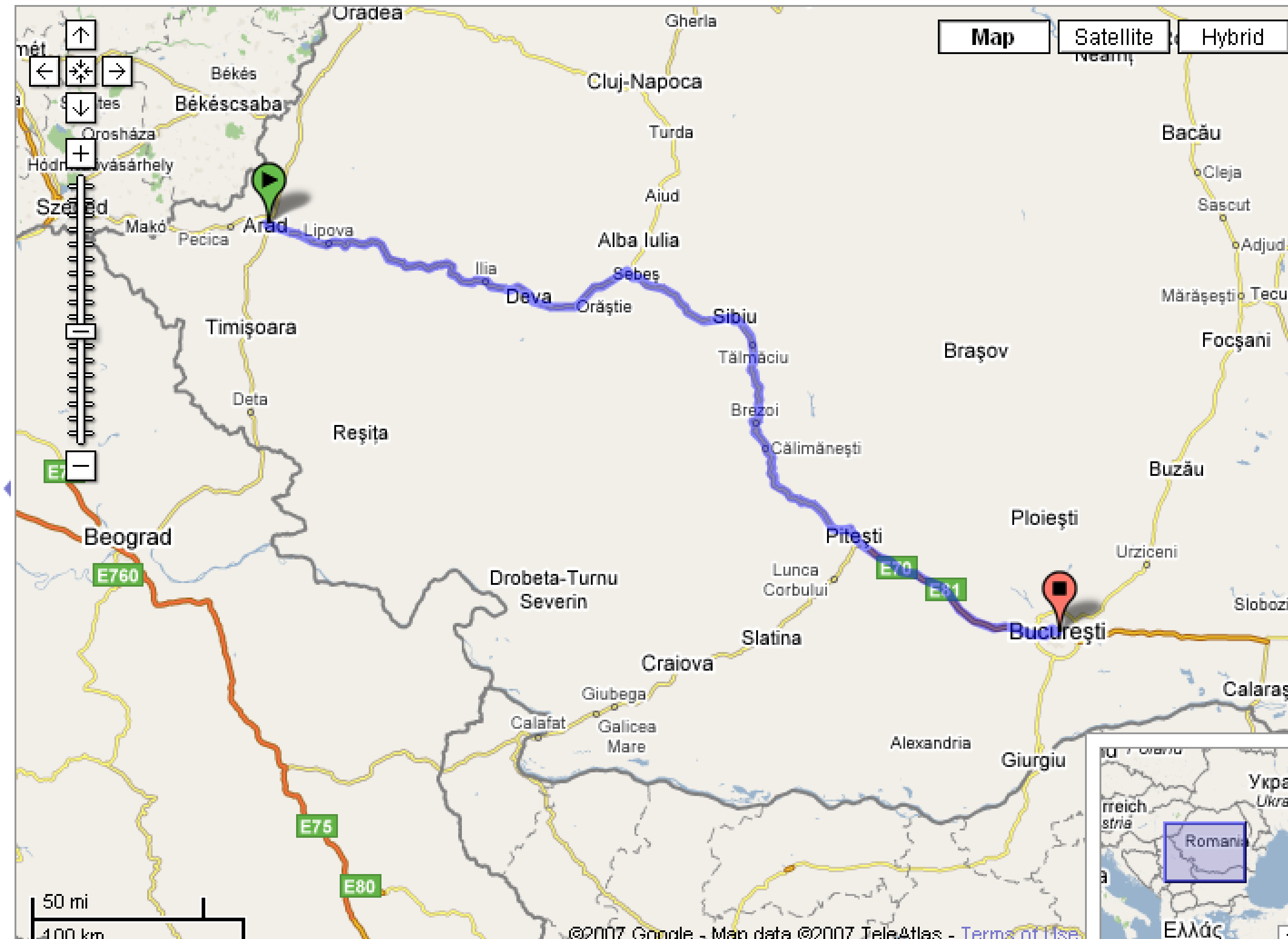
Get reverse directions

From: Arad Romania Edit

Drive: 549 km (about 9 hours 20 mins)

1. Head **south** on **79/E671** 0.7 km
2. Turn **left** toward **7/E68** 3.8 km
3. Slight **left** at **7/E68** (signs for **E68/DEVA**) 231 km
4. Turn **right** at **1/7/E68/E81** 0.5 km
5. Slight **left** to stay on **1/7/E68/E81** 32.2 km
6. Turn **right** at **1/7/E68** (signs for **BRAŞOV/RM. VALCEA**) 1.7 km
7. Turn **left** (signs for **BRAŞOV/RM. VALCEA**) 0.7 km
8. Turn **left** toward **7/E81** (signs for **BRAŞOV/RM. VALCEA**) 2.3 km
9. Turn **right** at **7/E81** 72.3 km
   Go through 1 roundabout
10. Turn **left** to stay on **7/E81** 6.2 km
11. Turn **left** at **E81** 84.3 km
12. Turn **left** at **65B/E81** 1.9 km
13. Turn **left** at **E70/E81** 104 km



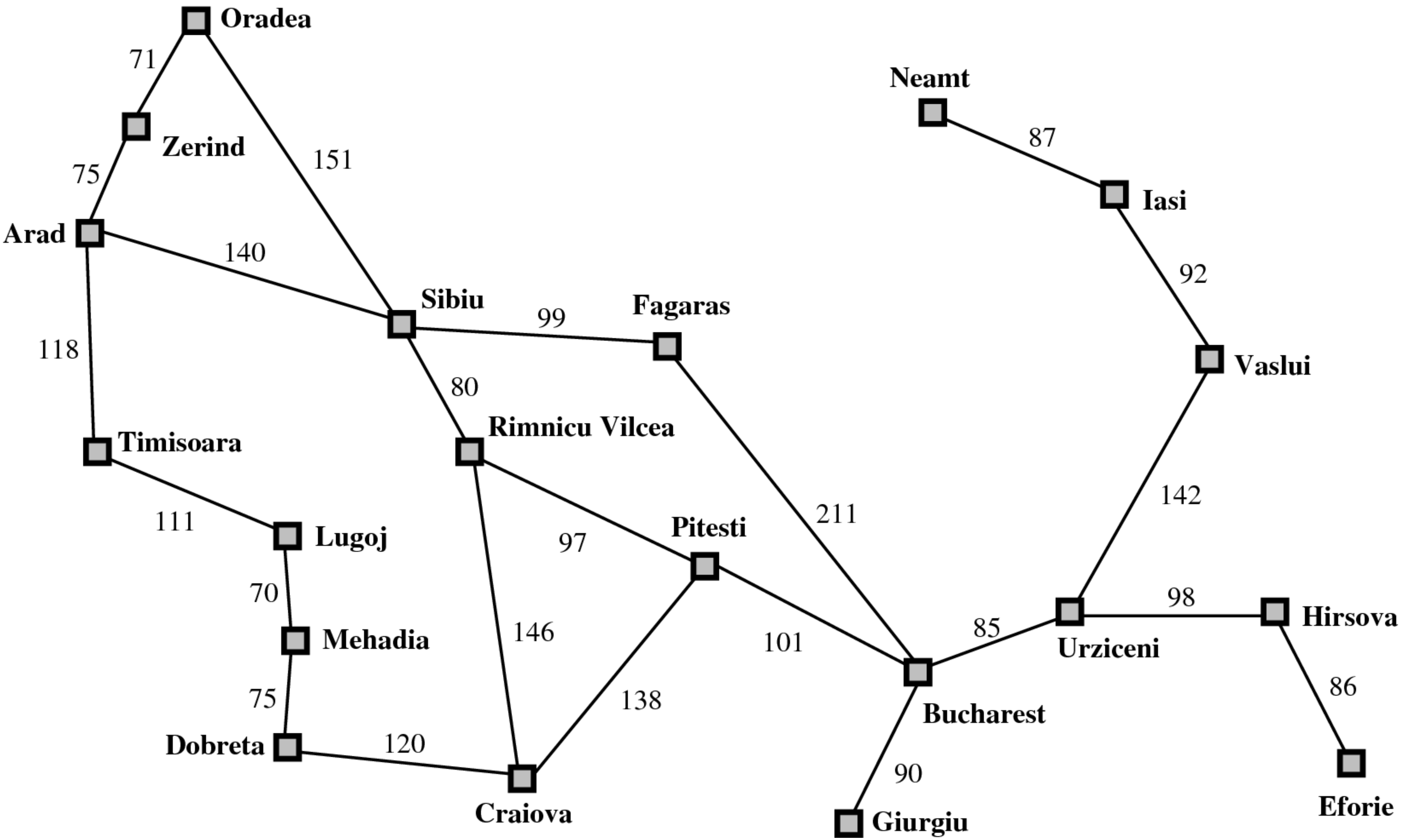
Selecting a State Space

Real world is absurdly complex

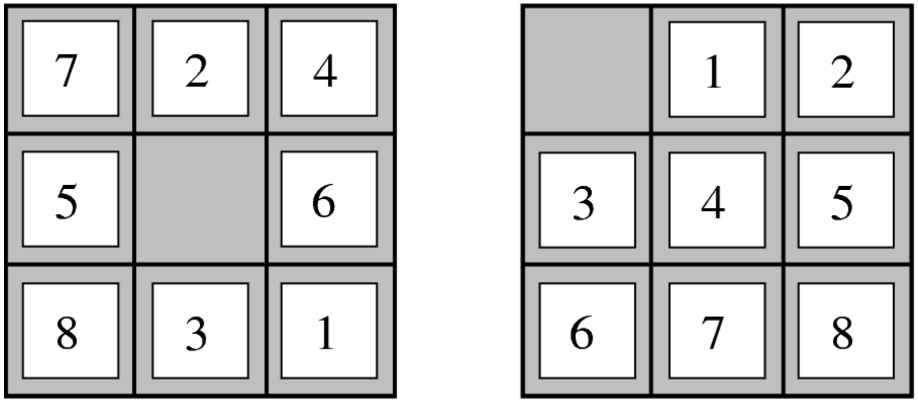
→ state space must be abstracted for problem solving

- (Abstract) state
  - corresponds to a set of real states
- (Abstract) action
  - corresponds to a complex combination of real actions
  - e.g., "go from Arad to Zerind" represents a complex set of possible routes, detours, rest stops, etc.
  - for guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind"
  - each abstract action should be "easier" than the original problem
- (Abstract) solution
  - corresponds to a set of real paths that are solutions in the real world

