Optimal Replacement under Partial Observations

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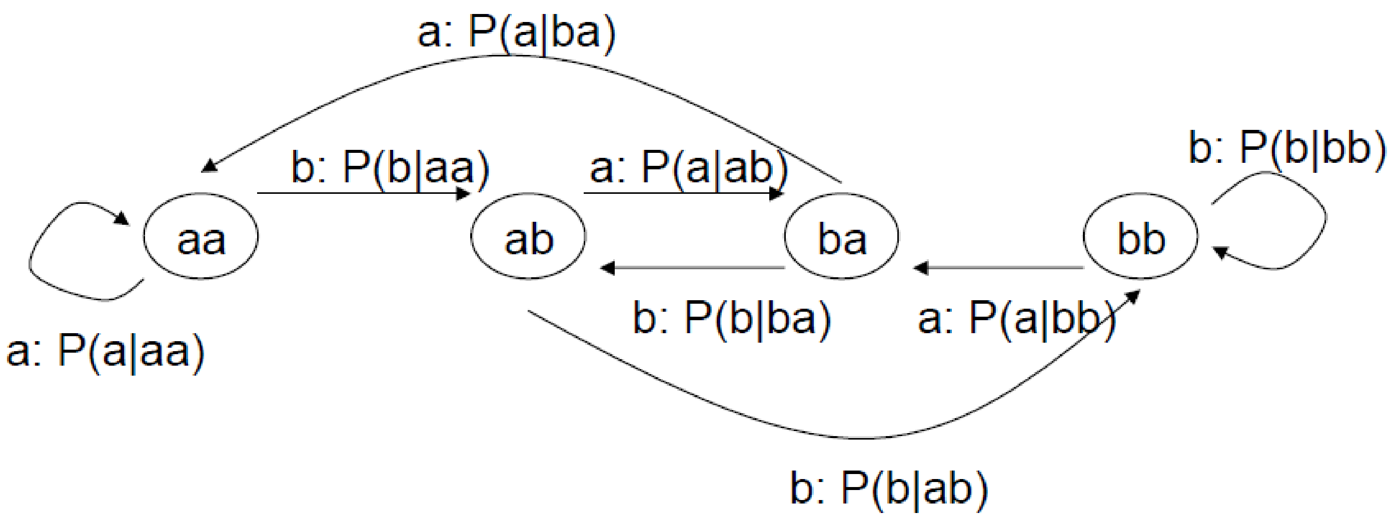
Outline

- Introduction: What is the paper about?
- Model: Markov Chain
- λ-Maximization Technique (Aven & Bergman 1986)
- Smooth Semimartingale Process (Jensen 1989)
- Projection Theorem (Bayes 1989)
- Example: Implementation
- Summary

Introduction

- Presented: a framework for the condition-based maintenance optimization
- Considered: a technical system which can be in one of N operational states or in a failure state
- Only the failure state is observable
- Stochastically related information is obtained through condition monitoring at equidistant inspection times
- System can be replaced at any time but preventive replacement is more costly than after a failure
- Aim: developing a replacement policy that minimizes the long run expected average cost per unit time

Model: Markov Chain



Assumptions

1. System dynamics: The state process is a continuous time homogeneous Markov chain with N unobservable working states and observable failure state N‘

2. Observation process: the system is monitored at times kL, k=1,2,…, and the information obtained at time kL is Y_k

3. Maintenance actions: Preventive replacement is to control, routine maintenance between observation epochs modeled as minimal repair is neglected. The system state is not affected by routine maintenance

4. Cost structure: The system installation cost is C_p (a constant). If failure occurs from state i, the replacement cost is $K_i + C_p$, preventive replacement cost is $C_p - C_{pi}$, the maintenance cost rate in state i is C_i

Step 1: λ-Maximization Technique

- Aven and Bergman (1986)
- Transformation of the stopping problem to an equivalent parameterized stopping problem → easier to analyze
- Originally developped to find numerical solutions for a general class of optimal stopping problems
- The optimal average cost is defined as the following:

$$\lambda^* = \sup\{\lambda \colon V^\lambda(P_0) < 0\}$$

- Optimal replacement policy if $\lambda = \lambda^*$

Step 2: SSM (Jensen 1989) and Projection Theorem (Brémaud 1981)

- SSM (Smooth Semimartingale Process)
- Semimartingale: historically defined as a process having the decomposition A=M+V for a local martingale M and a finite variation process V
- Needed for the indicator process
- Then: Projection theorem (Brémaud 1981) is applied
- Aim: Transformation of the stopping problem with partial information to a stopping process with complete information

Optimal Replacement Policy

(i) *If* $\overline{V}(\overline{P}_{kL}) \leq 0$, *replace the system immediately.*

(ii) *If* $\overline{V}(\overline{P}_{kL}) > 0$, *then*
if $\overline{V}^1(\overline{P}_{kL}) \geq \overline{V}^2(\overline{P}_{kL})$, *replace the system at time* $\tau^* = \arg\max(\int_0^t \langle r, \overline{P}_{kL+s}\rangle \, ds)$,
otherwise, run the system till the next observation epoch $(k+1)L$.

