Learning various classes of models of lexicographic orderings

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Introduction

Topic: learn to order objects of a combinatorial domain

E.g. computers, described by
Type: *d*esktop or *l*aptop
Color: *y*ellow or *b*lack
Dvd-unit: *r*eader or *w*riter . . .
Recommender system : learn how a user orders these objects, in order to suggest the "best" ones among those that are available / the user can afford.

If n variables, domains of m values : m^n objects, $!m^n$ orderings
$\Rightarrow$ need compact representation of the orderings :

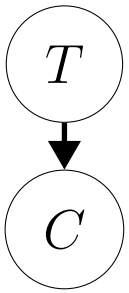
- local preferences on each attribute
- extra structure on the set of variables to "aggregate" to global preferences

Introduction

Lexicographic orderings :

local preferences over the domains of each variable

$+$ *importance ordering* of the variables



$l \succ d$

$y \succ b$

- Type is more important than Colour
- Prefer laptop to desktop
- Prefer yellow to black

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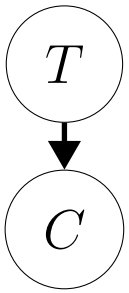

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$\mathbf{l}b \succ \mathbf{d}y$ (decided at node T)

$l\mathbf{y} \succ l\mathbf{b}$ (decided at node C)

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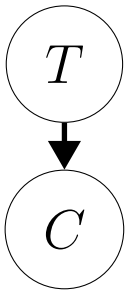

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$+$ comparisons in linear time

$+$ learning in polynomial time [SM06, DIV07]

$-$ very weak expressive power:

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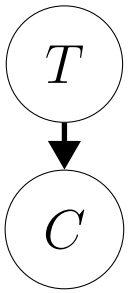

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Conditional Preference Networks (CP-nets) :

conditional local preferences (dependency graph)

e.g.: $l : y \succ b$ (for laptops: yellow pref. to black) $d : b \succ y$ $l \succ d$

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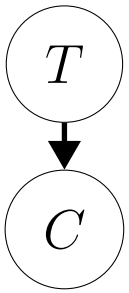

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$-$ hard to learn [session on CP-net learning at IJCAI'09]

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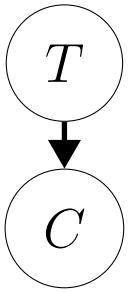

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(easy classes of CP-nets / examples, incomplete algorithms)

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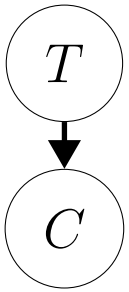

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$\Rightarrow$ **find something in between the two formalisms**

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Contribution of this paper:

it is possible to add conditionality in lexicographic prefence models without increasing the complexity of reasoning / learning

Learning unconditional lexicographic preferences

Sample complexity: VC dim $= n$ (when n variables, all binary)

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Active learning: a learner asks "user" queries of the form "What is preferred between ly and bd ?"
Goal : identify preference model of the user

$\Rightarrow$ If local pref. fixed, need $\log(!n)$ queries (worst case) [DIV07]
$\Rightarrow$ If local pref. to be learnt, need $n + \log(!n)$ queries

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Model optimization (less than k errors)
$\Rightarrow$ NP-complete with fixed local pref. [SM06]
$\Rightarrow$ NP-complete with unknown local pref.

Learning unconditional lexicographic preferences

Greedy algorithm [DIV07]

1. initialize seq. of var. with empty sequence;
2. while there remains some unused variable:
   (a) choose a variable and local pref. that does not wrongly order the remaining examples
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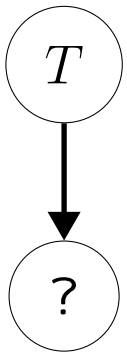

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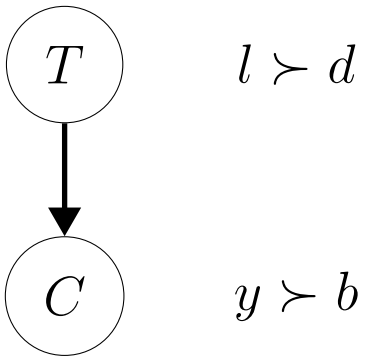

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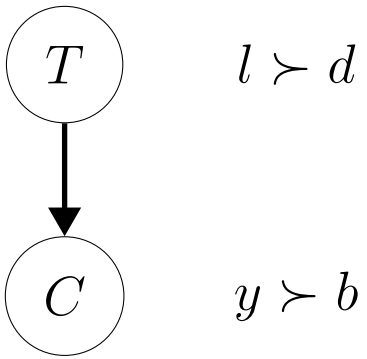


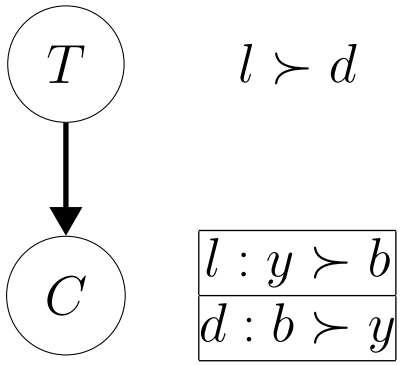
success !