Example: Romania – State Space



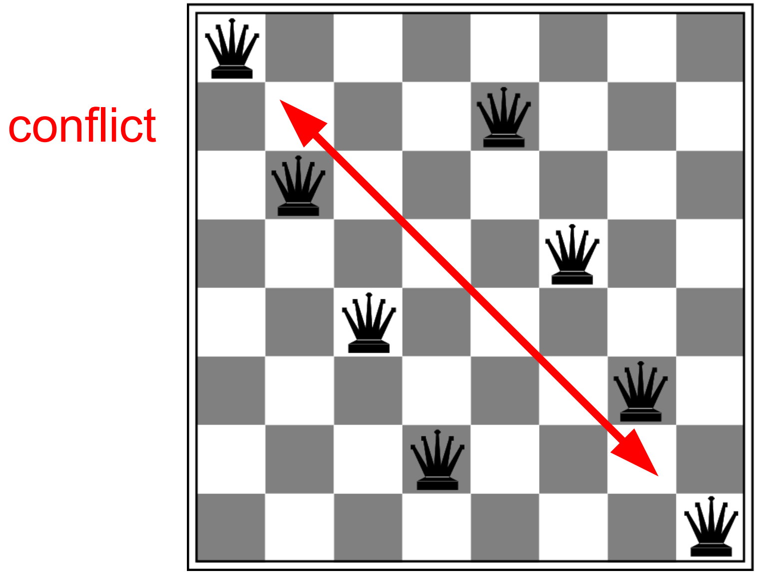
Example: The 8-puzzle

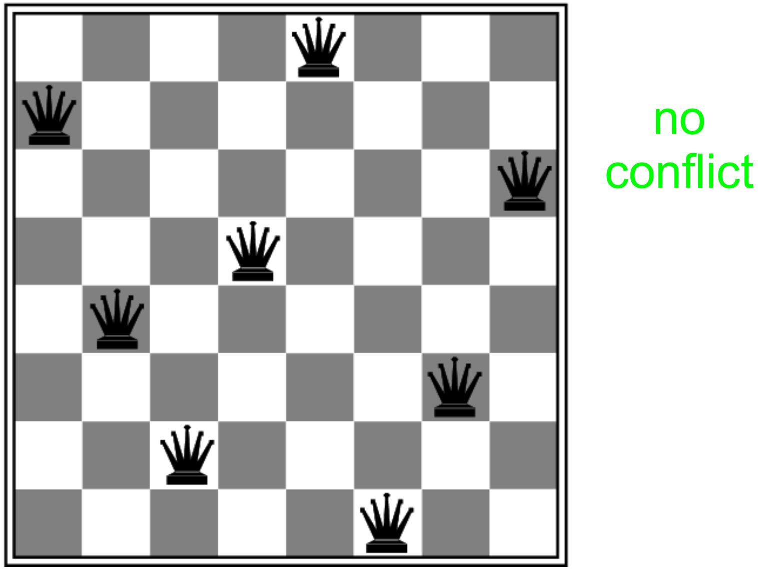


Start State Goal State

- states?
  - location of tiles
    - ignore intermediate positions during sliding
- goal test?
  - situation corresponds to goal state
- path cost?
  - number of steps in path (each step costs 1)
- actions?
  - move blank tile (left, right, up, down)
    - easier than having separate moves for each tile
    - ignore actions like unjamming slides if they get stuck

Example: The 8-Queens Problem



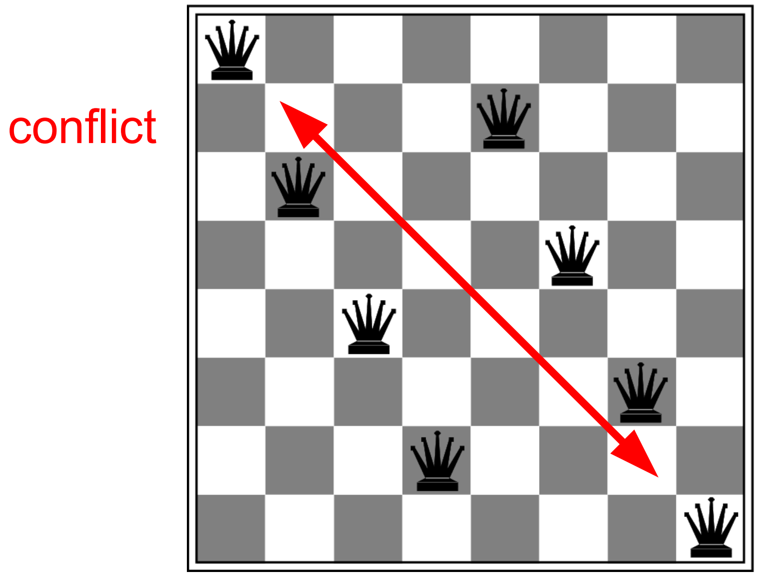


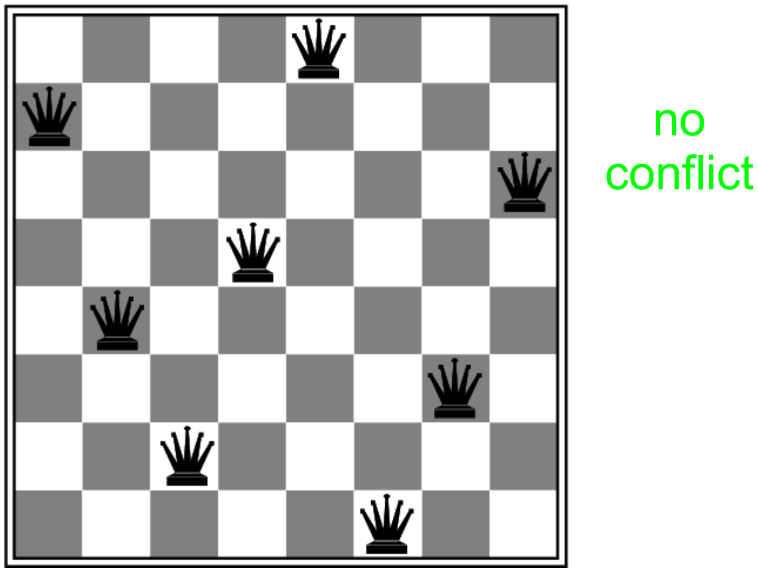
- states?
  - any configuration of 8 queens on the board
- goal test?
  - no pair of queens can capture each other
- actions?
  - move one of the queens to another square
- path cost?
  - not of interest here

inefficient complete-state formulation

$\rightarrow 64 \cdot 63 \cdot \ldots \cdot 57 \approx 3 \cdot 10^{14}$ states

Example: The 8-Queens Problem





- states?
  - n non-attacking queens in the left n columns
- goal test?
  - no pair of queens can capture each other
- actions?
  - add queen in column $n+1$
  - without attacking the others
- path cost?
  - not of interest here

more efficient incremental formulation

$\rightarrow$ only 2057 states

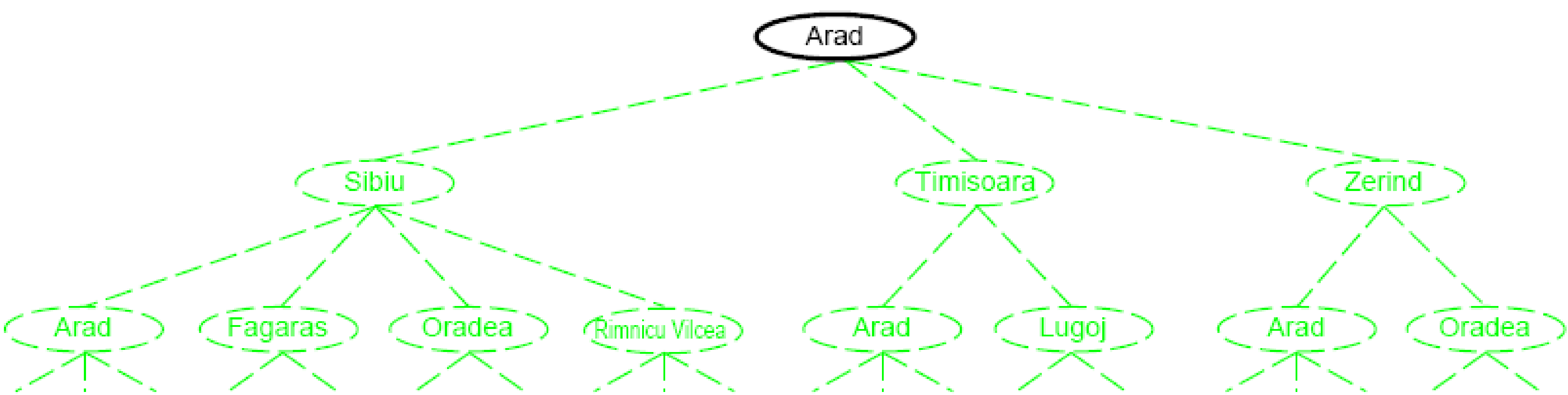
Tree Search Algorithms

- Treat the state-space graph as a tree
- Expanding a node
  - offline, simulated exploration of state space by generating successors of already-explored states (successor function)
- Search strategy
  - determines which node is expanded next
- General algorithm:

