

*Diploma thesis*

KRK Chess Endgame Database  
Knowledge Extraction and Compression

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Darmstadt, Juli 2006

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# Abstract

The chess endgames are a perfect test domain for Machine Learning algorithms. Reciprocally, Machine Learning offers efficient methods to analyse and compress chess tablebases. Such databases consist of informations at a low level of description and are often huge, what makes them unpractical. Our goal is to reduce the size of the King and Rook against King (KRK) database without information loss with the help of knowledge-based compression. Some studies already tried in the case of the KRK endgame to use specific patterns in order to build a knowledge with a higher level of description. Grouping such patterns together, we transform the primary tablebase in a more compact form containing only values of these attributes. The representation of this new database as a decision tree simplifies its description, enabling a better compression rate than standard mechanical methods.



# Chapter 1

## Introduction

The game of chess is one of the oldest and most complex game in the history of humanity. With a simple board, thirty two pieces and a few rules it achieves a complexity level that the current computers cannot master.. The huge amount of possibilities in the game make it a perfect test domain for Artificial Intelligence, and especially for Machine learning: if we are able to invent a computer program that can learn to play such a difficult game, then we can also create similar ones that can solve a large part of real life problems which usually have a much smaller complexity. Reciprocally, Machine Learning algorithms can be tested very well on chess problems. Actually, the chess endgame domain is already complex enough to be a standard test place for Machine Learning algorithms. In particular, the compression of data using knowledge-based methods coming from the Artificial Intelligence domain can be perfectly studied on chess endgames tablebases, which provide large amounts of information at a very low level of description.

Our work is precisely engaged in that subject: Having a database which lists all existing positions of a 3 figures endgame, is it possible to compress it with knowledge extraction methods? Could another form of the database expressed by concept attributes from the chess domain be smaller than the primary one? This is the topic we study in the present report, in the case of the King and Rook against King endgame. After explaining in the first chapter some basis concepts of the domain and our precise procedure, we summarize the literature our work is based on. We then look in detail at the new knowledge which we build to transform the database and obtain new ones, whose properties we analyse afterwards in the fourth chapter, before transforming the databases again to obtain the optimally compressed form presented in the fifth chapter.

## 1.1 Chess Programming

### 1.1.1 Complexity in Chess Programming

The main problem of chess programming is the very large number of continuations involved. In an average position there are about 40 legal moves. If you consider every reply to each move you have  $40 * 40 = 1600$  positions. This means that after two ply (half-moves), which is considered a single move in chess, 1600 different positions can arise. After two moves there are 2.5 million positions, after three moves 4.1 billion. The average game lasts 40 moves. The number of potential positions is to the order of  $10^{128}$ , which is vastly larger than the number of atoms in the known universe (about  $10^{80}$ ). The exhaustive listing of all possible chess games is therefore impossible. Nevertheless this process is possible if only few pieces stay on the chess board at the end of a game, i.e. in chess endgames.

### 1.1.2 Common Endgames

The simplest endgames contain 3 pieces. A two pieces endgame has no interest, because the two pieces are the two kings and one king alone cannot take advantage over the other, so the game is drawn. The number of positions in an endgame database is to the order of  $60^N$ , where N is the number of pieces. The combinatorial explosion makes an enumeration impossible for endgames of more than a few pieces. The complexity of common endgames is listed in the table 1.1.

No. of pieces	Complexity
3	262 144
4	1 677 216
5	1 073 741 824
6	68 719 476 736

Table 1.1: Amount of positions in chess endgames

Endgame databases were originally constructed by Thompson ([Tho86]). They exist for all 3, 4 and 5 pieces endgames. Some have already been generated for 6-pieces endgames, but they are very complex. This complexity can be reduced with basic rules: Two pieces cannot stand on the same square; or basic chess knowledge: The two kings cannot be on adjacent squares (because a king cannot put itself in chess). With these rules, some theoretical possible positions are illegal

and so they don't have to appear in the database. Furthermore, depending on the nature of the pieces that exist in the endgame, some symmetries can also reduce the complexity. Particularly, endgames without pawns have no concept of direction (since the pawn is the only piece in chess that *goes forward*, all the other pieces can move the same in any direction). That's why pawnless endgames have a smaller complexity, as more symmetries can be applied to reduce the number of interesting possible positions.

Common 3-pieces endgames are King and Pawn vs King (KpK), King and Queen vs King (KQK), King and Rook vs King (KRK, which we will consider here) or King and Knight vs King (KNK). In the 4-pieces category there are for example King, Knight and Bishop vs King (KNBK), King and Queen vs King and Rook (KQKR) or King, Knight and Pawn vs King (KNpK). As a 5-pieces chess endgame we can cite King and two Knights vs King and Pawn (KNNKp). The basic endgames are finished by the end of the game: By check mate (if a side has its king in check and cannot do anything to avoid it), or draw (ie stalemate, if a side has no legal move to do but its king is not in check, or if checkmate is impossible, i.e. for example if only the two kings remain on the board). Some more complex endgames can be considered as ended by getting into a simpler endgame, like KQKR ends as KQK if the black rook is taken. Such endgames are then also called *converted*.

