Handling Ties

Analysis of Ties in Input and Output Data of Rankings

TECHNISCHE
UNIVERSITÄT
DARMSTADT

Outline

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Genuine Input Ties

Incorporating Ties

Perturbation Analysis

Real Data Sets

Induced Input Ties

Output Ties

Summary

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Genuine Input Ties

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- Tied results in the input data
- Frequency depends on
  - data source
  - tie resolution policy

Incorporating Ties

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Colley's Method

Massey's Method

Markov's Method

Elo's, Keener's, OD Method

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Summary

- Colley's method does not account for ties
- Markov's methods depends on voting mechanism used
- Elo's, Keener's, Massey's and OD method account for ties

Incorporating Ties

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Summary

To compare methods with ties and methods without we need to

- derive a Colley's method accounting for ties
- create a Massey's method ignoring ties
- choose a Markov's method allowing for both
- modify Elo's, Keener's and the OD method

Colley's Method

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$C * r = b$, with

- $C_{n \times n}$ having entries as follows

$$C_{ij} = \begin{cases} 2 + t_i, & \text{i=j} \\ -n_{ij}, & \text{i}\neq\text{j} \end{cases}$$

- $r_{n \times 1}$ being the unknown Colley rating vector
- $b_{n \times 1}$ defined as $b_i = 1 + \frac{1}{2}(w_i - l_i)$

Colley's Method

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Ties in Colley's method

- are ignored
- represent an equal chance for either team winning - or losing
- can be emulated by creating two artificial games
- do not alter vector b
- increment C_{ij} by 1 and decrement C_{ji} by 1
- thus preserving the Colley property

$$\sum_{i=0}^{n} r_i = \frac{n}{2}$$

Massey's Method

$M * r = p$, with

- $M_{n \times n}$ having entries as follows

$$M_{ij} = \begin{cases} t_i, & \text{i=j} \\ -n_{ij}, & \text{i}\neq\text{j} \end{cases}$$

- $r_{n \times 1}$ being the unknown Massey rating vector
- $p_{n \times 1}$ being the vector of all teams' point differentials

Massey's Method

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Summary

Ties in Massey's method

- are naturally accounted for
- increment M_{ij} and M_{ji}
- do not change p
- can be ignored when forming M to create a No-Ties Method

Markov's Method

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Summary

Standard Markov voting procedures:

1. Loser casts one vote for each team lost against.
2. Loser casts one vote for each point lost to the other team.
3. Loser and winner cast one vote for each point lost to one another.

Markov's Method

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In the context of ties:

1. Tied teams cast half a vote each for the other team.
2. Loser casts one vote for each point lost on average.
3. Ignore tied events for a no-ties variant.

Elo's, Keener's, OD Method

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Ties in Elo's Method

- are explicitly taken care of
- can be ignored for a no-ties method

Elo's, Keener's, OD Method

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Ties in Keener's and the OD Method

- are naturally accounted for
- can be excluded by setting tied scores to 0

Perturbation Analysis

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Summary

- Apply variants with and without ties to a data set devoid of ties
- Introduce a single tie into data set
- Compare the rankings produced by the variant methods

Perturbation Analysis

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Let

- C be the Colley matrix for the no-ties variant
- $\widetilde{C}$ denote the Colley matrix for the variant including ties
- e denote a tied input
- r be the rating vector related to C
- $\tilde{r}$ be the rating vector related to $\widetilde{C}$

Perturbation Analysis

$$\widetilde{C} = C + (e_i - e_j)(e_i - e_j)^T$$

$$\tilde{r} = r - \left(\frac{r_i - r_j}{1 + [C^{-1}]_{ii} - 2[C^{-1}]_{ij} + [C^{-1}]_{jj}}\right) C^{-1}(e_i - e_j)$$