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

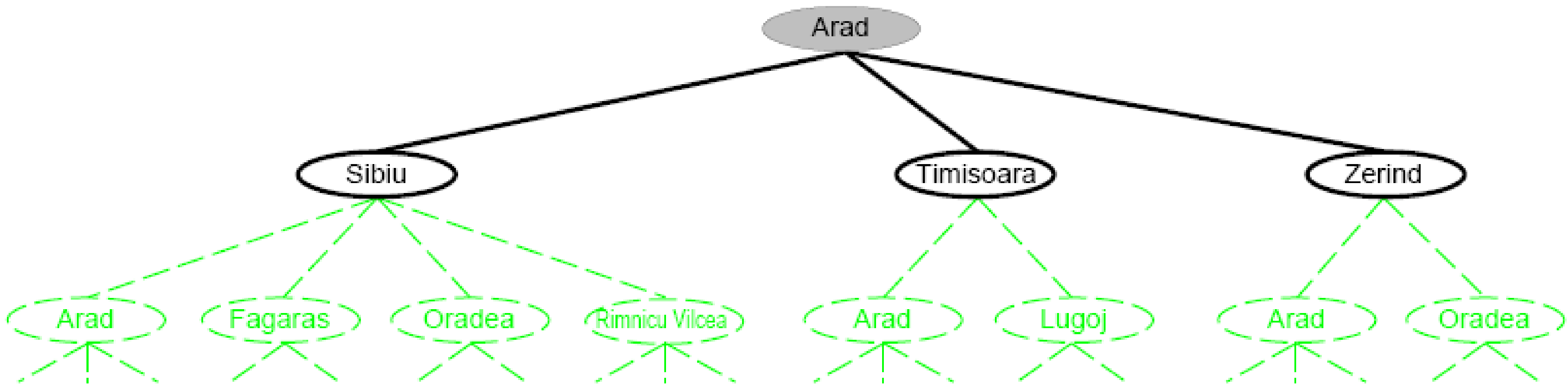
Tree Search Example

- Initial state: start with node Arad



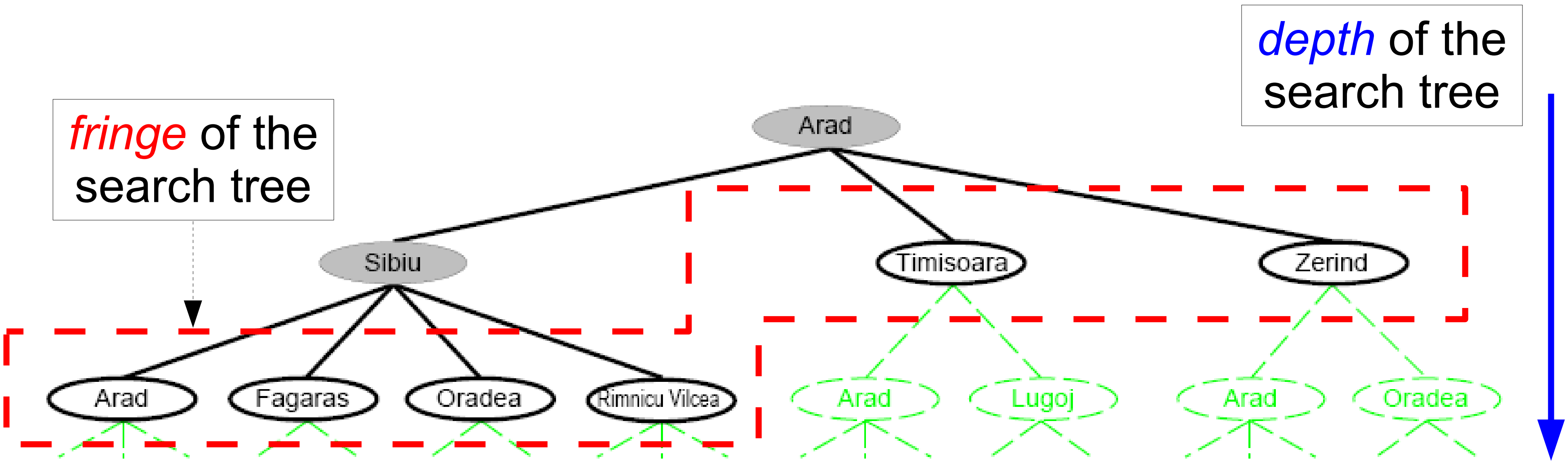
Tree Search Example

- Initial state: start with node Arad
- expand node Arad



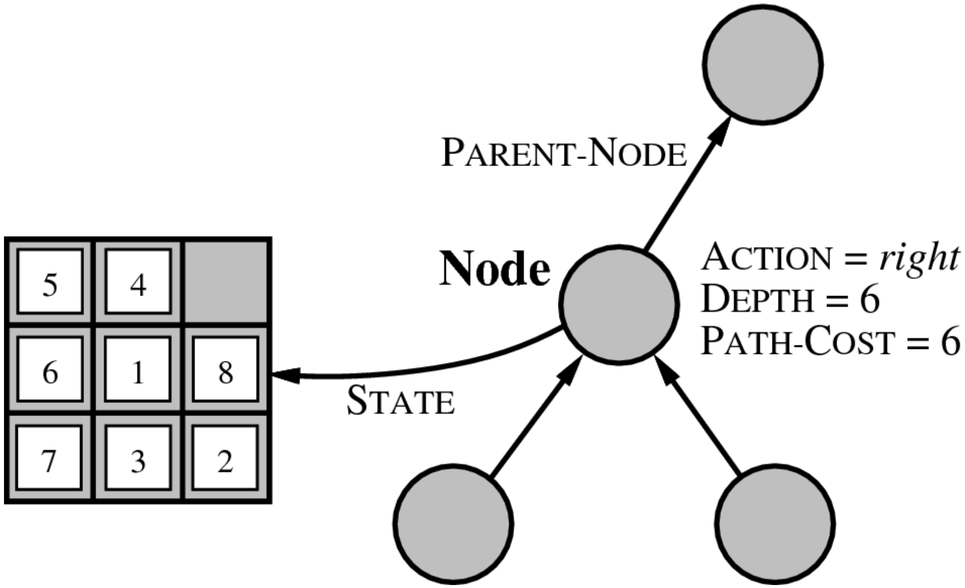
Tree Search Example

- Initial state: start with node Arad
- expand node Arad
- expand node Sibiu



States vs. Nodes

- State
  - (representation of) a physical configuration
- Node
  - data structure constituting part of a search tree
  - includes
    - state
    - parent node
    - action
    - path cost *g(x)*
    - depth
- Expand
  - creates new nodes
  - fills in the various fields
  - uses the successor function to create the corresponding states



Implementation: General Tree Search

function TREE-SEARCH(*problem*, *fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST(*problem*, STATE(*node*)) **then return** *node*
    fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

---

function EXPAND(*node*, *problem*) **returns** a set of nodes
  successors ← the empty set
  for each *action*, *result* **in** SUCCESSOR-FN(*problem*, STATE[*node*]) **do**
    s ← a new NODE
    PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
    PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node*, *action*, *s*)
    DEPTH[*s*] ← DEPTH[*node*] + 1
    add *s* to *successors*
  return *successors*

Search Strategies

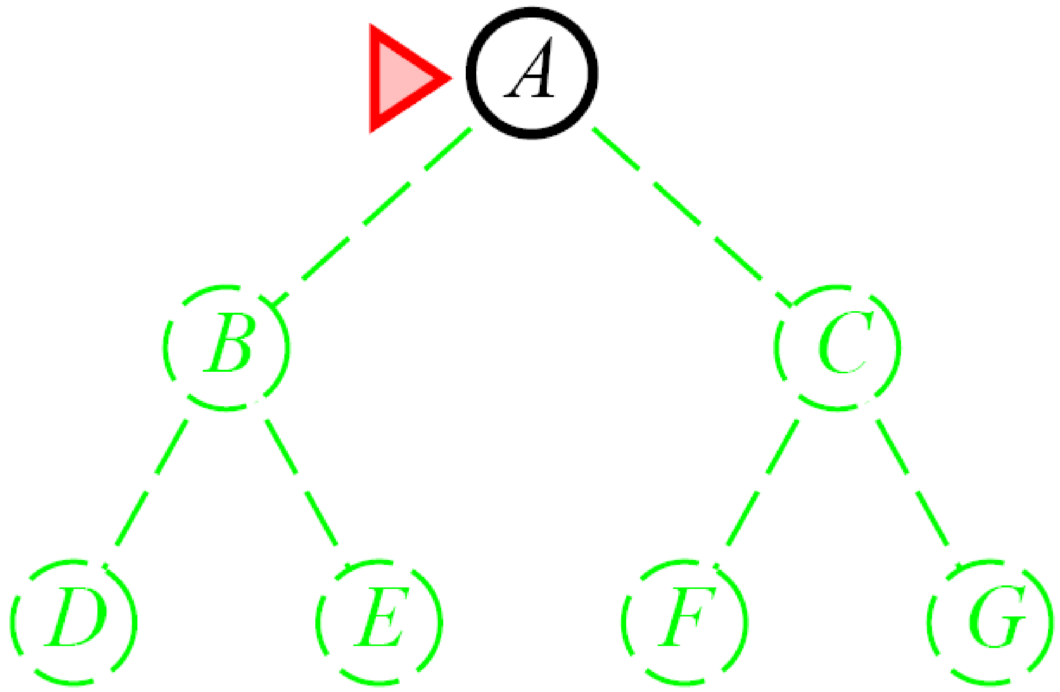
- A search strategy is defined by picking the order of node expansion
  - implementation in a queue
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

Search Strategies

- Uninformed (blind) search strategies use only the information available in the problem definition
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search
- Informed (heuristic) search strategies have knowledge that allows to guide the search to promising regions
  - Greedy Search
  - A* Best-First Search

Breadth-First Strategy

- Expand all neighbors of a node (breadth) before any of its successors is expanded (depth)
- Implemetation:
  - expand the shallowest unexpanded node
  - fringe is a FIFO queue (first-in-first-out, new nodes go to end of queue)



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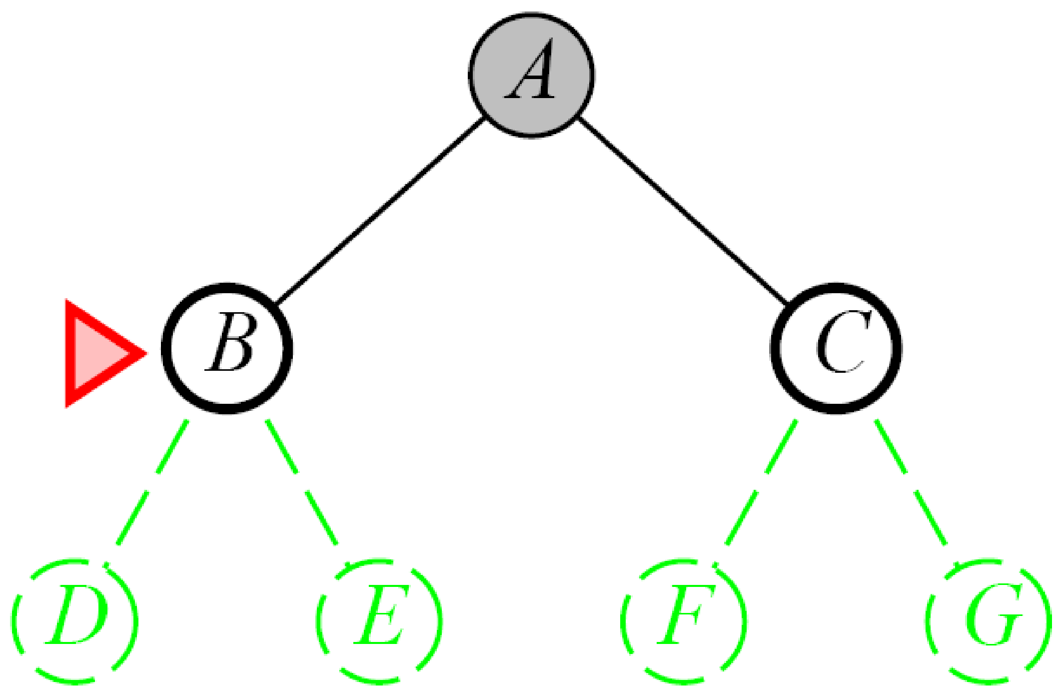