### 1.1.3 Endgame Databases

#### Different types of databases

Often also called tablebase, a chess endgame database is an ordered list of all positions in the endgame with interesting calculated values. There are different types of endgame databases. They usually list *black-to-move* positions, as the strongest side is often the white side, so the black side is mated in the simplest positions in the database, but they can also list *white-to-move* positions. All types of databases know whether a given position in the endgame is a win, loss or draw. If that is all the database contains, it is called a WLD-database (Win/Lose/Draw). If the database contains information on how long it will take until the game is over, it is called a distance-to-mate (DTM) database. This depth of win is calculated in case of an optimal play. If it contains only the information on how long it will take until a conversion takes place, it is called a distance-to-conversion (DTC) database. A conversion is either a pawn promotion or a piece being captured or checkmate. If a piece is being captured or a pawn promoted,

we come either to a draw if only the kings remain, or we have to look in another database with the new pieces set. These databases do not take into account the *fifty moves rule*<sup>1</sup>. To overcome that deficiency, Distance to zeroing (DTZ) and the superior Distance to rule (DTR) databases have been created, but only in theory.

The WLD database is smaller, but it has the problem that even though a program may be in a winning position, it might not be able to actually win the game. All the database tells is that it has a win, and it also tells which moves conserve the win. But some win-conserving moves may increase the distance to mate, and the program cannot easily decide which of these win-conserving moves to make. DTM databases are obviously better in this respect, since you just make the win-conserving move with the lowest DTM associated. DTC databases also solve the problem of winning a won position, however the program might take longer than necessary to do so. The main advantage of the WLD database is its size: storing WLD information only needs little space, therefore larger parts of the database can be kept in memory, if the database size is larger than the amount of memory of the computer (which is typically the case). Accessing the database when it is not loaded in the main memory is not really an option, as the speed is very slow compared to memory.

The best known databases are the ones of Ken Thompson ([[Tho86](#)]), Steven J. Edwards ([[Edw95](#)]) and Eugene Nalimov ([[NWH99](#)]). They are tablebases of type DTC, DTM and compressed DTM respectively. Most commonly used are those from Nalimov since they are free and more efficient. Nalimov tablebases are nearly "perfect" since they take into account *en Passant*<sup>2</sup>. However, they don't take *Castling*<sup>3</sup> into account, but it is usually ignored in a tablebase, because games in practice rarely reach the endgame without a king or rook moving, so castling almost never make sense here.

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<sup>1</sup>The *fifty move rule* in chess states that a player can claim a draw if no capture has been made and no pawn has been moved in the last fifty consecutive moves.

<sup>2</sup>*En passant* is a maneuver in the board game of chess. The *en passant* rule applies when a player moves a pawn two squares forward from its starting position, and an opposing pawn could have captured it if it had only moved one square forward. The rule states that the opposing pawn may then capture the pawn as if it had only moved one square forward.

<sup>3</sup>*Castling* is a special move in the game of chess involving the king and one of his original rooks. It consists of moving the king two squares towards a rook, and moving the rook onto the square over which the king crossed.

## Generation of chess databases

Endgame chess databases are generated via induction. The generation is easy if you have a good algorithm, but it can require huge amount of time and computer memory depending on the complexity of the databases which are generated. The generation consists in the following steps:

1. All possible positions are examined, and positions where one side is mated are marked. These are called *mated in 0* positions.
2. Once this is done, all positions are examined again, except those which are already marked, and if any of them can reach a “mated in 0” position, it is marked as *mate in 1*. The concept *possible position* here depends on the side which has the move: a position in which the black king is in check is not a legal white-to-move position.
3. Now we look at everything again, and the possible positions from which all possible moves lead to a “mate in 1” position are marked as *mate in 2*. It could seem weird to call these positions mate in 2, because mate appears after only 2 plies, but it is due to the fact that the losing side has the move.
4. Now we look at all the possible positions again, and we try to reach “mate in 2”: The found positions are also marked “mate in 2”.
5. The next step is a bit different: We look for cases where it is impossible to avoid “mate in 2” and “mate in 1” positions. These are marked *mate in 3*.
6. And so on, until no progress is made. The rest of the positions are proven to be draws.

The last step can be difficult to evaluate, because the concept “no progress is made” depends on possible conversion. For instance in the endgames with pawns, if a pawn converts, you can end in several different other endgames, what brings up new possibilities. For example, there might be a mate in 105 moves in KBP vs KN, but there might not be a mate in 95. This is because the mate in 105 moves might involve immediate conversion to a KBN vs KN.

## 1.2 Database Size Reduction

The utilisation of chess endgame databases require them to be loaded in the main memory of a computer. Since most of these databases are too big for the memory, they have to be reduced. As the database has to be accessed as fast as possible, the general data compression methods or any method that slows down the access time cannot be applied. The aim is to find a more compact representation of the database without information loss.

### 1.2.1 Reduction Through Symmetries

The first and simplest reduction of the complexity is achieved through properties particular to the chess game. The board is the same in every direction, and most of the pieces move the same way forward and backward (in fact all pieces except pawns). That is why the interesting part of the database can be reduce thanks to various symmetries, which are more numerous if there is no pawn in the endgame. To do that, we build position equivalence classes. Each class is represented by a canonical position. All the positions that can be retrieved by rotating or reflecting a canonical position belong to the same class. Only the canonical positions are stored in the database. The exact symmetries and rotations which can be used to determine the canonical positions depend on the pieces of the endgame. We will explain more precisely for the KRK ending.

The KRK endgame is quite simple and its pieces offer good symmetric properties, since the only three figures in the endgame are the white king, the white rook and the black king, and they all move symmetrically in all direction. These symmetries are resumed in the figure 1.1.

All the starting positions can be limited to those having the white king in a ten squares octant (a1, a2, a3, a4, b2, b3, b4, c3, c4 and d4). To obtain that, the following rules are applied (in that order):

1. **WK horizontal reflection:** All the positions in which the white king is on the right part of the board (file e or greater) are reflected through the central vertical axis so that it stands then on the left part of the board.
2. **WK vertical reflection:** All the positions in which the white king is on the upper part of the board (rank 5 or greater) are reflected through the central horizontal axis so that it stands then on the lower part of the board.