Perturbation Analysis

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With $\epsilon = r_i - r_{i+1}, \epsilon > 0$ denoting the difference in pre-disturbance ratings this implies

$$\tilde{r}_i < \tilde{r}_{i+1} \iff$$

$$\epsilon < \left(\frac{(r_i - r_j)([C^{-1}]_{ii} - [C^{-1}]_{ij} - [C^{-1}]_{i+1,i} + [C^{-1}]_{i+1,j})}{1 + [C^{-1}]_{ii} - 2[C^{-1}]_{ij} + [C^{-1}]_{jj}} * C^{-1}(e_i - e_j)\right)$$

Let $r_i < r_j$:

- if $r_i \approx r_j$ teams i and j are unlikely to change in rank
- if $r_i \gg r_j$ team i is likely to drop in rank

Real Data Sets

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Movies

NHL Hockey Teams

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Movies

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Summary

Rank	Ties Ignored	Ties Incorporated	Change
1	Casablanca	Casablanca	-
2	Schindler's List	Schindler's List	-
3	Shawshank Redemption	Rear Window	1 up
4	Rear Window	Shawshank Redemption	1 down
5	The Godfather	The Godfather	-
6	Citizen Kane	To Kill a Mockingbird	2 up
8	To Kill a Mockingbird	Citizen Kane	2 down
11	Pulp Fiction	Raging Bull	3 up
12	It's a Wonderful Life	It's a Wonderful Life	-
13	Taxi Driver	Taxi Driver	-
14	Raging Bull	Pulp Fiction	3 down

NHL Hockey Teams

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Induced Input Ties

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- Modifications of input data
- Change non-tied events to a tie

Induced Input Ties

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Ties are often broken

- seemingly at random
- irrespective of the teams' actual performance
- after regular match time

Induced Input Ties

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- Motivation
- Methods
- Analysis

Output Ties

Summary

Costa Rica	1 – 1 (5 – 3)	Greece

Scoring summary

Bryan Ruiz (#10) 52' ⚽ Socratis Papastathopoulos (#19) 90+1'

Team stats

6	Total shots	24
2	Shots on target	13
23	Fouls	16
43%	Possession (%)	57%
3	Corners	11
7	Saves	0
0	Offsides	10

Induced Input Ties

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Final results may

- fail to portray the teams' actual strength
- create a false sense of precision
- skew the ranking
- impede a ranking's predictive capabilities

Methods

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Induce a tie if

- a winner is only determined after regular play time
- points differ only by a small margin
- match statistics indicate comparable performance