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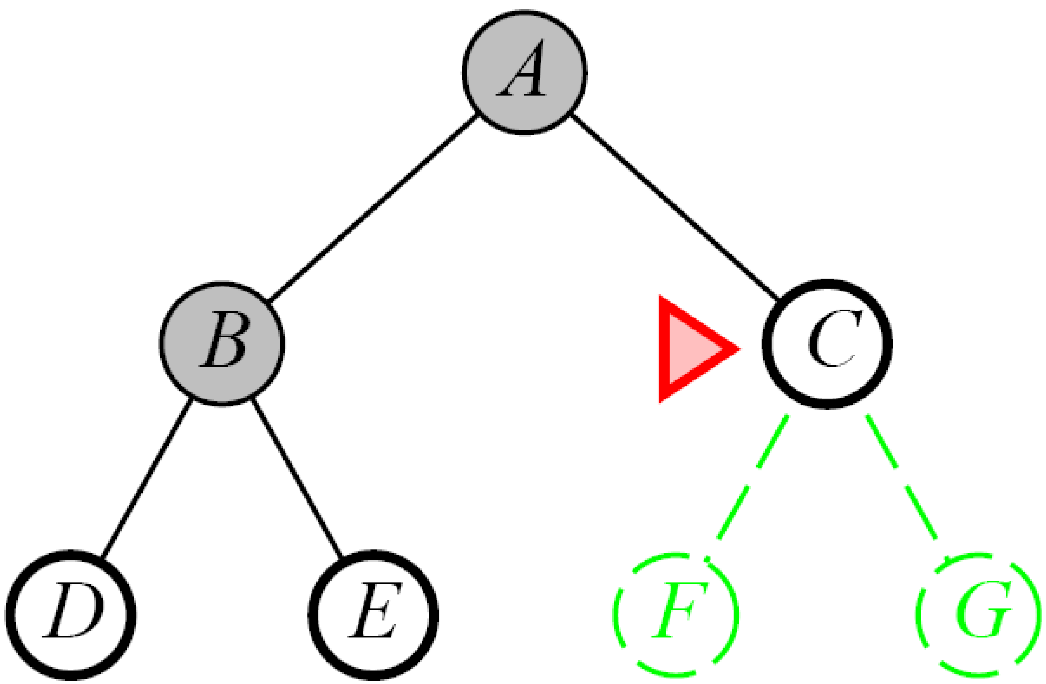

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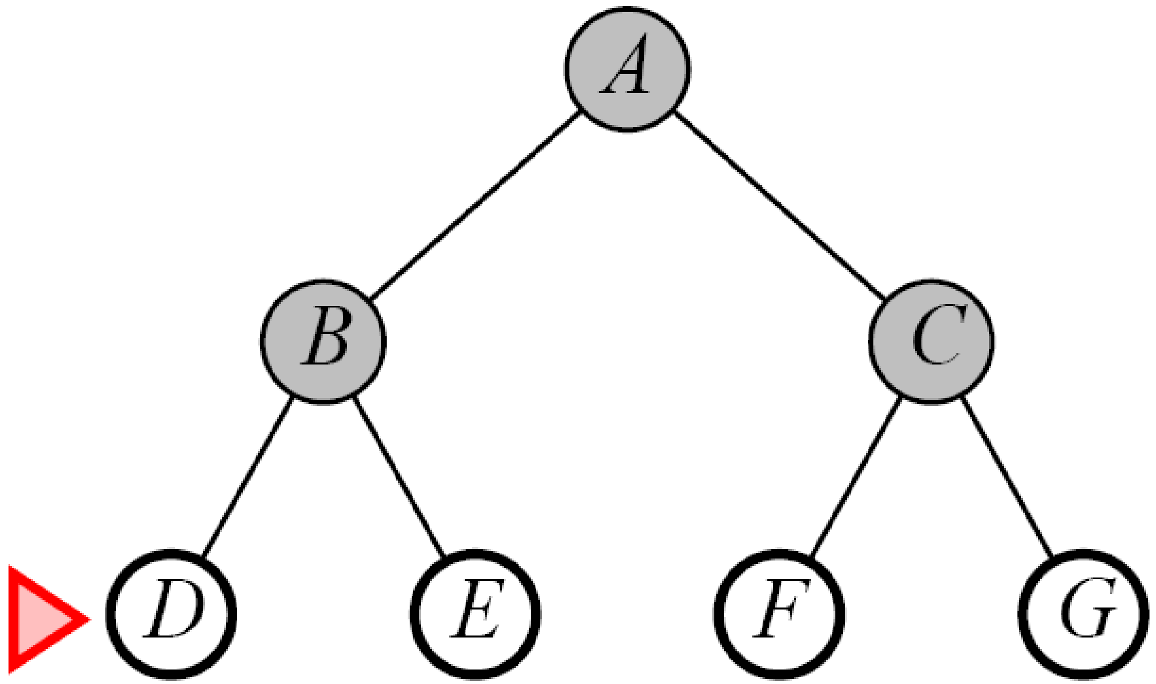


Properties of Breadth-First Search

- Completeness
  - Yes (if b is finite)
- Time Complexity
  - each depth has b times as many nodes as the previous
  - each node is expanded
  - except the goal node in level d
    - worst case: goal is last node in this level

$$\Rightarrow 1+b+b^2+b^3+\ldots+b^d+(b^{(d+1)}-b)=O(b^{d+1})$$

- Space Complexity
  - every node must remain in memory
    - it is either a fringe node or an ancestor of a fringe node
    - in the end, the goal will be in the fringe, and its ancestors will be needed for the solution path

$$\Rightarrow O(b^{d+1})$$

- Optimality
  - Yes, for uniform costs (e.g., if cost = 1 per step)

Combinatorial Explosion

- Breadth-first search
  - branching factor $b = 10$, 10,000 nodes/sec, 1000 bytes/node

Depth	Nodes	Time	Memory
2	1100	.11 secs	1 MB
4	111 100	11 secs	106 MB
6	10^7	19 mins	10 GB
8	10^9	31 hours	1 TB
10	10^{11}	129 days	101 TB
12	10^{13}	35 years	10 PetaBytes
14	10^{15}	3523 years	1 ExaByte

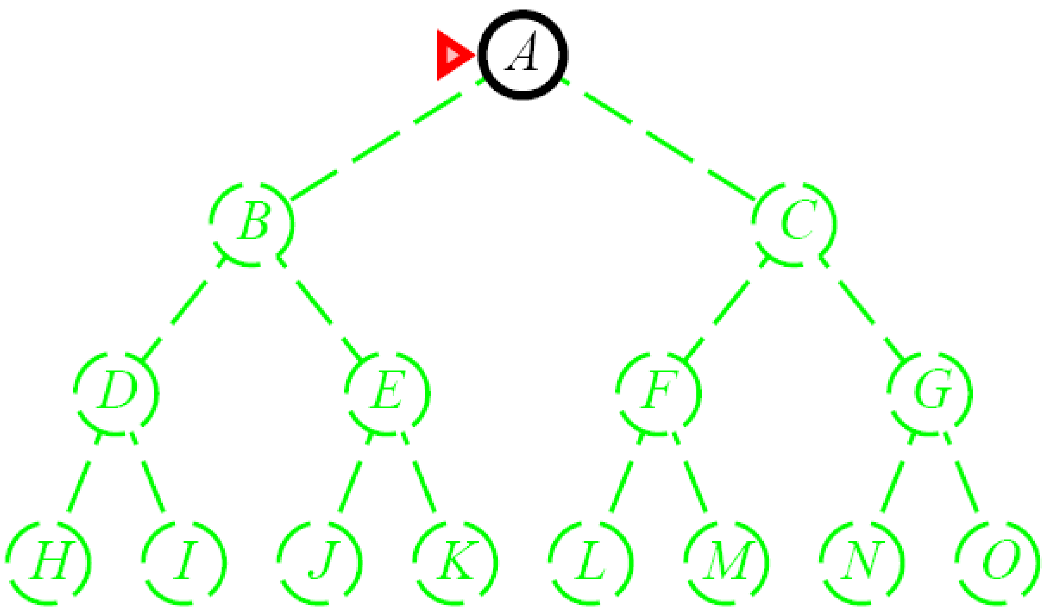
  - Space is the bigger problem
    - can easily generate nodes at 100MB/sec $\Rightarrow$ 24hrs = 8640 GB

Uniform-Cost Search

- Breadth-first search can be generalized to cost functions
  - each node now has associated costs
  - costs accumulate over path
  - instead of expanding the shallowest path, expand the least-cost unexpanded node
  - breadth-first is special case where all costs are equal
- Implementation
  - fringe = queue ordered by path cost
- Completeness
  - yes, if each step has a positive cost (cost ≥ ε)
  - otherwise infinite loops are possible
- Space and Time complexity $b^{1+O(\lfloor C^*/\epsilon \rfloor)}$
  - number of nodes with costs < costs of optimal solution C^*
- Optimality
  - Yes – nodes expanded in increasing order of path costs

Depth-First Strategy

- Expand all successors of a node (depth) before any of its neighbors is expanded (breadth)
- Implemetation:
  - expand the deepest unexpanded node
  - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)



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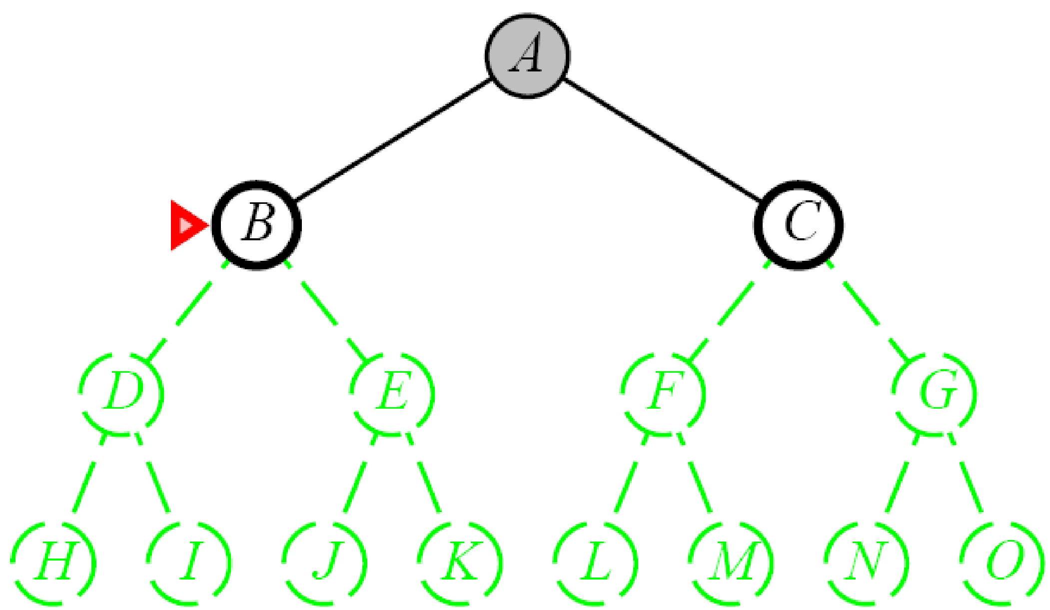

