# Point Spreads

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## Sports Data Mining

Topic : Point Spreads

# Overview

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- Introduction
- Method of Spread Ratings
- Example
- Comparison
- Conclusion

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# Introduce to Point Spreads

**Spread betting -** any of various types of wagering on the outcome of an event, including in financial market(spread betting is regulated by the Financial Conduct Authority in UK)

**Point spreads -** the scoring differences of a competition ( mainly in sport area)

- Handicap

**How can bookmaker makes money?**

- Vigorish/Juice: a **fee** that charged for the services from bookmaker
- By setting the **payout rate** under 100% probabilities
- If there is an **equal amount** of money that betting on each teams

# Odds VS Point Spreads

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If only odds betting

- the payoff on a winning bet should tied to the **odds in favor of winning**(hard to predict)
- less fovor on a weak team(skillful gambler)
- →decreased betting activity & less vig
- **Competition** between bookmakers

point spreads betting:

For gambler: Great **volatility** → excitement, substantial losses and rewards

For bookmaker: Maximize the betting activity

**Balance** the amount that betting on either teams( make them equally attractive)

# Point Spread Betting

Early age: Charles K. Mcneil, a mathematics teacher, became bookmaker and offerd betting against a published spread around 1940s in U.S.

Target: maximize and balance the activity on both sides

**Operations**:

- bookmakers create a handicap against the favored team;
- bookmakers decide the point spread that may result in an equal amount of wager;
- gamblers betting that, the scoring differences is more/less than the offered spread;
- adjust the spreads

# Types of Point Spread Betting - Handicap

Handicap(beat the spread)

- Underdog & favorite - the underdog(weaker/away team) receives points and favorite give points;
- The result of wager is then determind by the adjusted score: the favorite team with original score and the underdog plus spread on basis of primitive score;
- „Push“ - using half-point fractions to avoid tie
- Example of World Cup from bet365.com, in which team Holland receives 1 additional point: if you bet on Argentina to win and Argentina wins exceeding 2 additional score, then you beat the spread;

Asian handicap

- range from one-quarter goal to several goals
- originated in Indonesia

**ASIAN HANDICAP**

| HOLLAND | ARGENTINA |
|---|---|
| 0.0,+0.5 1.850 | 0.0,-0.5 2.075 |

**ALTERNATIVE ASIAN HANDICAP**

| HOLLAND | ARGENTINA |
|---|---|
| -1.5 7.200 | +1.5 1.100 |
| -1.0,-1.5 6.600 | +1.0,+1.5 1.110 |
| -1.0 6.000 | +1.0 1.130 |

**HANDICAP RESULT**

| | | |
|---|---|---|
| Holland (+1) 11/20 | Tie - (Argentina -1) 11/4 | Argentina (-1) 15/4 |

# Types of Point Spread Betting - Over/Under Betting

Over/Under Betting

- Another popular spread betting – bet on the total score
- Usually with a half-point fraction to avoid tie
- Example of over/under bet of World Cup ½ final (Hollland VS Argentina) from bet365.com and oddset.de showed below
- Other stastistics – total passing yards in football, turnovers in basketball, ect.

**GOALS OVER/UNDER**

| | OVER | UNDER |
|---|---|---|
| 2.5 | 27/20 | 4/7 |

**Weniger oder Mehr Tore**

Wie viele Tore werden erzielt?

| | | | |
|---|---|---|---|
| Weniger als 2,5 | **1,45** | Mehr als 2,5 | **2,15** |

Over/Under Betting of a soccer game

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# Predicting Spreads with Ratings?