Analysis

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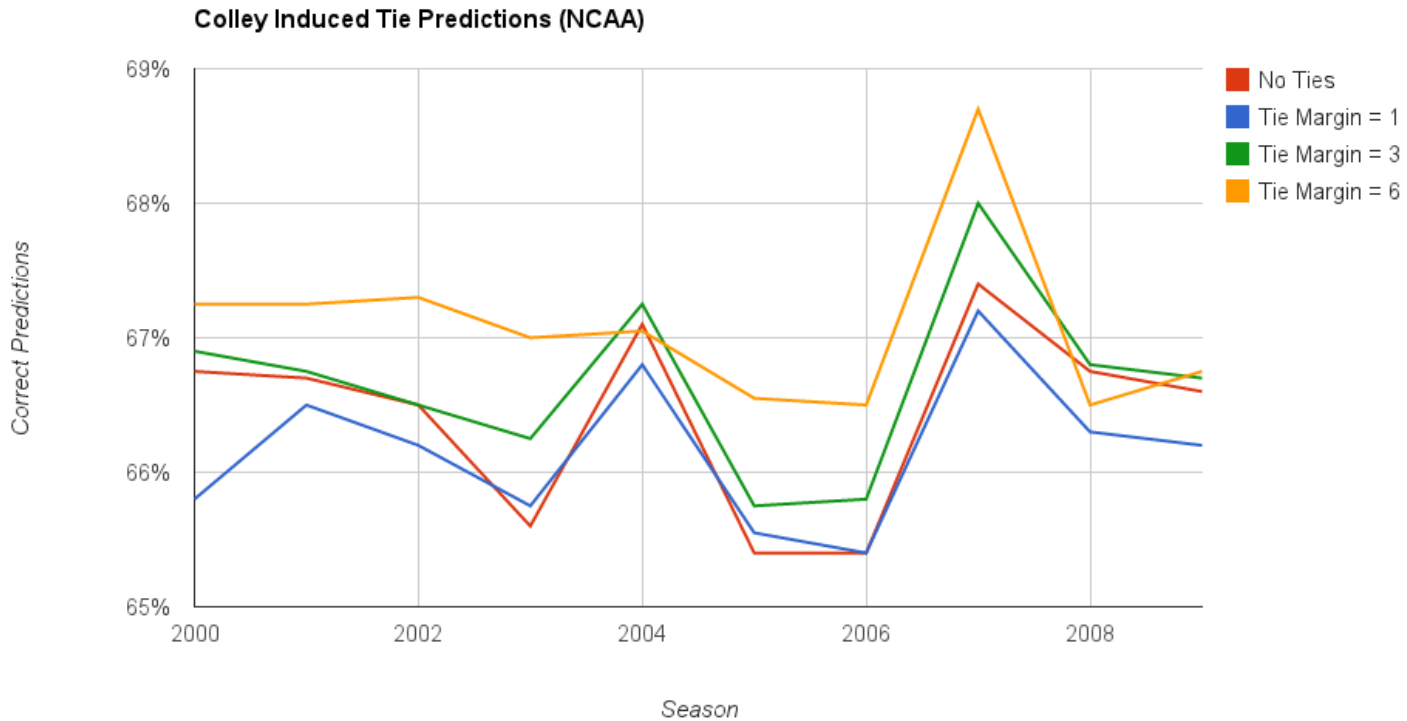

Output Ties

Outline

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Output Ties

- Recapitulation
- Resolution Methods
- Ramifications

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Recapitulation

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Recapitulation
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Rankings are total preorders, i.e. relations on a set S that are

- total $(\forall x, y \in S : x \leq y \vee y \leq x)$
- transitive $(\forall x, y, z \in S : x \leq y \wedge y \leq z \Rightarrow x \leq z)$
- but not antisymmetric $(\exists x, y \in S : x \leq y \wedge y \leq x \wedge x \neq y)$

Resolution Methods

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Resolution Methods
Ramifications
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- Standard Competition Ranking ("1 2 2 4")
- Modified Competition Ranking ("1 3 3 4")
- Dense Ranking ("1 2 2 3")
- Ordinal Ranking ("1 2 3 4")
- Fractional Ranking ("1 2.5 2.5 4")

Ramifications

TECHNISCHE UNIVERSITÄT DARMSTADT

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- Psychological Effects
- Statistical Effects