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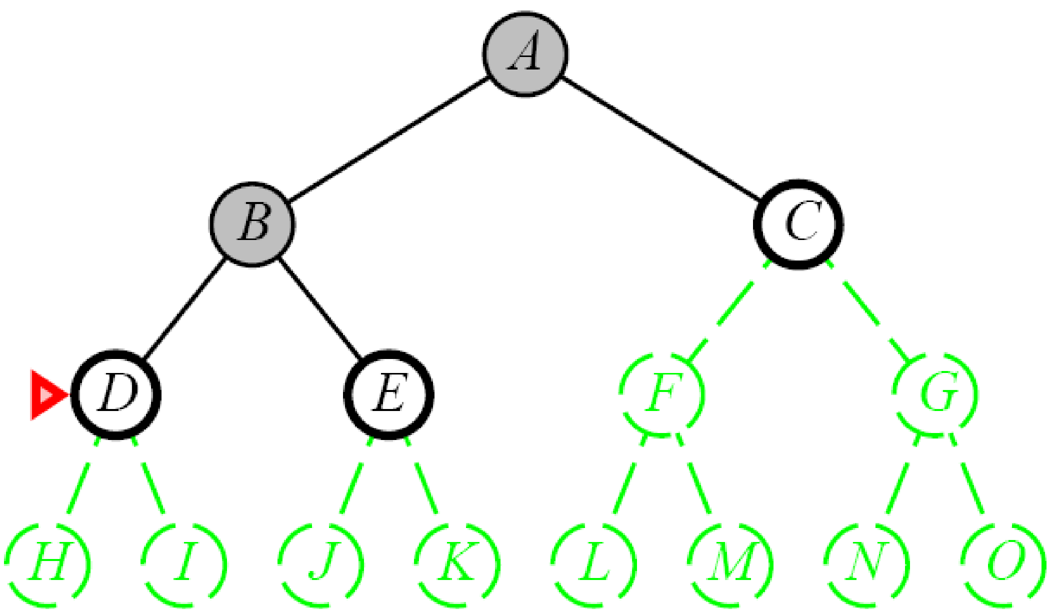

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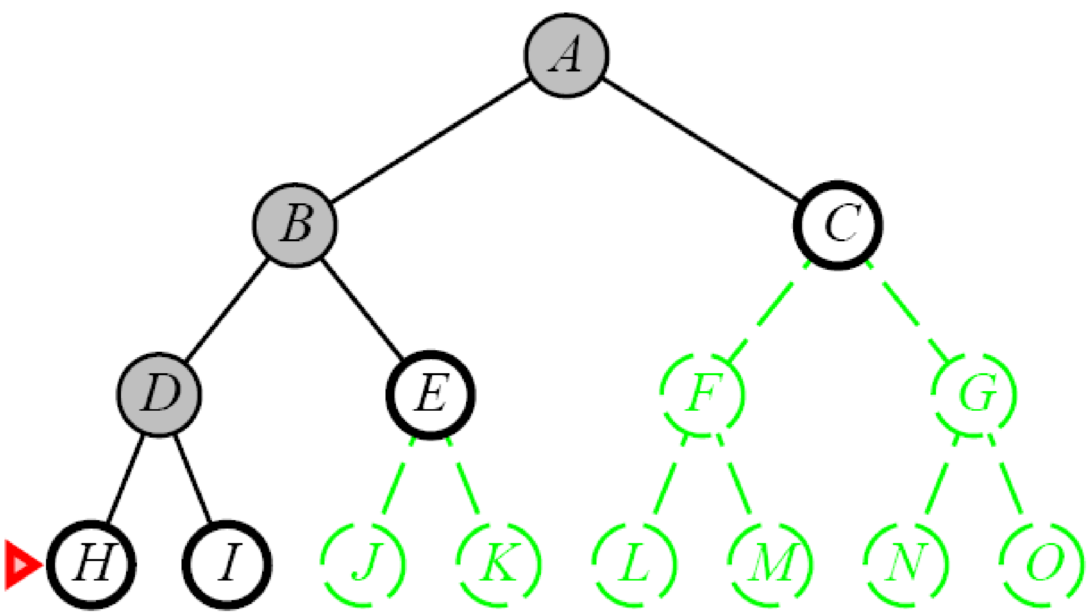

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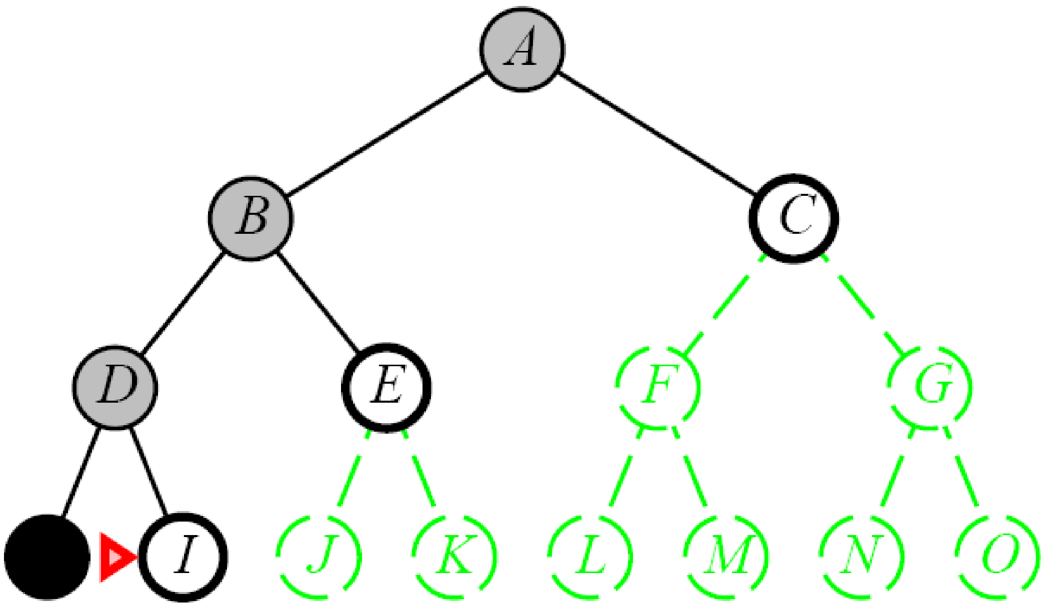

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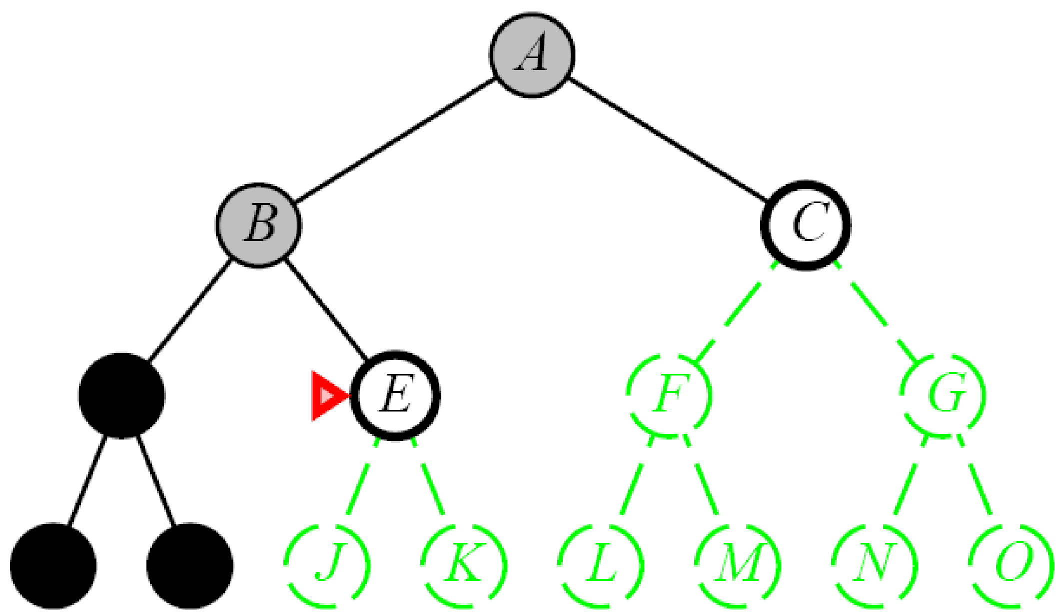

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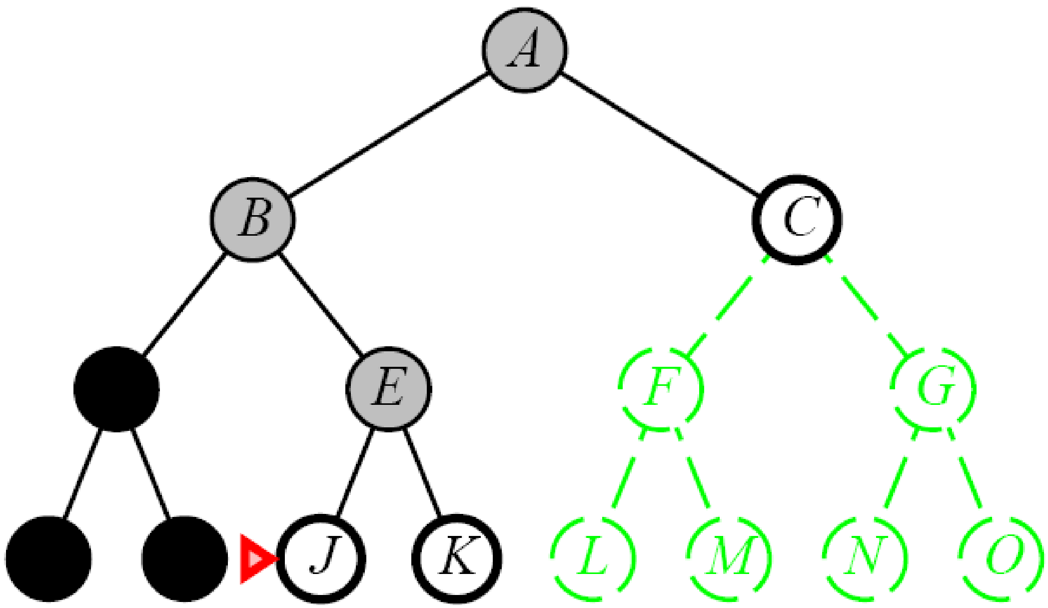

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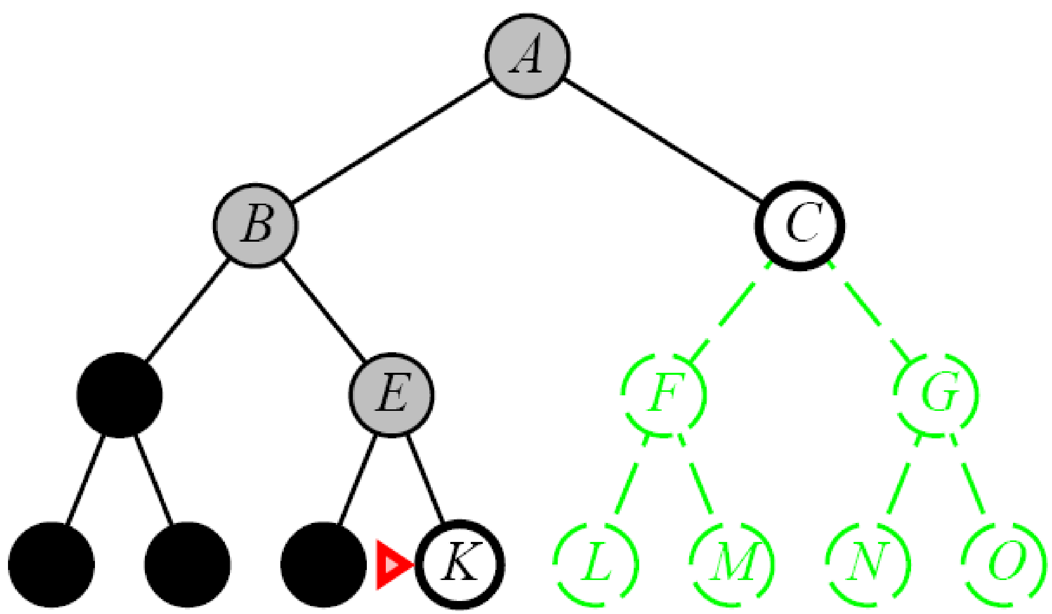