Difficulties:

- The purpose of ratings are different – usually the **overall strengh** of teams that related to others
- Details may filtered out during distillation
- Lack of the seprate analyses of individual aspects , such as offensive/defensive

# Method - Predicting Spreads with Spread data

- Use the historical information to build rating of spreads
- Similar to Massey‘s method
- Build an optimal rating system that closely reflect the point spreads

Suppose that there is a perfect rating vector

$$\mathrm{r} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}$$

that rating difference $r_i - r_j$ is equal to each scoring difference $S_i - S_j$

$$K = \begin{pmatrix} 0 & S_1 - S_2 & \cdots & S_1 - S_n \\ S_2 - S_1 & 0 & \cdots & S_1 - S_n \\ \vdots & \vdots & \ddots & \vdots \\ S_n - S_1 & S_n - S_2 & \cdots & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & r_1 - r_2 & \cdots & r_1 - r_n \\ r_2 - r_1 & 0 & \cdots & r_2 - r_n \\ \vdots & \vdots & \ddots & \vdots \\ r_n - r_1 & r_n - r_2 & \cdots & 0 \end{pmatrix}$$

⇒ K = R = r$e^T$ - e$r^T$ (e is a column of 1)

# Method - Predicting Spreads with Spread data

$$K = \begin{pmatrix} 0 & S_1 - S_2 & \cdots & S_1 - S_n \\ S_2 - S_1 & 0 & \cdots & S_1 - S_n \\ \vdots & \vdots & \ddots & \vdots \\ S_n - S_1 & S_n - S_2 & \cdots & 0 \end{pmatrix}$$

- Score-differential matrix K is skew symmetric $\Rightarrow$ $K^T = -K$
- Rank of $re^T - er^T$ is 2 $\Rightarrow$ Rank of K is 2
- But actually we can only build ratings vector **r** that:

  minimize the distance between K and R

Build a function of vector x that is minimal for Frobenius norm:

$$f(x) = \| K - R(x) \|^2 = \| K - (xe^T - ex^T) \|^2 \qquad (1)$$

as rank(K) = 2 and it is simplest and most standard

# Method - Predicting Spreads with Spread data

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Frobenius norm is

$$\| A \|_F = \sqrt{\sum_{i,j} a_{ij}^2} = \sqrt{trace(A^T A)}$$

In which the trace function have these proterties:

$$trace(\alpha A + B) = \alpha\ trace(A) + trace(B), \text{ and}$$
$$trace(AB) = trace(BA)$$

Set $\frac{\partial f}{\partial x_i} = 0$ for each $i$ and solve the resulting system of equations for $x_1, x_2, \ldots, x_n$ to minimize the function (1)

$$f(x) = \| K - R(x) \|^2 = \| K - (xe^T - ex^T) \|^2$$

yields $\longrightarrow$

$$f(x) = trace\,[K - (xe^T - ex^T)]^T\,[K - (xe^T - ex^T)]$$
$$= trace\,K^T K - trace[K^T(xe^T - ex^T) + (xe^T - ex^T)^T K] +$$
$$trace\,[(xe^T - ex^T)^T (xe^T - ex^T)]$$

As K is skew symmetric, then: $f(x) = trace\,(K^T K) - 4x^T Ke + 2n(x^T x) - 2(e^T x)^2$

# Method - Predicting Spreads with Spread data

$$f(\mathrm{x}) = trace\,(\mathrm{K}^T\mathrm{K}) - 4\mathrm{x}^T\mathrm{Ke} + 2\mathrm{n}(\mathrm{x}^T\mathrm{x}) - 2(\mathrm{e}^T\mathrm{x})^2$$

differntiate $f$ with respect to $x_i$, we got:

$$\frac{\partial f}{\partial x_i} = -4\mathrm{e}_i^T\mathrm{Ke} + 4nx_i - 4\mathrm{e}^T\mathrm{x}$$

set $\frac{\partial f}{\partial x_i} = 0$ then:

$$-4(\mathrm{e}_i^T\mathrm{Ke} - nx_i + \mathrm{e}^T\mathrm{x}) = 0$$

$$nx_i - \mathrm{e}^T\mathrm{x} = \mathrm{e}_i^T\mathrm{Ke}$$

$$nx_i - \sum_{j=i}^{n} x_j = (\mathrm{Ke})_i, for\ i = 1, 2, \ldots, n$$

This linear equation can be transfored into a matrix equation:

$$\begin{pmatrix} (\mathrm{n}-1) & -1 & \cdots & -1 \\ -1 & (\mathrm{n}-1) & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & (\mathrm{n}-1) \end{pmatrix} \begin{pmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \vdots \\ \mathrm{x}_\mathrm{n} \end{pmatrix} = \begin{pmatrix} (\mathrm{Ke})_1 \\ (\mathrm{Ke})_2 \\ \vdots \\ (\mathrm{Ke})_\mathrm{n} \end{pmatrix}, \text{each side divide n} \rightarrow \left(\mathbf{I} - \frac{\mathrm{ee}^\mathrm{T}}{\mathrm{n}}\right)\mathrm{x} = \frac{\mathrm{Ke}}{\mathrm{n}}$$

which **I** is a ideantity matrix

# Method - Predicting Spreads with Spread data

$$\left(\mathbf{I} - \frac{\mathrm{ee^T}}{\mathrm{n}}\right)\mathrm{x} = \frac{\mathrm{Ke}}{\mathrm{n}} \qquad (2)$$

As $\mathrm{e}^T\mathrm{Ke}$ is a scalar and $\mathrm{K}^T = -\mathrm{K} \xrightarrow{\text{yields}} \mathrm{e}^T\mathrm{Ke} = 0$,
So one solution of formula (2) is the vector x that:

$$\mathrm{x} = \frac{\mathrm{Ke}}{n}$$

Further more, $(\mathbf{I} - \frac{\mathrm{ee^T}}{\mathrm{n}})\,\mathrm{e} = 0$ and $rank\left(\mathbf{I} - \frac{\mathrm{ee^T}}{\mathrm{n}}\right) = \mathrm{n} - 1$,
Then all solutions of the formula (2) are:

$$\mathrm{x} = \frac{\mathrm{Ke}}{\mathrm{n}} + \alpha\mathrm{e}\,, \quad \alpha \in \mathcal{R}$$

With help of some constraint such as sum of ratings equals 0, we can determine a unique solution:

$$\sum x_i = \mathrm{e}^T\mathrm{x} = 0$$

$$\mathrm{e}^T\left(\frac{\mathrm{Ke}}{\mathrm{n}} + \alpha\mathrm{e}\right) = \frac{\mathrm{e}^T\mathrm{Ke}}{\mathrm{n}} + \mathrm{e}^T\alpha\mathrm{e} = \mathrm{e}^T\alpha\mathrm{e} = 0 \xrightarrow{\text{yields}} \alpha = 0$$

# Method - Predicting Spreads with Spread data

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- check if r= $\frac{Ke}{n}$ is the actual minimizer using the Cauchy-Schwarz (or CBS) inequality method

Then the only critical point for *f(x)* is:

$$r = \frac{Ke}{n} \quad \text{(the centroid of the columns in K)}$$

In which the set ot ratings $\{r_1, r_2, \cdots, r_n\}$ with constraint $\sum_i r_i = 0$, that the (Frobenius) distance between K = $[S_i - S_j]$ and R = $[r_i - r_j]$ is minimized.

Take the result of World Cup in group G for example:

| | Germany | Portugal | Ghana | United States |
|---|---|---|---|---|
| Germany | 0 | 4 - 0 | 2 - 2 | 1 - 0 |
| Portugal | 0 - 4 | 0 | 2 - 1 | 2 - 2 |
| Ghana | 2 - 2 | 1 - 2 | 0 | 1 - 2 |
| United States | 0 - 1 | 2 - 2 | 2 - 1 | 0 |

$$K = \begin{bmatrix} 0 & 4 & 0 & 1 \\ -4 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$r = \begin{bmatrix} 1.2 \\ -0.75 \\ -0.5 \\ 0 \end{bmatrix}$$