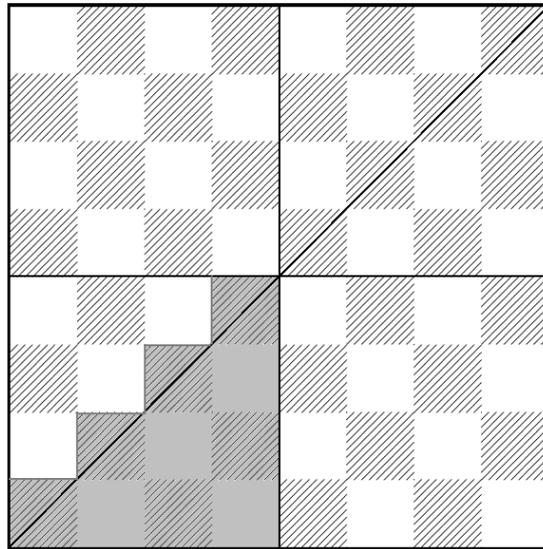


Figure 1.1: Symmetries used to reduce the KRK database

3. **WK diagonal reflection:** All the positions in which the white king is above the diagonal a1 to h8 are reflected through the diagonal axis so that it stands then below the diagonal.
4. **BK reflection:** All the positions in which the white king is on the diagonal a1-h8 and the black king is above this diagonal are reflected through the diagonal axis so that the black king stands then below the diagonal.
5. **WR reflection:** All the positions in which the two kings are on the diagonal a1-h8 and the rook is above this diagonal are reflected through the diagonal axis so that the rook stands then below the diagonal.

As summarised in the table 1.2, these symmetry operations reduced the database to 28056 positions. Compared to the rough KRK database, they make it almost 8 times smaller.

Rough 3 Figures database:	262 144 positions	(64*64*64)
Rough KRK database:	223 944 positions	(only legal positions)
KRK database:	28 056 positions	(with symmetry reductions)

Table 1.2: Amount of positions in the different forms of the KRK database

### 1.2.2 Knowledge-based Reduction

The research of higher level knowledge can help reducing the size of the database. For example, a rule that summarises many lines of the database takes less memory to be stored than all the corresponding lines, even if some exceptions also have to be stored. Depending on the size of a line and the number of exceptions, the induced rule is more or less efficient to compress the database. In our case of the KRK endgame for example, every position where the white rook can be taken is a draw. So, a rule characterising this fact can replace a lot of positions, and help reducing the database.

Some systems that perform Data Mining Compression exist already. For example, SPARTAN ([BGR01]) is such a system that exploits attribute semantics and data-mining models to effectively compress massive data tables. It takes advantage of predictive correlations between the table attributes and the specified error tolerances to construct concise and accurate *Classification and Regression Tree* (CaRT) models for entire columns of the table. The key is to achieve a good compression while staying at a tolerable computation time. To restrict the huge search space of possible CaRTs, SPARTAN explicitly identifies strong dependencies in the data by constructing a Bayesian network model on the given attributes, which is then used to guide the selection of promising CaRT models. The CaRT-building component also employs integrated pruning strategies that take advantage of the prescribed error tolerances to minimise the computational effort involved.

Our method is similar to the one of SPARTAN, but best adapted for our case. We used attributes which were partly automatically detected by existing systems (described in chapter 2). They are generally invented by systems which are helped by chess masters: the system is fed with chess positions, and depending on what it already learnt, it either saves a new pattern or learns a new rule. All these operation are based on Logic Programming (ILP) methods and Data Mining algorithms.

## 1.3 Data Mining

Data Mining, also known as Knowledge-Discovery in Databases (KDD), is the process of automatically searching large volumes of data for patterns. It is a fairly recent and contemporary topic in computing. However, Data Mining applies many older computational techniques from statistics, machine learning and pattern recognition. It can be defined as "The nontrivial extraction of implicit,

previously unknown, and potentially useful information from data”[FPSM92] and “The science of extracting useful information from large data sets or databases” [HMS01]. Although it is usually used in relation to analysis of data, data mining is an umbrella term and is used with varied meaning in a wide range of contexts. It is usually associated with a business or other organisation’s need to identify trends.

Once we have built our new database with patterns which may help significantly in the prediction of the distance of win, we need tools to analyse the database and test which dependencies exist between the pattern values and the distance of win. The Data Mining should provide us with some tools to achieve this goal. We especially use Decision Trees to predict the depth of win of a position as efficiently as possible.

In machine learning, a decision tree is a predictive model, that is, a mapping of observations about an item to conclusions about the item’s target value. The machine learning technique for inducing a decision tree from data is called decision tree learning. It is commonly based on the concept of Top Down Induction of Decision Tree (TDIDT, see [Qui86]). Here, a decision tree describes a tree structure wherein leaves represent classifications and branches represent conjunctions of features that lead to these classifications. A decision tree can be learned by splitting the source set into subsets based on an attribute value test. This process is repeated on each derived subset in a recursive manner. The recursion is completed when splitting is either non-feasible, or a singular classification can be applied to each element of the derived subset.