Conditional local pref. / Unconditional variable importance

"I always prefer laptops to desktops"

"For desktops, I prefer black to yellow"

"For laptops, I prefer yellow to black"



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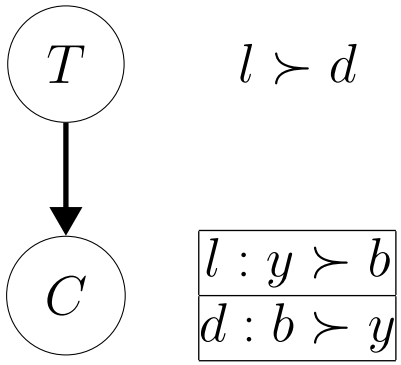

Sample complexity : VC dim $= 2^n - 1$

Active learning : $2^n - 1 + \log(!n)$ queries needed (worst case)

Passive learning : in P (Greedy algorithm still works)

Model optimization : NP-hard

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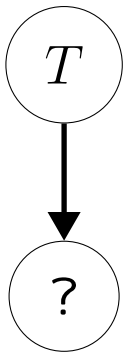

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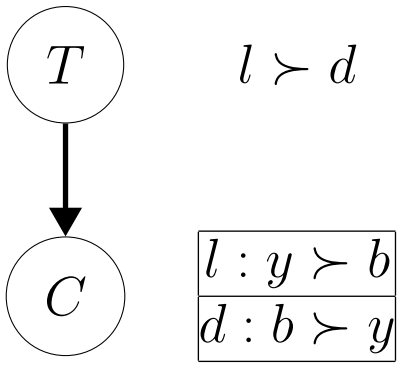

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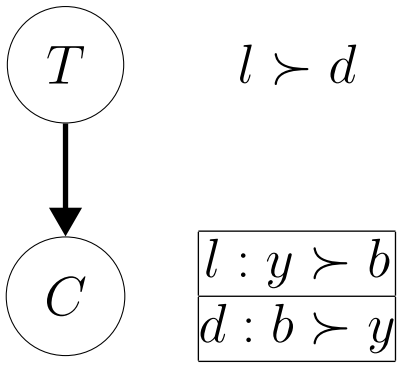

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Conditional local pref. & Conditional variable importance

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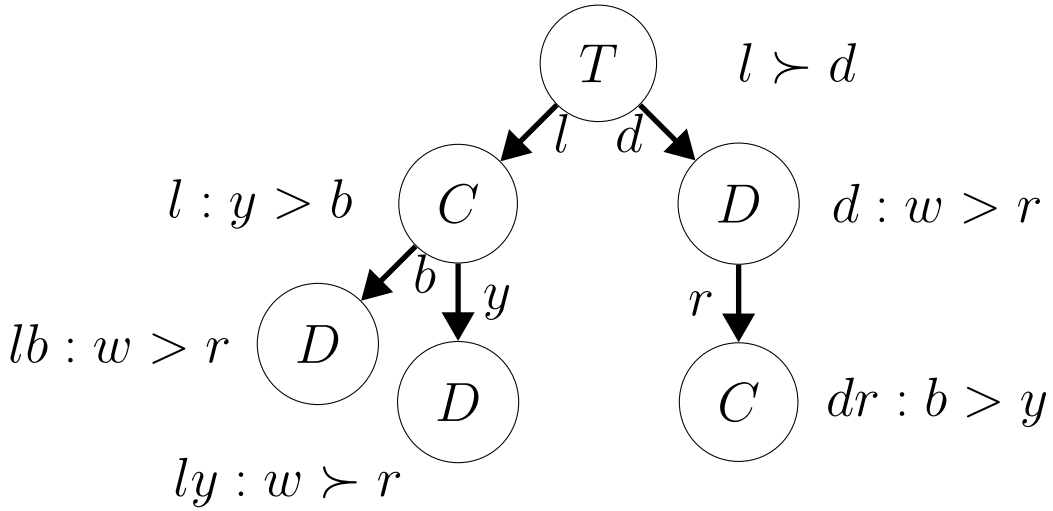
"For laptops, color is more important than the type of Dvd unit"