# Overview

# Example : NFL 2009 - 2010

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- 267 games in 2009 – 2010 NFL season
- If two teams meet more than once, use the average score respectively
- Hindsight prediction percentage : 71.54%
- High efficiency– only averaging the score differences
-

| Rank | Team | Rating | Rank | Team | Rating |
|---|---|---|---|---|---|
| 1. | SAINTS | 6.2187 | 17. | PANTHERS | -0.1094 |
| 2. | VIKINGS | 4.7187 | 18. | BRONCOS | -0.1250 |
| 3. | PACKERS | 3.9687 | 19. | TITANS | -0.9531 |
| 4. | RAVENS | 3.6875 | 20. | GIANTS | -1.0625 |
| 5. | PATRIOTS | 3.6250 | 21. | REDSKINS | -1.0937 |
| 6. | JETS | 3.0000 | 22. | DOLPHINS | -1.1094 |
| 7. | COLTS | 2.8594 | 23. | BEARS | -1.5781 |
| 8. | CHARGERS | 2.7031 | 24. | BILLS | -1.7812 |
| 9. | EAGLES | 2.5469 | 25. | JAGUARS | -2.9531 |
| 10. | TEXANS | 2.0781 | 26. | CHIEFS | -3.0156 |
| 11. | COWBOYS | 2.0156 | 27. | BROWNS | -3.0469 |
| 12. | STEELERS | 1.4219 | 28. | SEAHAWKS | -3.3281 |
| 13. | FALCONS | 1.1719 | 29. | BUCS | -3.9687 |
| 14. | CARDINALS | 0.3906 | 30. | RAIDERS | -5.1562 |
| 15. | BENGALS | 0.3281 | 31. | LIONS | -5.4219 |
| 16. | NINERS | 0.1875 | 32. | RAMS | -6.2187 |

Spread ratings NFL 2009-2010

# Example : NFL 2009 - 2010

**Absolute-spread error – *with linear regression approach***

Parameters: „home-field advantage“ , $\alpha, \beta, \gamma$

$$\text{Expected[(home-team score) – (away-team score)]} = \alpha + \beta r_h - \gamma r_a$$

$r_h$ and $r_a$ are the ratings for home and away teams;

In this euqation:

$$\alpha + \beta r_{h_i} - \gamma r_{a_i} = S_{h_i} - S_{a_i},\ i = 1,2,\ldots,267$$

$\alpha, \beta, \gamma$ are determined by computing least squares solution within equation above,

Where $r_{h_i}$ and $r_{a_i}$ are the ratings for home and away team in game *i*, $\boldsymbol{S_{h_i}}$ **and** $\boldsymbol{S_{a_i}}$ is the scores at home or away in game i,

Computing LS of linear system Ax = b → the 3 X 3 system of normal equations $A^T$Ax = $A^T$b,

Where

$$A_{267\times 3} = \begin{pmatrix} 1 & r_{h1} & r_{a1} \\ 1 & r_{h2} & r_{a2} \\ \vdots & \vdots & \vdots \\ 1 & r_{h267} & r_{a267} \end{pmatrix}, x = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}, \text{and } b = \begin{pmatrix} S_{h1} - S_{a1} \\ S_{h2} - S_{a2} \\ \vdots \\ S_{h267} - S_{a267} \end{pmatrix}$$

# Example : NFL 2009 - 2010

Result:

$\alpha = 2.3671$

$\beta = 2.4229$

$\gamma = 2.2523$

$\rightarrow$ $tocal\ absolute$-spread error:

$$\sum_{i=1}^{267} |(2.3671 + 2.4229 r_{h_i} - 2.2523 r_{a_i}) - (S_{h_i} - S_{a_i})| = 2827.6$$