Amongst other data mining methods, decision trees is a method that has several advantages: Decision trees are simple to understand and interpret. People are able to understand decision tree models after a brief explanation. They require little data preparation. Other techniques often require data normalisation, dummy variables need to be created and blank values to be removed. They can handle both nominal and categorical data. Other techniques are usually specialised in analysing datasets that have only one type of variable. For example, relation rules can be used only with nominal variables while neural networks can be used only with numerical variables. They make it possible to validate a model using statistical tests. That makes it possible to account for the reliability of the model. They are robust, perform well with large data in a short time. Large amounts of data can be analysed using personal computers in a time short enough to enable stakeholders to take decisions based on its analysis.

The open source Data Mining software WEKA<sup>4</sup>[WF05] allows to apply lots of algorithms on data, like decision trees. That is the software we used in our work to analyse our database. In particular, we used the c4.5 algorithm.

c4.5 is a software extension of the basic decision tree generating algorithm ID3. ID3 (short for Iterative Dichotomiser 3) is a basic algorithm from Ross Quinlan to build decision trees. The ID3 algorithm can be summarised in three steps: It first takes all unused attributes and counts their entropy concerning test samples. Then it chooses one of the attribute with the smallest entropy, and eventually makes a node containing that attribute. The c4.5 algorithm contains several improvements, especially needed for software implementation. The improvements contain the possibility to choose an appropriate attribute selection measure, a way of handling training data with missing attribute values. They also allow the handling of attributes with differing costs and continuous attributes. A decision tree can also be pruned or not. Pruning a decision tree means simplifying it by removing some parts. Another important property of a decision tree is the minimum number of instance per leaf (short `minNumObj`). This parameter defines the level of detail of the tree. See [Qui93] and [Mit97] for more details.

## 1.4 Our Approach

The aim of this work is the compression of the KRK endgame database with the help of attributes. We worked on the standard KRK database with 28056 positions distributed as shown in table 1.3. The attributes should represent a knowledge of higher level than pure position description. They can be combined in rules, that are automatically build by a Data Mining Program named WEKA to optimise efficiency.

The first step was to find attributes and to implement them in a script. The script was implemented in ruby<sup>5</sup> and designed to execute the transformations of the attributes practically and efficiently. The source code is available in Appendix B. The attributes are grouped together to form theoretically a consistent set, so that the attributes in the set represent the initial KRK database as good as possible. In fact we have used several sets and then compared the results. Each set of attributes is of the form:

$$S = (\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n)$$

---

<sup>4</sup><http://www.cs.waikato.ac.nz/ml/weka/>

<sup>5</sup><http://www.ruby-lang.org/>

Depth	Positions	Depth	Positions
draw	2796	8	1433
0	27	9	1712
1	78	10	1985
2	246	11	2854
3	81	12	3597
4	198	13	4194
5	471	14	4553
6	592	15	2166
7	683	16	390

Table 1.3: Distribution of the depths of win in the KRK database

Each attribute  $\mathcal{A}$  is in fact a transformation that returns a value  $v$  from a position vector  $P$ :

$$P = (w_{kf}, w_{kr}, w_{rf}, w_{rr}, b_{kf}, b_{kr}, d_{ow})^6$$

$$v = \mathcal{A}(P)$$

The distance to win is in fact not used to calculate the attributes' values, but it is written in the original database besides the positions.

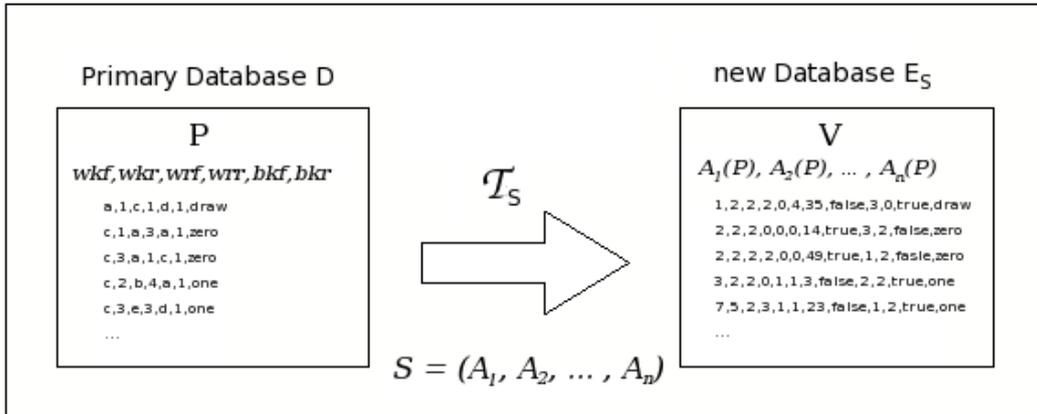


Figure 1.2: General approach: step 2

The second step was to build a new database for each set of attributes, and to check its properties. The new databases are composed of combinations of

<sup>6</sup>white king file, white king rank, white rook file, ..., depth of win

attributes' values, with a distance to win. Let  $D$  be the original database with the positions  $P$  of the pieces on the board:  $P \in D$ . For a set of attributes  $S$ , the vector  $P$  is transformed into a vector  $V$  so that:

$$\forall i \in [1, n], v_i = \mathcal{A}_i(P)$$

$$V = (v_1, v_2, \dots, v_n) = (\mathcal{A}_1(P), \mathcal{A}_2(P), \dots, \mathcal{A}_n(P)) = S(P)$$

As the depth of win should appear at last in the new database, we have

$$\mathcal{A}_n((w_kf, w_kr, w_rf, w_rr, b_kf, b_kr, dow)) = dow$$

The new database  $E_S$  is then the list of all the combinations  $V$ :

$$E_S = \{S(P), P \in D\}$$

So we define a transformation  $\mathcal{T}_S$  from  $D$  to  $E_S$ , like drawn in figure 1.2.