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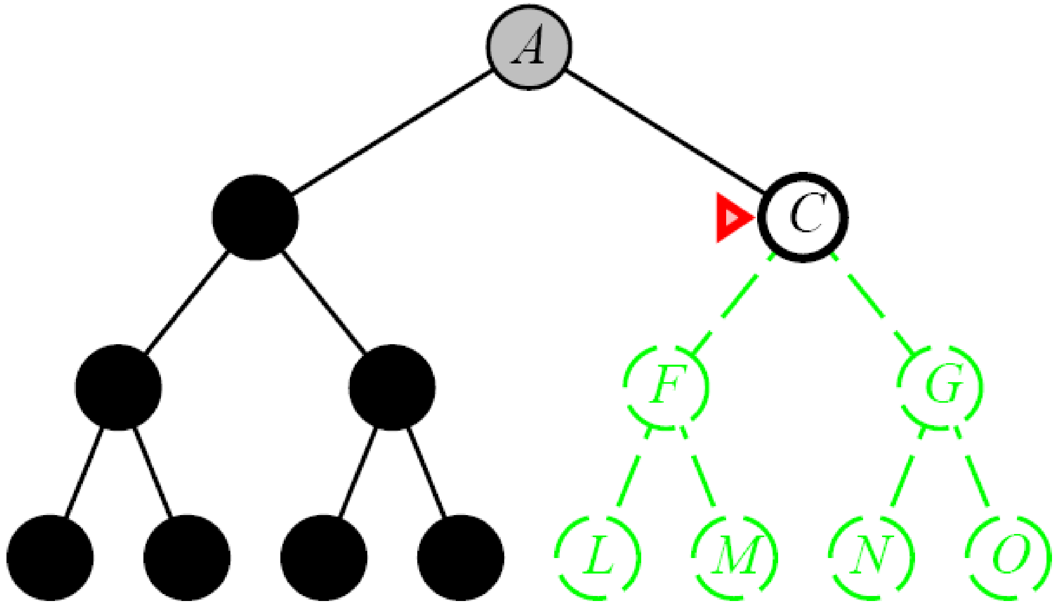

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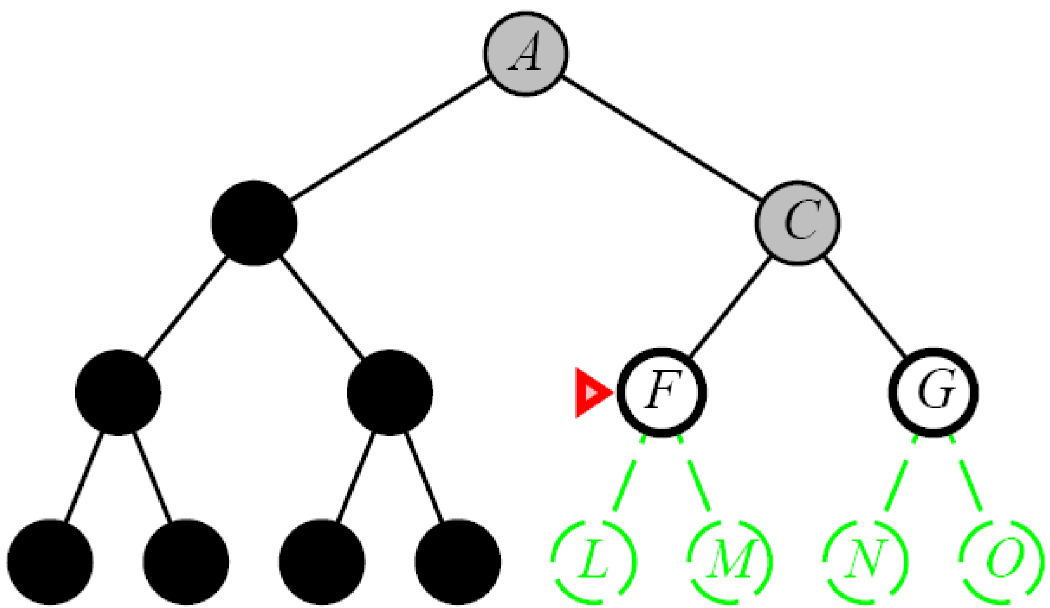

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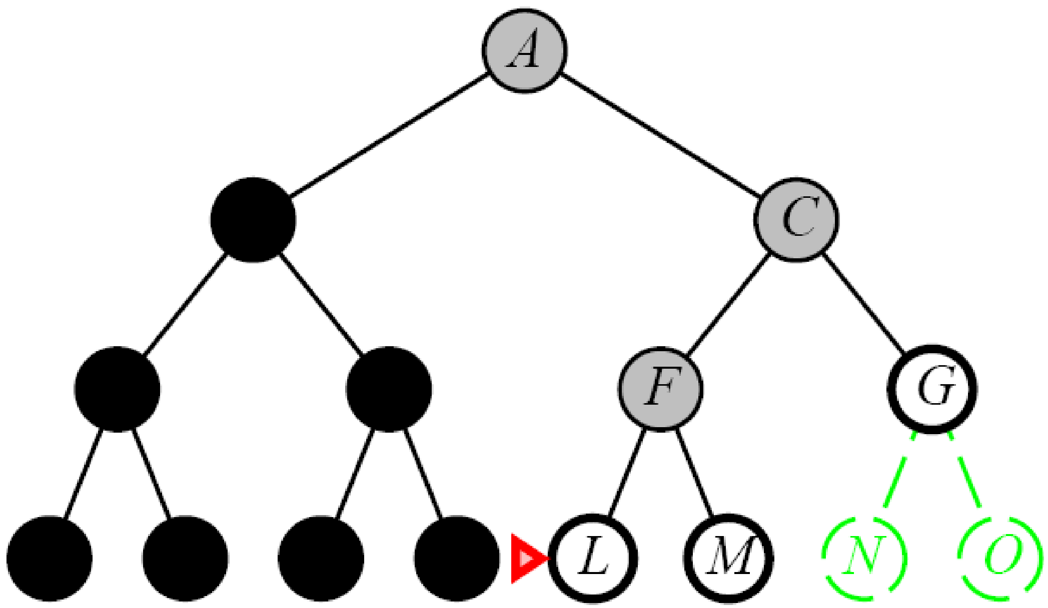

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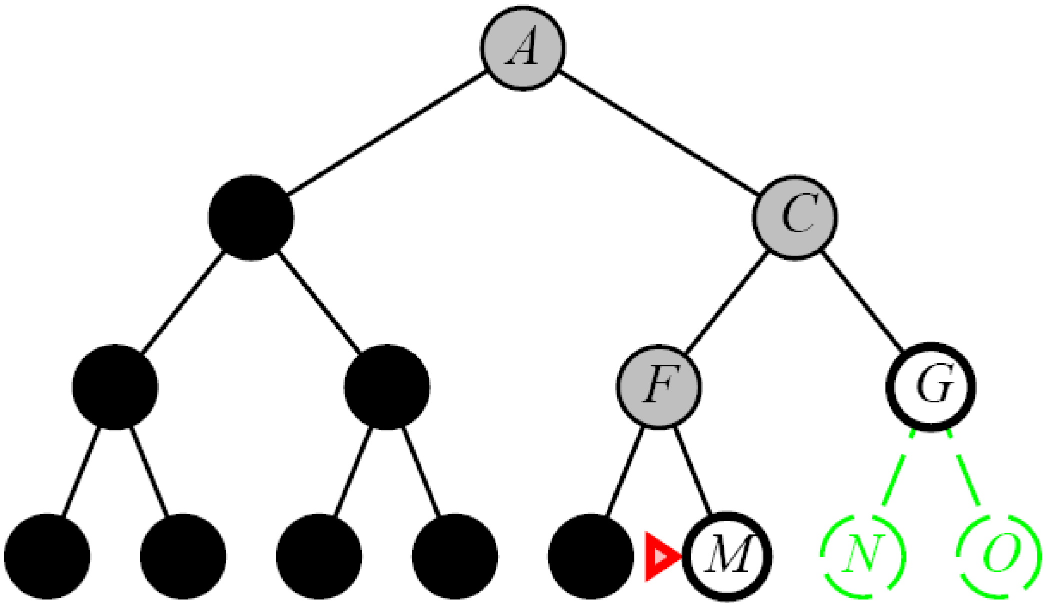
  - fringe is a LIFO queue (last-in-first-out, new nodes at begin of queue)



Properties of Depth-First Search

- Completeness
  - No, fails in infinite-depth search spaces and spaces with loops
  - complete in finite spaces if modified so that repeated states are avoided
- Time Complexity
  - has to explore each branch until maximum depth $m \Rightarrow O(b^m)$
  - terrible if $m > d$ (depth of goal node)
  - but may be faster than breadth-first if solutions are dense
- Space Complexity
  - only nodes in current path and their unexpanded siblings need to be stored

  $\Rightarrow$ only linear complexity $O(m \cdot b)$
- Optimality
  - No, longer (more expensive) solutions may be found before shorter (cheaper) ones

Backtracking Search

Even more space-efficient variant

- does not store all expanded nodes, but only the current path
  $\Rightarrow O(m)$
  - if no further expansion is possible, go back to the predecessor
  - each node is able to generate the *next* successor
- only needs to store and modify one state
  - actions can do and undo changes on this one state

Depth-limited Search

- depth-first search is provided with a depth limit l
  - nodes with depths $d > l$ are not considered → incomplete
  - if $d < l$ it is not optimal (like depth-first search)
  - time complexity $O(b^l)$, space complexity $O(bl)$

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST(problem, STATE[node]) then return node
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
      result ← RECURSIVE-DLS(successor, problem, limit)
      if result = cutoff then cutoff-occurred? ← true
      else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

Iterative Deepening Search

- Main problem with depth-limited search is setting of l
- Simple solution:
  - try all possible $l = 0, 1, 2, 3, \ldots$

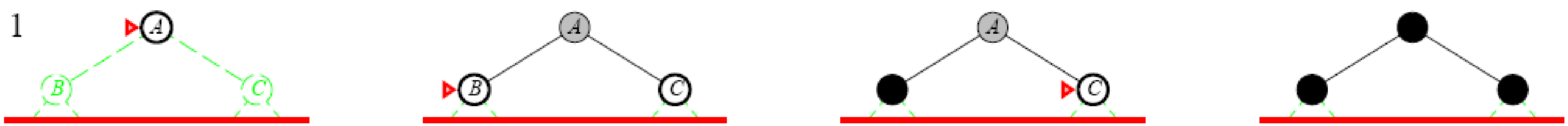
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