Statistical Effects - Rank Sum Test

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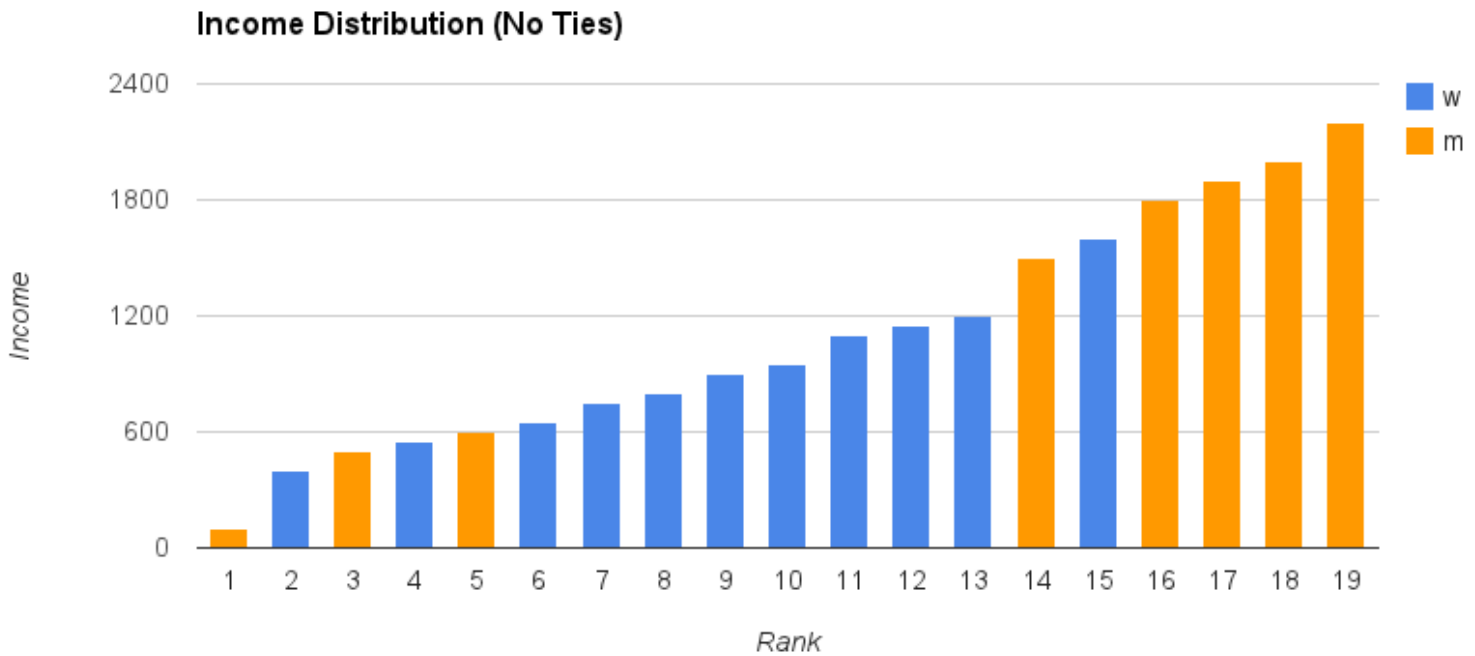

Statistical Effects - Rank Sum Test

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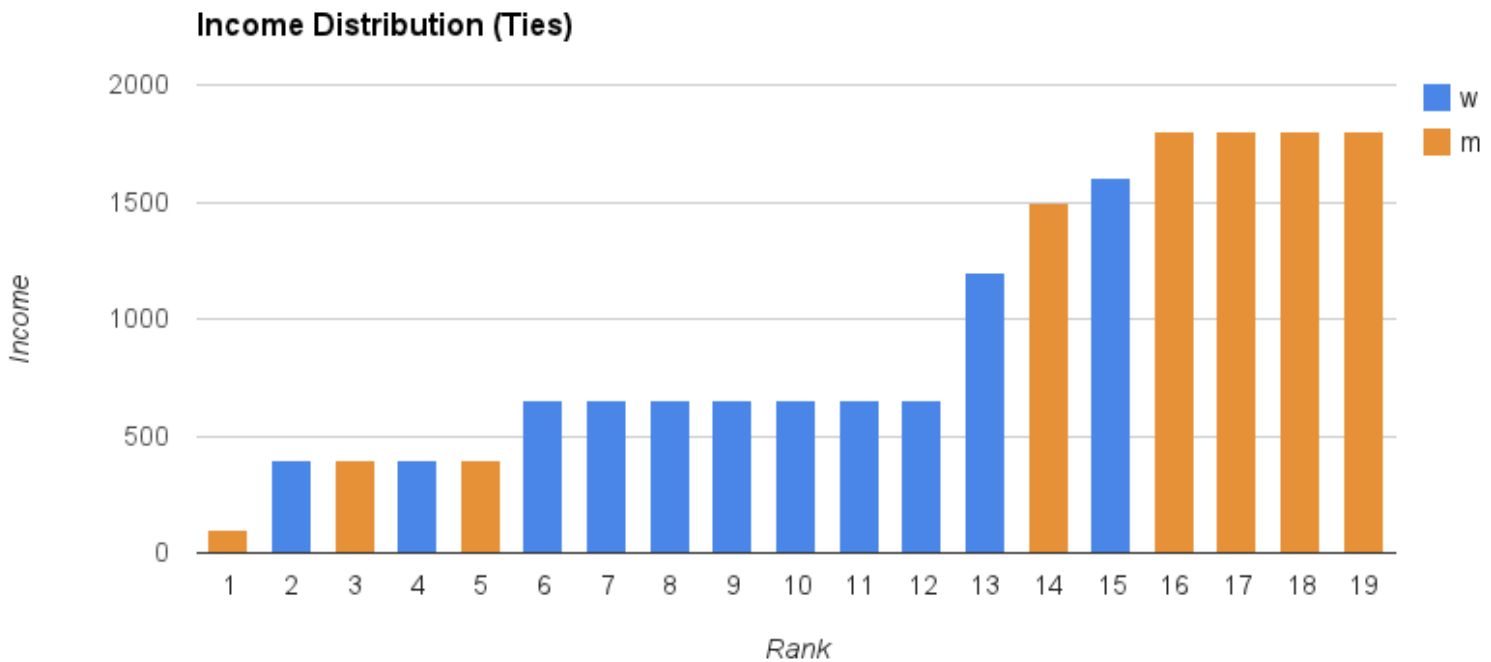