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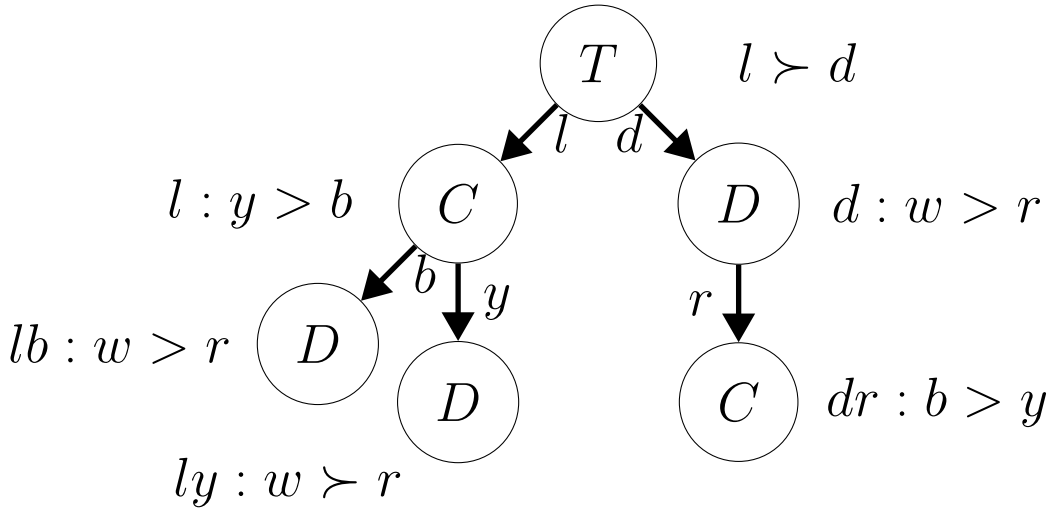

$\Rightarrow$ variable importance tree
$+$ conditional local preference tables

Note : tree need not be complete (but then partial ordering)

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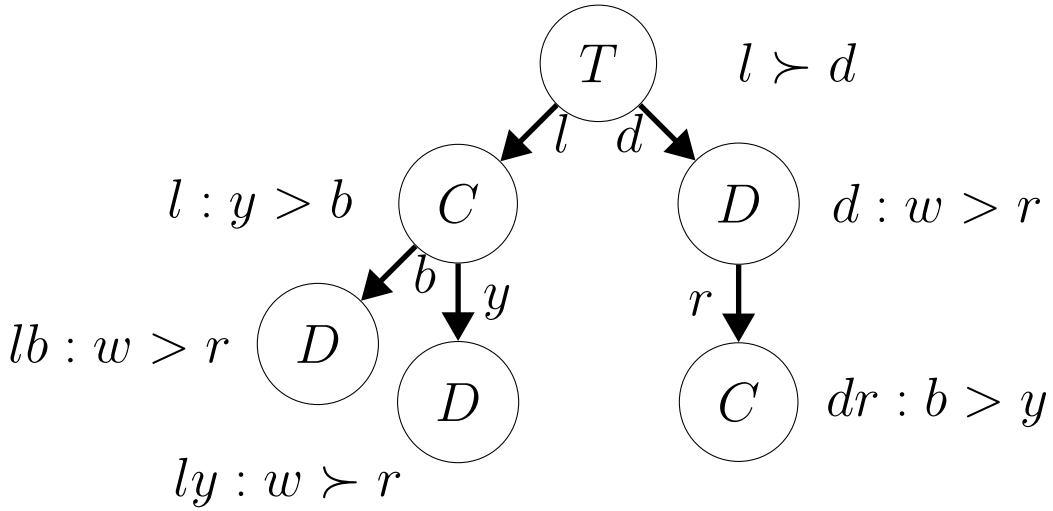

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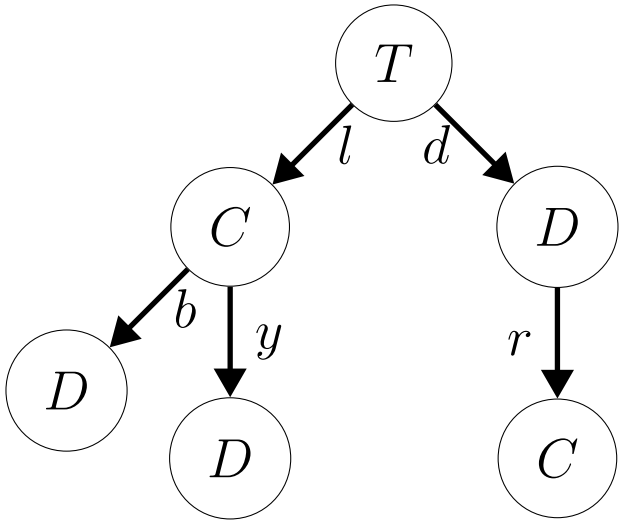