(iii) *The system is immediately replaced at failure time.*

The Algorithm:

Step 1. Choose $\varepsilon > 0$ and the lower and upper bound of λ, λ_L, and λ_U.

Step 2. Put $\lambda = (\lambda_L + \lambda_U)/2$, $\overline{V}_0^\lambda = 0$, $n = 1$.

Step 3. Calculate $\overline{V}_n^\lambda = T^\lambda(\overline{V}_{n-1}^\lambda)$. Stop the iteration of $\overline{V}_n^\lambda$ when $\|\overline{V}_n^\lambda - \overline{V}_{n-1}^\lambda\| \le \varepsilon(1-\beta)$

Step 4. If $V^\lambda(P_0) < C_p - EC_{p1} - \varepsilon$, put $\lambda_L = \lambda$ and go to Step 2.

If $\overline{V}^\lambda(P_0) > C_p - EC_{p1} + \varepsilon$, put $\lambda_U = \lambda$ and go to Step 2.

If $C_p - EC_{p1} - \varepsilon < \overline{V}^\lambda(P_0) < C_p - EC_{p1} + \varepsilon$, stop and put $\lambda^* = \lambda$.

Example with 3 working states (N=3) and 3 different observation values (M=3)

$$Q = \begin{pmatrix} -0.4 & 0.3 & 0 & 0.1 \\ 0.1 & -0.8 & 0.5 & 0.2 \\ 0 & 0.1 & -0.4 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad D = \begin{pmatrix} 0.7 & 0.2 & 0.1 & 0 \\ 0.3 & 0.5 & 0.2 & 0 \\ 0.1 & 0.1 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Illustration

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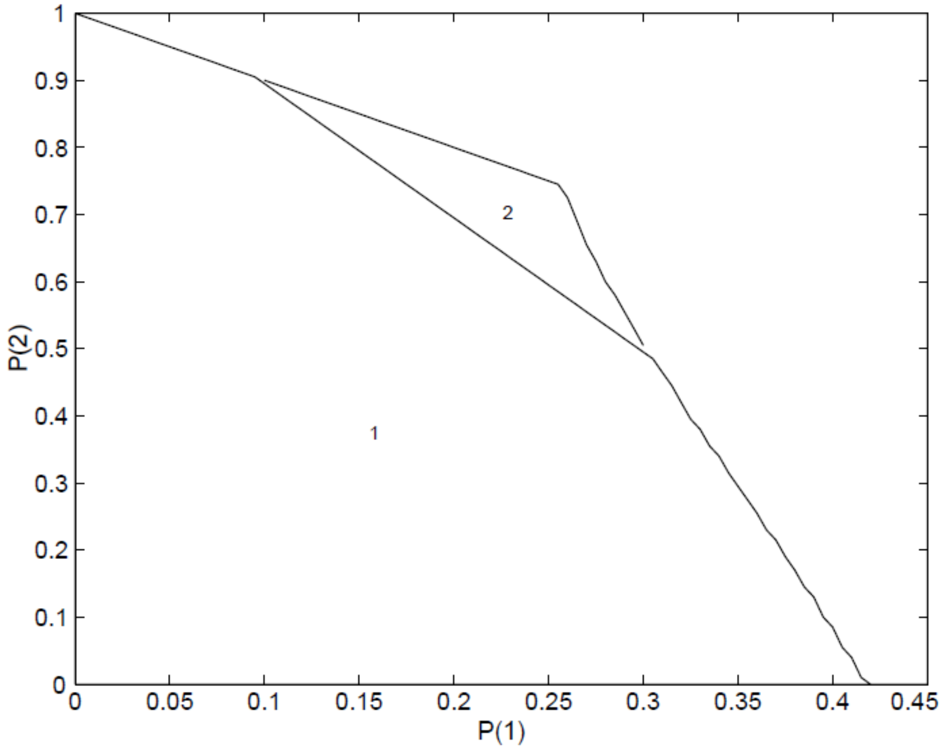

FIGURE 2. Optimal preventive replacement policy.

Summary

- Type of the problem: Stopping problem
- State process was a continuous Markov Chain
- Using maximization, the stopping problem was transformed to a parameterized stopping problem
- A projection theorem and a smooth semimartingale process (SMM) were used to transform the stopping problem with partial information into a stopping problem with complete information
- Finally, a dynamic optimality equation was derived to define when to replace the system

References

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- Makis, V., & Jiang, X. (2003). Optimal Replacement Under Partial Observations. *Mathematics of Operations Research*, *28*(2), 382–394. http://doi.org/10.1287/moor.28.2.382.14484