$$\begin{array}{ccc} & \mathcal{T}_S & \\ D & \longrightarrow & E_S \end{array}$$

Depending on the attributes' set, it can be that 2 different positions  $P_1$  and  $P_2$  with a depth of win  $dow_1$  and  $dow_2$  have the same image through all the attributes of the set except the last one (that encodes the depth of win):

$$\forall i \in [1, n-1], \mathcal{A}_i(P_1) = \mathcal{A}_i(P_2)$$

If the depths of win are the same ( $\mathcal{A}_n(P_1) = \mathcal{A}_n(P_2)$ ), which should be the case if the attributes perfectly describe the board, the 2 positions are summarised by one image:

$$dow_1 = dow_2 \implies S(P_1) = S(P_2)$$

But if they are different ( $\mathcal{A}_n(P_1) \neq \mathcal{A}_n(P_2)$ ), we have the following problem: 2 different positions with distinct depths of win are described by the same combination of attributes' values. This may be solved with the help of exception handling. Since we need to be able to retrieve perfectly the initial database, we have to store separately the exceptions, so that the transformation is a bijection. To achieve that, we extract from the initial database all the positions which would lead to an exception and store them in a separated file  $D_{err}$ . The original database without these positions is called  $\tilde{D}$ , and the transformation  $\mathcal{T}_{S|_{\tilde{D}}}$  from  $\tilde{D}$  to  $\tilde{E}_S = \mathcal{T}_S(\tilde{D})$  is a bijection.

$$\begin{array}{ccc} & \mathcal{T}_S & \\ \tilde{D} & \longrightarrow & \tilde{E}_S \end{array}$$

So the new database is then composed of two files: The error file containing all the ambiguous positions, and the real new database with the attribute values, which will then be compressed with data mining algorithms. Our aim is of course to have an error file as small as possible, since it cannot be compressed efficiently (only standard compression like zip). In the perfect case, all the positions have a unambiguous image, so there are no errors.

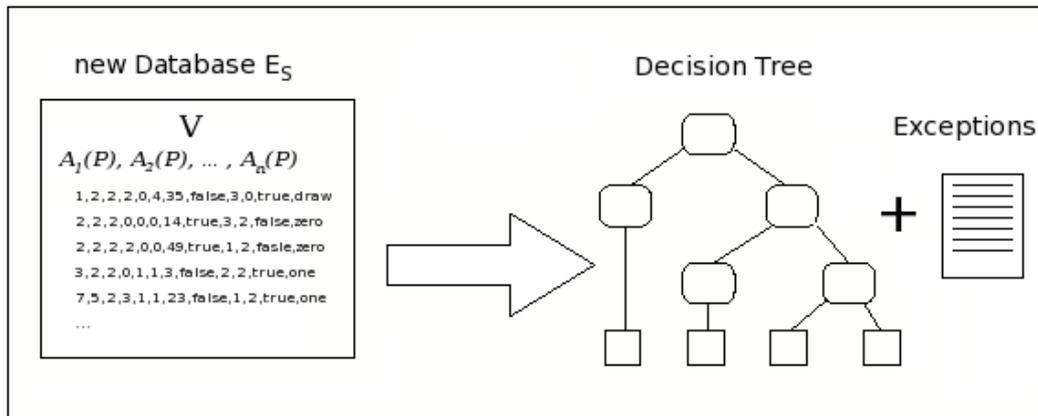


Figure 1.3: General approach: step 3

The third step consisted in trying data mining algorithms to compress the databases. It is illustrated on figure 1.3. We worked with decision trees, in which the depth of win of the pattern value combinations in the new database is analysed. As the tree can generalise and regroup some attribute combinations together, it needs less disc space when saved in a file than the database. Moreover the properties of the tree can be adjusted to have the best results. But it can happen that some instances are incorrectly classified in the tree, depending on its properties. Therefore we need to store beside the tree a list of the incorrectly classified combinations.



# Chapter 2

## Review of related work

The KRK endgame was already the subject of a lot of studies, especially in Machine Learning. Some of these studies particularly deal with patterns, which are supposed to represent some chess knowledge, what is exactly our subject. Our study is based on three papers and on the description of a chess playing machine. These four sources contain interesting elements for our work, therefore I will give an overview of their content. Please refer to the original papers for more details.

### 2.1 Generalising Closed-World Specialisation: A Chess End Game Application, by M. Bain ([BMS95])

In his paper Bain describes his experiments with a system based on an algorithm named Closed-World Specialisation (CWS). He tested his system on the KRK chess endgame. His aim was to develop an agent that, given access to the complete list of pre-classified positions, can learn rules to correctly classify each position by the optimal number of moves to checkmate, or as drawn. This system is based on a few simple patterns, and creates new ones to build recognisers for each depth of win.

#### 2.1.1 Introduction

Theory discovery is a branch of symbolic learning that has a good application in chess endgames: An agent has to learn from a chess database a set of rules to

classify each position of the endgame. The rules are expected to be more compact than the database and have to be transparent to a human. The example of KRK typically offers a large amount of data at a low level of description, from which a simple set of comprehensible rules with good predictive accuracy has to be learned

In many real-world problems the size of the database grows continuously even after the start of the learning phase of the algorithms. That leads to possibly too big databases for batch learning. Therefore a sampling approach is needed as well as the possibility for the learned theory to change continuously. Thus Bain and Muggleton were interested in incremental learning systems, with methods of correcting theories in a framework of minimal specialisation. It was shown that minimal correcting specialisations are not always obtainable within classical logic. Consequently they presented an algorithm carrying out a specialisation scheme based on a non-monotonic logic formalism used in logic programming: Closed-World Specialisation. After having implemented and tested it in chess endgame domains and on real-world data, its specialisation scheme showed some issues, what led them to combine it to a generalisation method, what produces a system called Generalising Closed-World Specialisation(GCWS).