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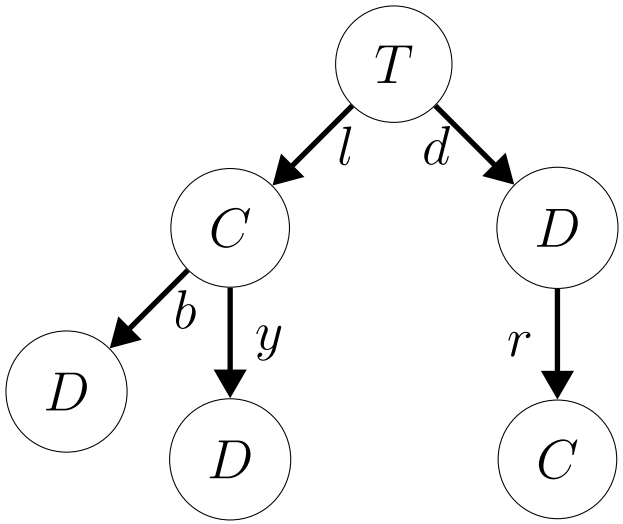

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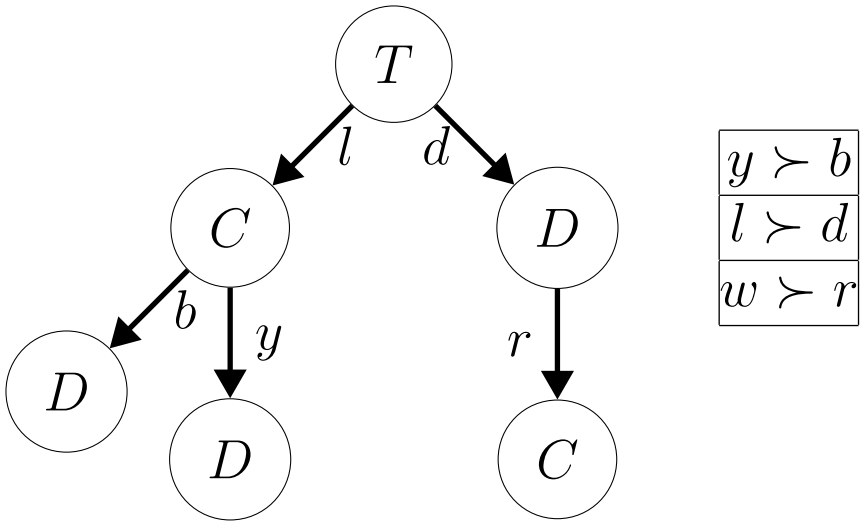

Sample complexity: ?

Active learning: .

$n + \sum_{k=0}^{n-1} 2^k \log(n-k)$ queries needed (unknown pref.)

$\sum_{k=0}^{n-1} 2^k \log(n-k)$ queries needed (fixed pref.)

Passive learning: NP-complete !! (Greedy algorithm still works)

Model optimization: NP-complete

Quick recap

	VC-dim	active l.	passive l.	approx
UI - FLP		$log(!n)$	P	NP-C
UI - ULP	n	$n + log(!n)$	P	NP-C
UI - CLP	$2^n - 1$	$2^n - 1 + log(!n)$	P	NP-hard
CI - FLP		$g(n)$	P	NP-C
CI - ULP	$\geq n$	$n + g(n)$	NP-C	NP-C
CI - CLP	$2^n - 1$	$2^n - 1 + g(n)$	P	NP-C

$$g(n) = \sum_{k=0}^{n-1} 2^k \log(n-k)$$

Related and future work

- Conditional lexic. orderings introduced by [Wilson, ECAI'06]
  $\Rightarrow$ approximate CP-nets
- Need to explore heuristics to choose variables during execution of the greedy algorithm
- Problem if tree not complete : the ordering is only partial
  $\Rightarrow$ Need to explore mixtures of conditional / unconditional structures
- Need to test algorithms on real / generated data
  $\Rightarrow$ How to deal with noisy date ?