Statistical Effects - Rank Sum Test

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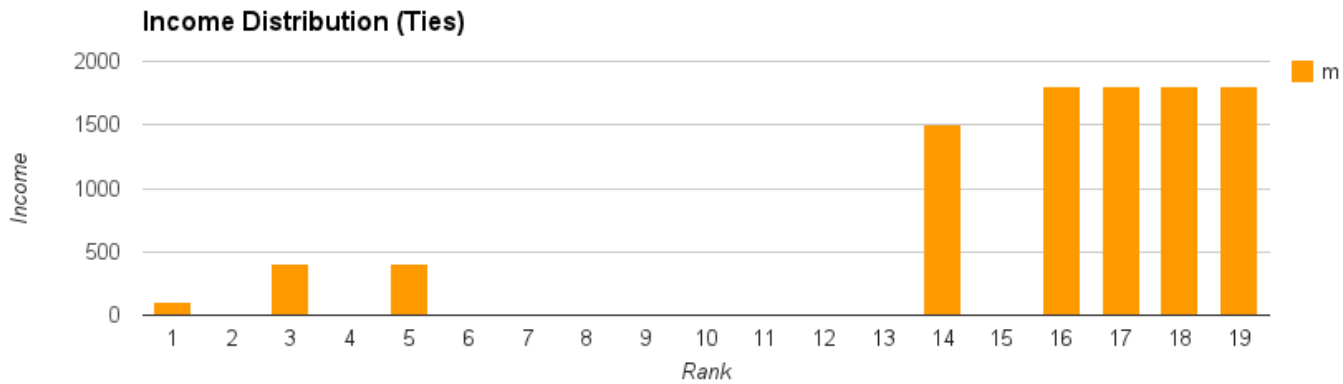