Which the spread error per game is 10.59

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# Compare with other Methods

| Rank | Team | Rating | Rank | Team | Rating |
|---|---|---|---|---|---|
| 1. | SAINTS | 1.693 | 17. | TITANS | 0.110 |
| 2. | COLTS | 1.416 | 18. | DOLPHINS | 0.062 |
| 3. | CHARGERS | 1.291 | 19. | GIANTS | 0.036 |
| 4. | COWBOYS | 1.002 | 20. | NINERS | -0.093 |
| 5. | VIKINGS | 0.909 | 21. | BRONCOS | -0.182 |
| 6. | JETS | 0.897 | 22. | BILLS | -0.232 |
| 7. | EAGLES | 0.814 | 23. | BEARS | -0.478 |
| 8. | PACKERS | 0.783 | 24. | JAGUARS | -0.621 |
| 9. | FALCONS | 0.671 | 25. | BUCS | -0.706 |
| 10. | PATRIOTS | 0.667 | 26. | REDSKINS | -0.938 |
| 11. | PANTHERS | 0.639 | 27. | BROWNS | -0.947 |
| 12. | RAVENS | 0.602 | 28. | RAIDERS | -0.986 |
| 13. | TEXANS | 0.340 | 29. | CHIEFS | -1.050 |
| 14. | CARDINALS | 0.318 | 30. | SEAHAWKS | -1.451 |
| 15. | BENGALS | 0.230 | 31. | LIONS | -2.385 |
| 16. | STEELERS | 0.178 | 32. | RAMS | -2.591 |

NFL 2009–2010 Massey ratings

| Rank | Team | Rating | Rank | Team | Rating |
|---|---|---|---|---|---|
| 1. | SAINTS | 32.18 | 17. | BENGALS | 21.07 |
| 2. | COLTS | 29.86 | 18. | GIANTS | 20.52 |
| 3. | VIKINGS | 27.91 | 19. | BRONCOS | 20.27 |
| 4. | JETS | 27.69 | 20. | NINERS | 20.21 |
| 5. | CHARGERS | 25.99 | 21. | TITANS | 19.76 |
| 6. | PATRIOTS | 25.89 | 22. | BILLS | 18.18 |
| 7. | COWBOYS | 25.65 | 23. | BEARS | 16.48 |
| 8. | RAVENS | 25.58 | 24. | JAGUARS | 16.2 |
| 9. | FALCONS | 24.3 | 25. | BUCS | 13.06 |
| 10. | PACKERS | 24.19 | 26. | REDSKINS | 12.84 |
| 11. | EAGLES | 23.79 | 27. | BROWNS | 12.66 |
| 12. | PANTHERS | 22.87 | 28. | RAIDERS | 12.56 |
| 13. | TEXANS | 22.49 | 29. | CHIEFS | 11.93 |
| 14. | DOLPHINS | 21.43 | 30. | SEAHAWKS | 11.61 |
| 15. | CARDINALS | 21.37 | 31. | LIONS | 6.41 |
| 16. | STEELERS | 21.24 | 32. | RAMS | 3.82 |

NFL 2009–2010 Sagarin ratings

# Compare with other Methods

| Hindsight Win Accuray: | Average Spread Error(Points per Game) |
|---|---|
| Spread Ratings = 71.5% | Spread Ratings = 10.59 points per game |
| Sagarin Ratings = 70.8% | Sagarin Ratings = 10.53 points per game |
| Elo Ratings = 75.3% | Elo Ratings = 10.71 points per game |
| Keener Ratings = 73.4% | Keener Ratings = 10.54 points per game |
| Massey Ratings = 72.7% | Massey Ratings = 10.65 points per game |

Comparisons of popular rating systems with 2009-2010 NFL data

Compare the hindsight win accuracy and spread error

- Spread ratings method is in the similar efficiency area as the other 4 rating systems
- Spread (centroid) ratings are more **easier** to compute

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# Conclusion

Gambling is sometimes an art (of psychology) – gamblers, bookmakers are betting on the expectation that their „predieted“ point spread will generate an equal amount of money,

Using simple method may also receive a good result.

# Questions?

Thank you for your attentions!