  - costs are dominated by the last iteration, thus the overhead is marginal

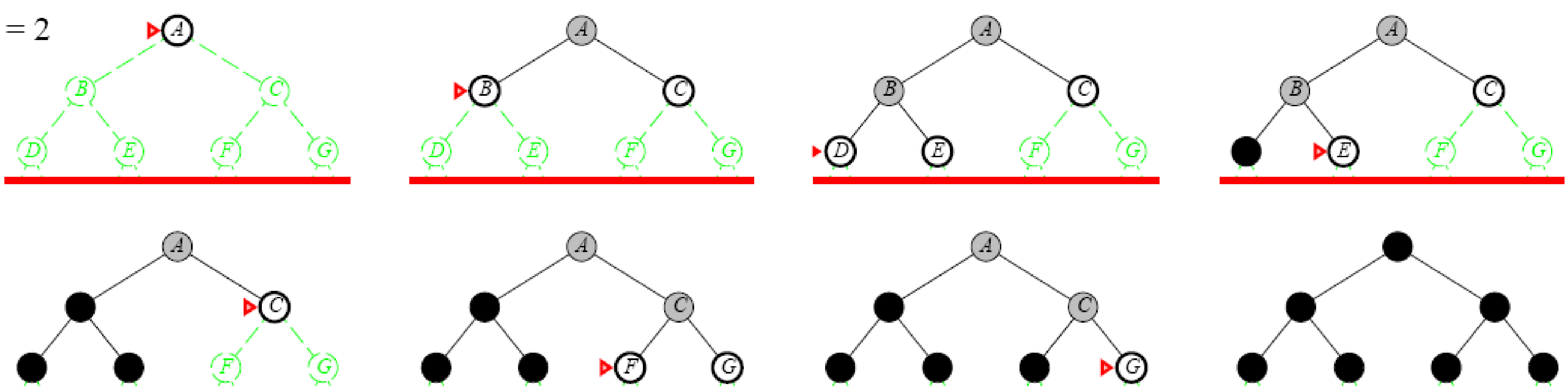
Iterative Deepening Search

Limit = 0

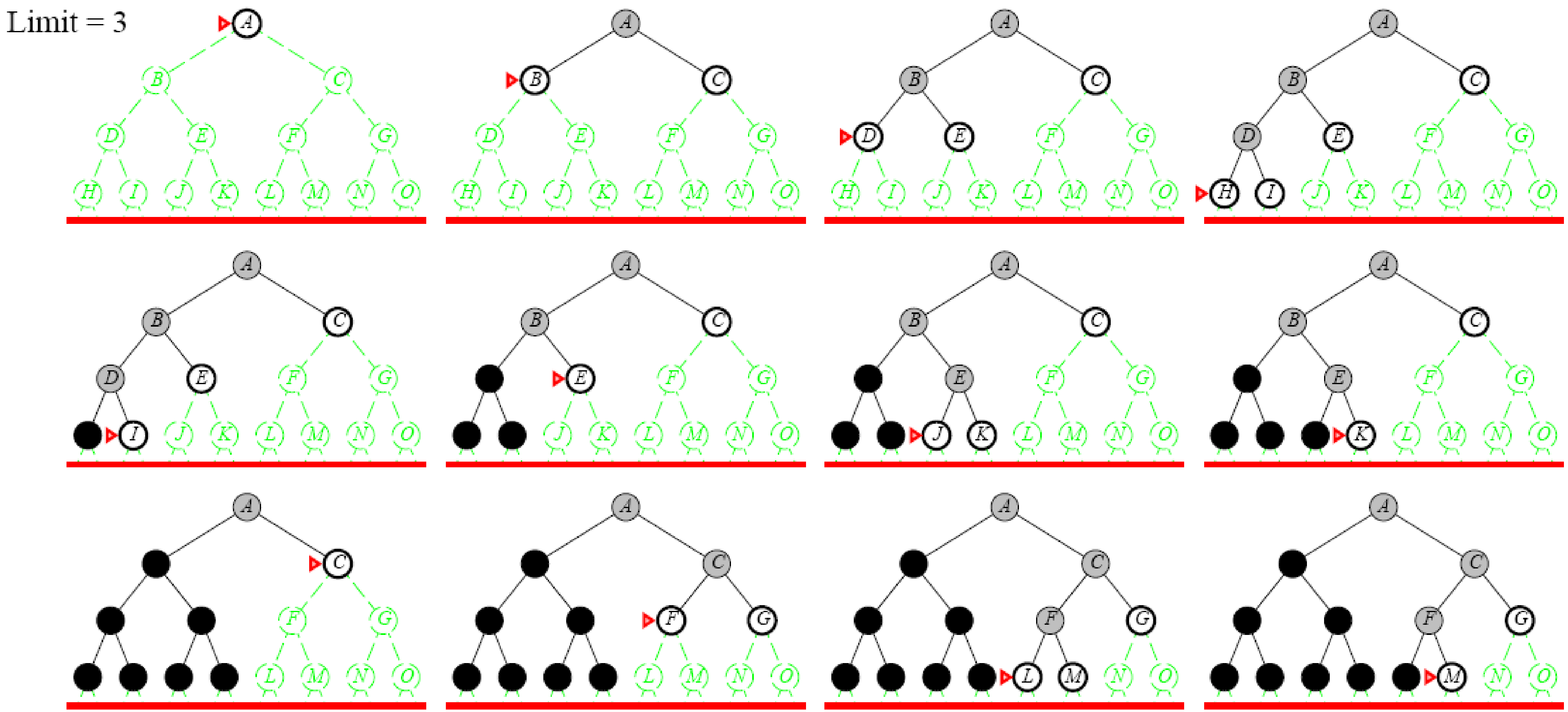
Limit = 1



Limit = 2



Iterative Deepening Search



Properties of Iterative Deepening Search

- Completeness
  - Yes (no infinite paths)
- Time Complexity
  - first level has to be searched d times
  - last level has to be searched once

$$\Rightarrow d \cdot b + (d-1) b^2 + \ldots + 1 \cdot b^d = \sum_{i=1}^{d} (d-i+1) \cdot b^i$$

- Space Complexity

  $\Rightarrow$ only linear complexity $O(bd)$
- Optimality
  - Yes, the solution is found at the minimum depth

$\Rightarrow$ combines advantages of depth-first and breadth-first search

Comparison of Time Complexities

Worst-case (goal is in right-most node at level d)

- Depth-Limited Search

$$N_{DLS}=b+b^2+\ldots+b^d=\sum_{i=1}^{d} b^i$$

- Iterative Deepening

$$N_{IDS}=d\cdot b+(d-1)b^2+\ldots+1\cdot b^d=\sum_{i=1}^{d}(d-i+1)\cdot b^i$$

Example: $b = 10$, $d = 5$

$$N_{DLS}=10+100+1000+10{,}000+100{,}000=111{,}110$$

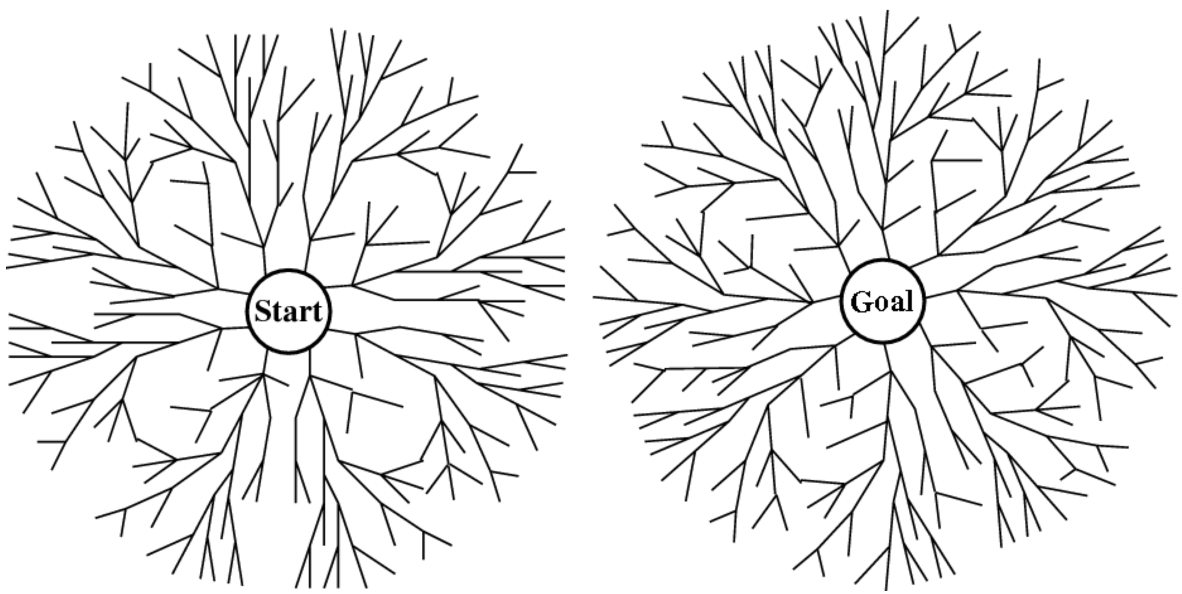
$$N_{IDS}=50+400+3000+20{,}000+100{,}000=123{,}450$$

Overhead of IDS only ca. 10%

Bidirectional Search

- Perform two searches simultaneously
  - forward starting with initial state
  - backward starting with goal state

  check whether generated node is in fringe of the other search



- Properties
  - reduction in complexity $(b^{d/2} + b^{d/2} \ll b^d)$
  - only possible if actions can be reversed
  - search paths may not meet for depth-first bidirectional search

Summary of Algorithms

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

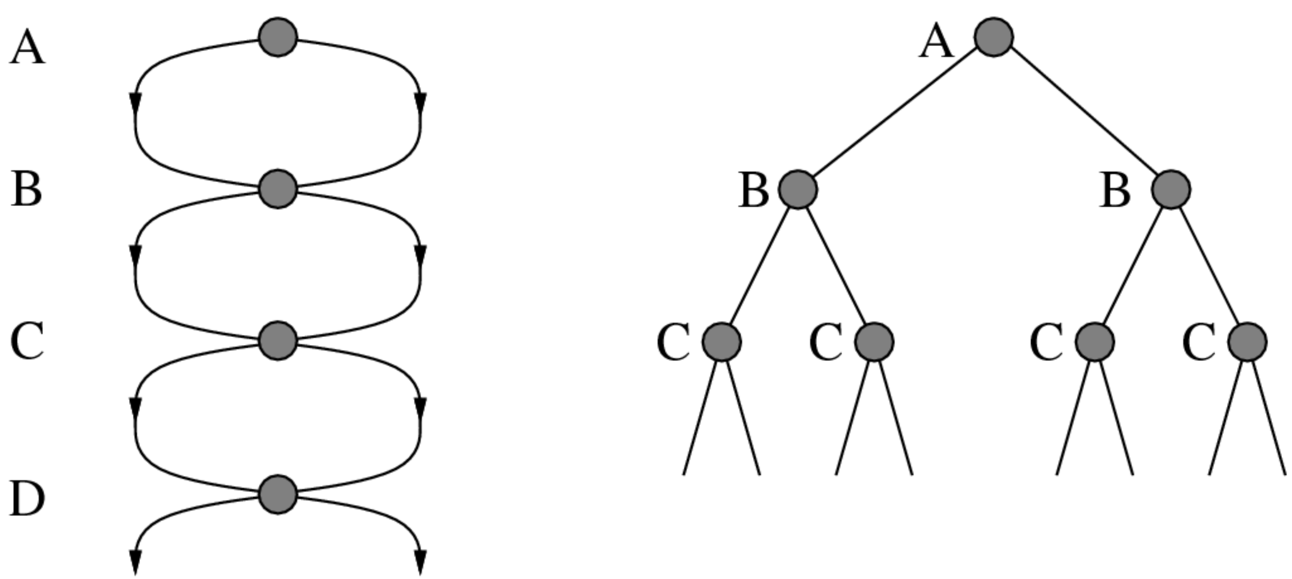
Repeated States

- Failure to detect repeated states can turn a linear problem into an exponential one!