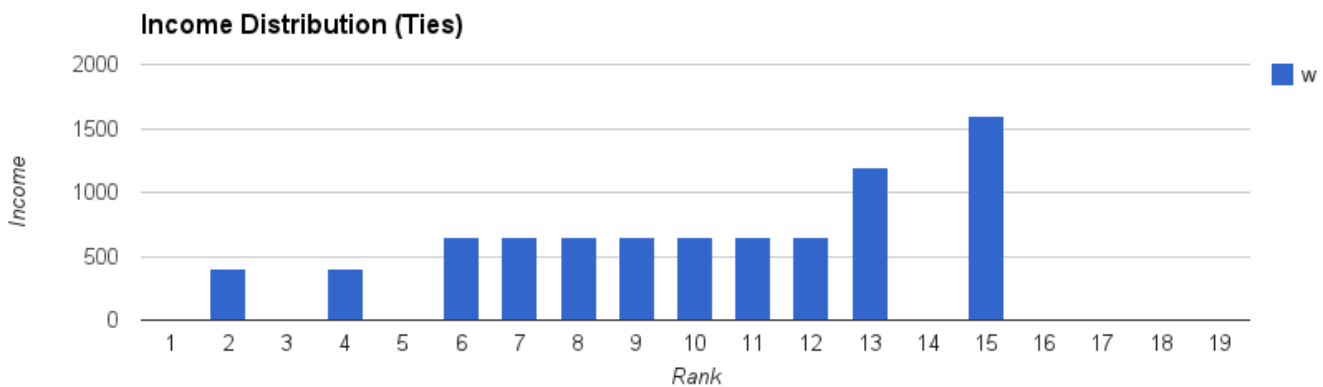

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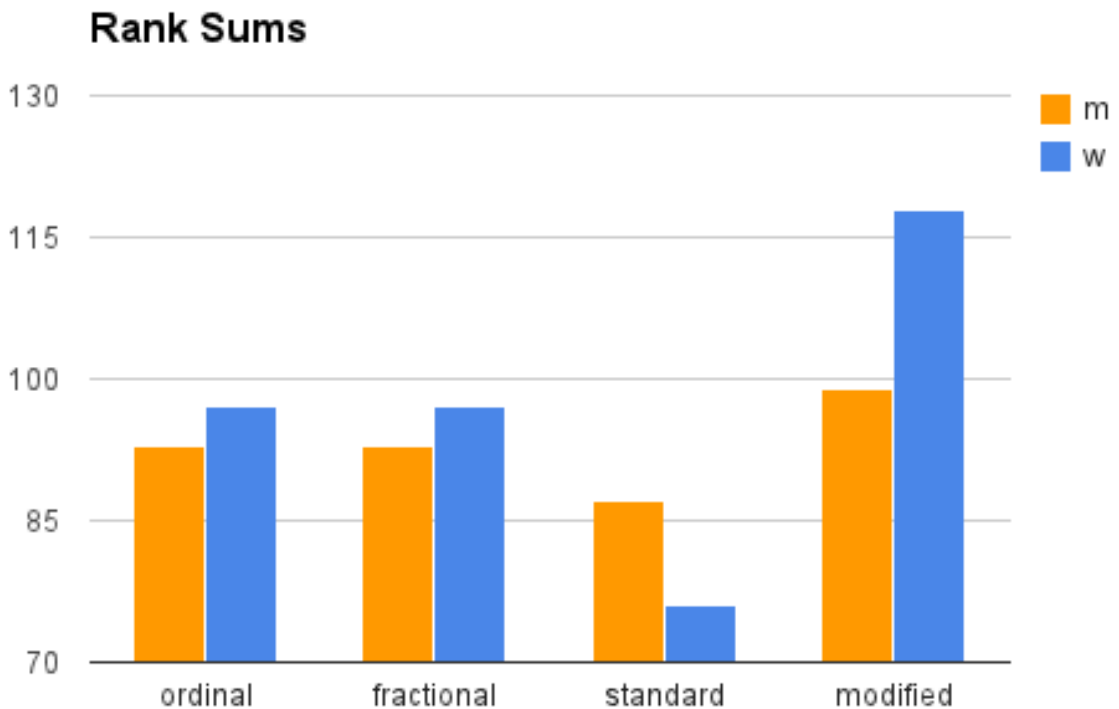

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- Accounting for ties is easy
- Input ties influence ranking order
- Inducing ties is beneficial
- Output ties require special care