A key feature of CWS is its ability to augment the hypothesis language by predicate invention. GCWS is designed to learn general rules which may have one or more levels of exceptions, what is suited for chess. In addition, incremental learning coupling with theory-guided sampling allows to deal with large data sets, since the system operates at any given time with only a subset of the total problem domain.

### 2.1.2 Closed World Specialisation

Minimal specialisation, i.e. most general correct specialisation, can lead to theories which are not finitely axiomatisable. To avoid this problem, Bain and Muggleton used the “negation as failure” rule in the context of normal logic programs. Following [Llo87] they develop their method of CWS with the help of semantic definitions.

#### **Closed-World Specialisation algorithm and incremental learning**

The CWS transforms a normal logic program given a single negative example. In practice, a set of negative examples has to be respected. This is often the case

within an incremental learning process which includes alternate training and testing phases. Such testing may also detect uncovered positive examples in addition to covered negative examples. So CWS is incorporated in an incremental learning system with the implementation of learning from batches of examples. The batch transformations return from a possibly incorrect and incomplete program another program which will be correct and complete with respect to the examples of the test set. In practical systems, these batch transformations are not used. Other transformations for generalisation and specialisation are used in GCWS. These are implemented using a number of ILP algorithms: The generalisation method uses versions of Golem ([MF92]), and the specialisation method uses different “test-then-specialise” versions of CWS.

To avoid over-specialisation, the incremental learning of programs where the target predicate is invoked by some other predicate is not considered here.

The examples used by GCWS in the incremental learning process are selected through theory-guided sampling: Only examples found to be exceptions towards the current program are used in subsequent learning. So much larger example sets can be handled than by a “one-shot” learning system.

### **Discussion of Closed-World Specialisation and Generalising Closed-World Specialisation**

Two recent approaches (about machine learning and logic programming) related to the CWS algorithm are discussed here.

[Wro93] proposed the use of criteria other than minimal specialisation for theory revision. Unfortunately this framework depends on a classical logic representation, in the belief of which sets are closed theories, and thus minimal specialisations may not be finitely axiomatisable.

In Logic Programming the specialisation problem may be expressed in terms of updating logic databases. For instance, [GL90] and [GL91] have presented results for provably correct updates on normalised logic programs. Deletion and insertion are carried out by update procedures which return a set of database transactions. The specialisation operation is then the deletion of an atom from the program. This method though does not guarantee minimal specialisation.

Future work should investigate in more detail the relationship between belief revision in AGM logic and other frameworks and the method of closed-world specialisation. Perhaps in practice minimal specialisation is too conservative for machine learning applications where the main focus is on theory construction.

And it may be more suited for applications where the task is to find small adjustments to a substantial theory, which is already largely correct.

### 2.1.3 A classifier of KRK by optimal depth of win

As mentioned before, chess endgames have highlighted knowledge representation issues for learning systems. A well studied case is the KRK endgame, with the target predicate “White-to-move position is illegal”. An incremental ILP algorithm already enabled the induction of a complete and correct solution for the KRK illegality problem ([Bai91]). A harder problem is to know if a machine could learn to play the game optimally from samples of the database and simple facts about the board geometry.

#### KRK BTM database

Chess endgames are complex enumerable domains, what enables the construction of databases: tables of legal positions with the number of moves until a side wins assuming minimax-optimal play. Such databases provide positive and negative examples and also an oracle ([Roy86]) for testing induced rules. These databases are generated using a single iterative process using Shannon’s standard backup algorithm. They can be used to play the endgame optimally using only a legal move generator and a database look-up program. As the order of an endgame database is  $64^N$  (with N the number of pieces) without removal of redundancies, the exhaustive enumeration is impossible for endgames with more than a few pieces. Various symmetries can be used to reduce the size of the databases, so that only the canonical positions appear. Only information on Black-to-move (BTM) positions was extracted from the database, what is sufficient to play optimally with a 2-ply (i.e. 1 move) legal move generator. In the KRK database, the symmetries enable to reduce the size of the database from potentially 262144 to 28056.

#### Structure of an optimal classifier

The framework of this work, called classifier, handles Black-to-moves KRK legal, canonical positions and returns an integer from 0 to 16 included or a symbol denoting draw. Its structure is shown in figure 2.1. Each separate recogniser should be induced by the GCWS algorithm, and should detect if the given position is of its depth of win. By this process of elimination a position is classified by

the minimum number of moves necessary for White to win, assuming minimax-optimal play.