Ribbon Example

- two connections from each state to the next

d states but state space is 2^d



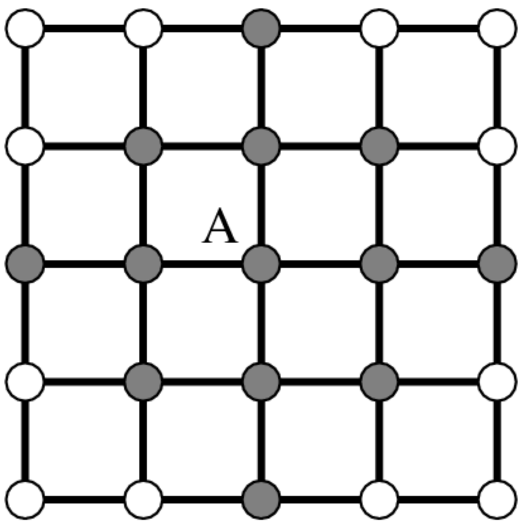
(a) (b)

Repeated States

- Failure to detect repeated states can turn a linear problem into an exponential one!

(more realistic) Grid Example

- each square on grid has 4 neighboring states in
- thus, game tree w/o repetitions has 4^d nodes
- but only about $2d^2$ different states are reachable in d steps



Graph Search

- remembers the states that have been visited in a list *closed*
  - Note: the fringe list is often also called the open list

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end
```

- Example:
  - Dijkstra's algorithm is the graph-search variant of uniform cost search

Assumptions about the Environment

- static
  - we do not pay attention to possible changes in the environment
- observable
  - we can at least observe our initial state
- discrete
  - possible actions can be enumerated
- deterministic
  - the expected outcome of an action is always identical to the true outcome
  - once we have a plan, we can execute it „with eyes closed“

→ easiest possible scenario

Problems with Partial Information

- Single-State Problem

  deterministic, fully observable
  - agent knows exactly which state it will be in
  - solution is a sequence
- Conformant Problem (sensorless problem)

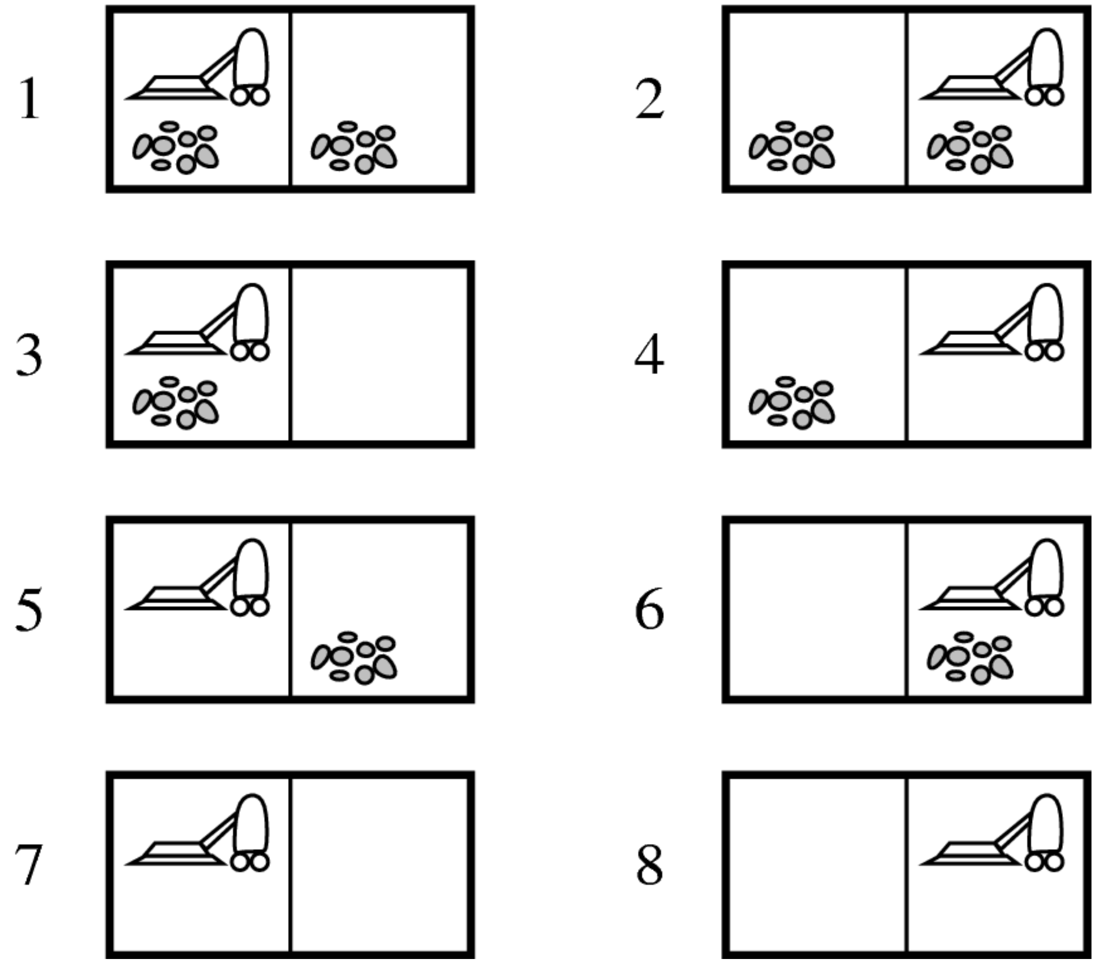
  non-observable
  - agent may have no idea where it is
  - solution (if any) is a sequence
- Contingency Problem

  nondeterministic and/or partially observable
  - percepts provide new information about current state
  - solution is a contingent plan (tree) or a policy
  - search and execution often interleaved
- Exploration Problem

  state-space is not known → Reinforcement Learning

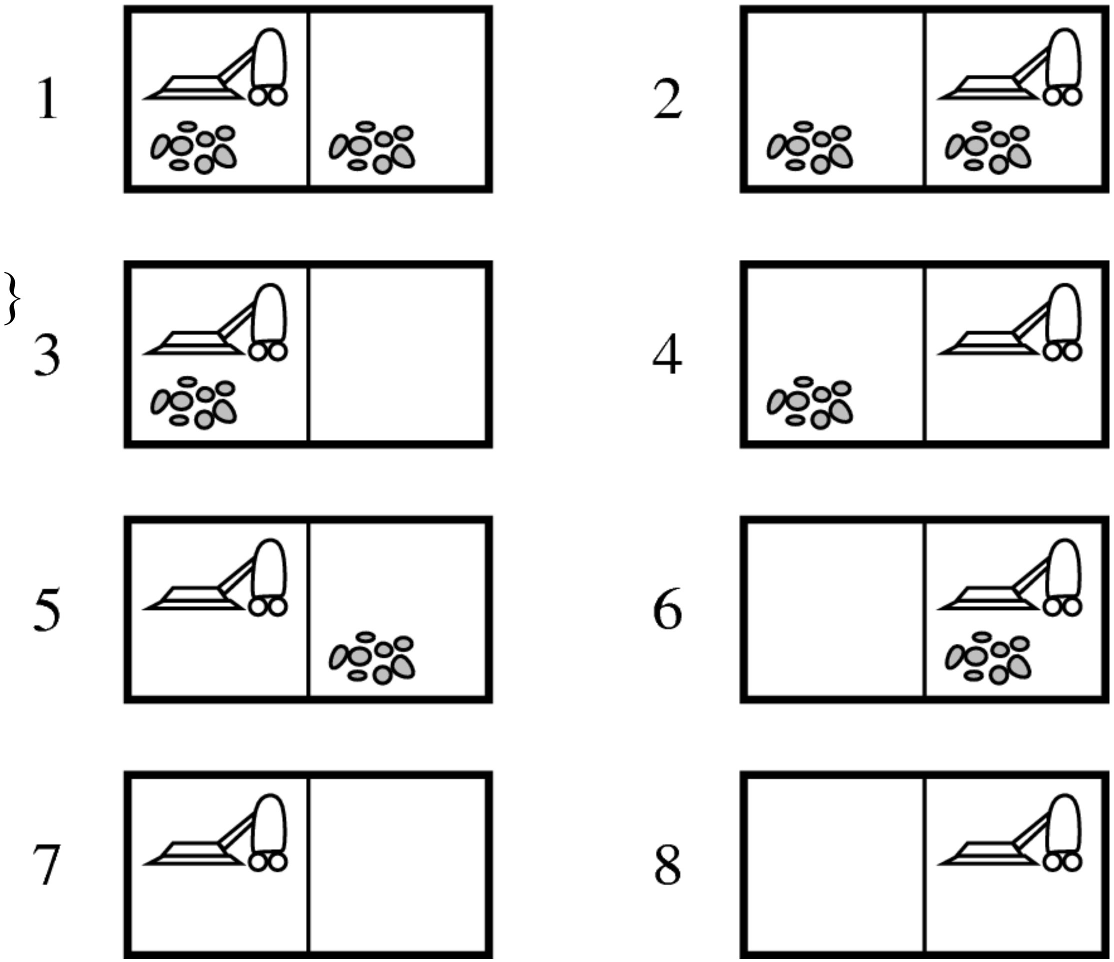
Example: Vacuum World

- Single-state Problem
  - start in #5
  - goal
    - no dirt
- Solution
  - [*Right*, *Suck*]



Example: Vacuum World

- Conformant Problem
  - start in any state (we can't sense)
    - *start* ← {1,2,3,4,5,6,7,8}
  - actions
    - e.g., *Right* goes to {2,4,6,8}
  - goal
    - no dirt
- Solution
  - [*Right, Suck, Left, Suck*]



Example: Vacuum World

- Contingency Problem
  - start in #5
  - indeterministic actions
    - *Suck* can dirty a clean carpet
  - sensing
    - dirt at current location?
  - goal
    - no dirt
- Solution
  - [*Right*, **if** *dirt* **then** *Suck*]