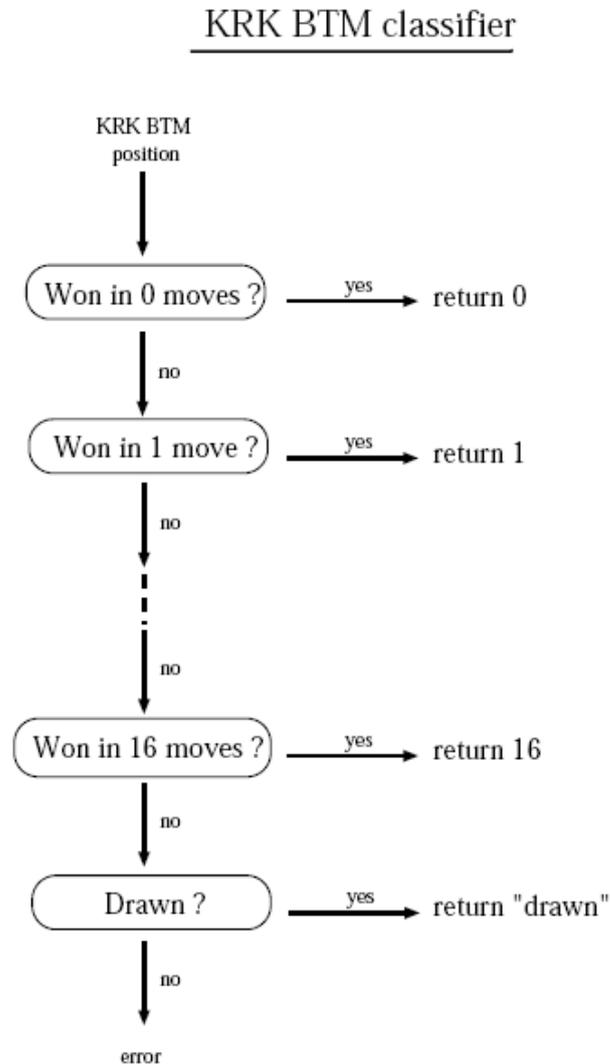


Figure 2.1: [BMS95]: Sequentially-ordered KRK BTM position recognisers from a single global classifier.

The ordering of the recognisers constraints the example sets from which each recogniser is induced and determines which exceptions have to be excluded, since the negative examples at depth  $D$  comprise only positions won at depth  $> D$  plus draw. Therefore when the completeness and correctness of a recogniser is described in this paper, this is to be understood only in reference to the whole classifier structure. The classifier is implemented by employing the representation

mechanism of Quintus Prolog predicates. The predicate `krk/7` with the depth of win  $N$  as first argument and the file and rank positions of the White King, the White Rook and the Black King respectively as other arguments mean that the given position can be won optimally in  $N$  moves. As Prolog indexes the clauses of a predicate in order of the first argument, this representation implements the ordering of recognisers as in figure 2.1, as long as draw positions are assigned to an integer greater than 16. The background knowledge of the algorithm was restricted to contain only one specifically chess-oriented geometrical relation, namely the symmetric difference between files and between ranks:  $abs(V_1 - V_2)$  ( $V_1$  and  $V_2$  being the two files or the two ranks). Other background predicates available were `num/1`, specifying the ranks 1-8 and files a-h, and `edge/1`, specifying the edges of the board along rank and file<sup>1</sup>. These background predicates were selected as basic building blocks for the expression of piece relations in terms of geometry of the chess board. They facilitate the expression of higher-level chess concepts such as capture, safety, check, etc. Thus they supply some of the raw material for relevant predicate invention in chess domains.

### Induction of complete recognisers not guaranteed correct

The learning tasks for the recognisers were created with Golem ([MF92]). For each depth of win  $D$ , the positive examples set are the positions with depth of win  $D$ , and the negative set is chosen randomly amongst the positions of depth of win  $> D$ . A generaliser is expected to compress, i.e. the hypothesis should be of less complexity than the original examples. Any gain in compression may be at the expense of introducing some incorrectness into the hypothesis. The method produces generalised recognisers for each depth of win, which were tested over the whole database. For large example sets (like for depth of win 14), Golem may fail to find a generalisation, what increases the complexity of the theory. The draw positions are classified by default: they are the positions which were not classified by any recogniser.

The system was tested as a compound classifier regrouping the sequentially-ordered recognisers. For each depth of win  $D$ , the tests were defined by using as a set of positive examples all the positions won at depth  $D$  and as a set of negative examples all the positions won at depth  $> D$ . The results are shown in figure 2.1.3. The depth 14 is perfectly classified, but very complex. The depths 16 and draw have no position, because they were classified before by another recogniser.

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<sup>1</sup>Some other simple predicates are used to detect valid positions.

Depth	Correctly Predicted	Incorrectly Predicted	Description Complexity
0	27	99	2
1	78	433	3
2	178	178	5
3	48	418	4
4	63	1098	8
5	181	2022	9
6	67	2793	9
7	90	3427	11
8	76	1118	23
9	183	4441	23
10	221	2276	25
11	143	634	53
12	359	2517	77
13	473	1693	109
14	1153	0	4553
15	1146	410	22
16	0	0	12
draw	0	0	-
Total	4496	23560	7774

Table 2.1: [BMS95]: Sequentially-ordered BTM recognisers by depth of win. The total lack of cases for Depth=16 and draw is the result of their having erroneously been claimed by recognisers earlier in the sequence.

### Induction of complete and correct recognisers

To develop complete recognisers which are also correct, a radical extension of Golem was built according to GCWS. The generalisation step is based on Golem and is called *not(Golem)* due to his ability to perform Closed-World Specialisation. The specialisation step was implemented in two variants. In the Strategy 1 specialisation, CWS modifies the current hypothesis with respect to the exception. In the Strategy 2 specialisation, the faulty clause is deleted and rebuilt with

Theory	No. of clauses	No. of invented predicates	Size of encoding in bits
Depth 0 (Strategy 1)	9	5	3491(+88935=92426)
Depth 0 (Strategy 2)	6	3	3174(+77524=80698)
Depth 0 (All Examples)	6	3	3177(+77524=80701)
Examples			278893
Depth 1 (Strategy 1)	16	7	4310(+112116=116426)
Depth 1 (Strategy 2)	11	4	3633(+96964=100597)
Depth 1 (All Examples)	12	4	3588(+100482=104070)
Examples			278608
Depth 2 (Strategy 1)	31	7	4931(+138474=143405)
Depth 2 (Strategy 2)	23	7	4381(+126438=130819)
Depth 2 (All Examples)	20	6	4329(+120802=125131)
Examples			277833
Depth 3 (Strategy 1)	61	17	7322(+164311=171633)
Depth 3 (Strategy 2)	44	12	5784(+151253=157037)
Examples			275388
Depth 4 (Strategy 1)	116	38	12608(+189444=202052)
Depth 4 (Strategy 2)	89	22	9035(+178885=187920)
Examples			274583
Depth 5 (Strategy 1)	338	88	30974(+230402=261376)
Depth 5 (Strategy 2)	185	40	15445(+20655=222000)
Examples			272614

Table 2.2: [BMS95]: Comparison of specialisation strategies.

respect to the exceptions. Each depth of win  $D$  was treated as a separate learning problem, with all the positions won at  $D$  as positive examples and randomly chosen position won at  $> D$  as negative examples.

The GCWS incremental algorithm was run to induce six separate complete and correct recognisers for depths of win 0 to 5 moves with both strategies. The induced theories begin to pass the human horizon of comprehension by depth 2. The results for both strategies are shown in figure 2.1.3. For depth 0 to 2, the results for both strategies are compared with the results of a batch learning of *not(Golem)* of all examples. The complexity is first measured in number of Prolog's clauses and the number of the invented predicates, and second in the encoding size of the recognisers. The encoding size is calculated as encoding size of the hypothesis plus the encoding size of the proof (a specification of the derivation of the examples from the hypothesis). It can be compared to the size of all the learning examples.

Some common concepts that are present in different recognisers cannot be detected with this method. So a test was made to combine the recognisers for depth 0 and 1, to see if the idea could bring better results. A common depth 0 and 1 recogniser was build and contains 13 clauses. No more tests were made to deepen this method.

## Discussion

The depth 0 and depth 1 recognisers are similar to those of an earlier study ([Bai94]) and were validated as meaningful by a chess expert. At depth 2, the recogniser has 2 clauses which cover two thirds of the total positions. Moreover at this depth, exceptions to exceptions appear, what was not seen before.

As seen in figure 2.1.3, the recognisers are more compact than the examples set. But the compression is small for depth 5, and the complexity trend suggests that this approach would be bad for depth 6 and more. Strategy 2 is more compressive and produces less complex theories, so it is here more suitable than Strategy 1.

The test for a common concept in different depths could be an interesting idea to push on, because the depth 0 and 1 recogniser contains 13 clauses, compared to 17 clauses for the 2 recognisers for depth 0 and depth 1.

### 2.1.4 Concluding Remarks

The KRK problem is a good example of a large-scale and unstructured data problem, particularly with the aim of transparent theories. The results of the current work surpass our best previous solution, what shows the applicability of our method of incremental learning in a non-monotonic representation. However our method may fail for depth bigger than 5 in the KRK problem. It seems that further progress will depend on approaching the learning task in a different way. The decline in compression for recognisers as depth of win increases suggest that more various predicates should be used, either with user-supplied background or with other predicates invention methods. A tree could replace the sequential organisation of the recognisers, in which the depth of a position is framed more precisely at each node. The optimal criteria could be also adapted, to another metric for example.

### 2.1.5 Benefit for our Work

The system GCWS explained in the paper and *Golem* are based on several chess patterns used to describe the board and to test some properties of the position (like if the position is valid or not), as recalled in the paper of J. Fürnkranz: [Fue93]. These patterns are general chess patterns or simple description patterns, and they are exactly the types of attributes we need, since we aim to describe the board within the KRK ending using chess knowledge. The considered patterns are listed in the table 2.3. Other patterns were found by the system, but they are too complex to be simply used.

distance(X, Y)	returns the symmetric difference between the files or the ranks of 2 figures
adjacent(X, Y)	determines if the files or the ranks of 2 figures are side by side or the same
between(X, Y)	determines if the file (or the rank) of a figure is between the files (or the ranks) of 2 others

Table 2.3: Useful Attributes on which Bain's paper ([BMS95]) is based.

## 2.2 On Learning How To Play, by E. Morales ([Mor97])

In his paper, Morales describes *Pal-2*, an extension of the *Pal* ([Mor92]) learning algorithm. The both systems *Pal* and *Pal-2* learn chess patterns from traces of games. *Pal-2* also learns strategy rules defined with the help of the learned patterns. Morales has tested *Pal-2* on the KRK endgame, and the results include some learned patterns, what we are in interest in.

### 2.2.1 *Pal-2*

Perception and recognition of patterns seems to be a good way to play chess efficiently. The key is to detect good patterns and to learn how to combine them in rules to build a valid game strategy. *Pal-2* can both acquire patterns from board positions like *Pal* and build a playing mechanism based on learned condition-action rules using the detected patterns.

#### Learning rules

*Pal-2* follows traces of games, and for each move made by the winning side, it tests all its known patterns before and after the move and constructs playing rules. A rule is a list of conditions before the move (in form of detected patterns), a move to do, and a list of conditions after the move. Patterns that do not change as a consequence of the move are eliminated from the conditions of the rule after the move. The positions of the pieces are replaced by variables in constructed rules. The coincidences in the values have to be eliminated later by more general rules.

For each move of the winning side, *Pal-2* compares the move made in the trace with the moves suggested by his applicable rules. If a rule suggests the good move, this rule becomes the first one in the rules' list. If no rule is applicable or suggests the good move, a new rule has to be learned and *Pal-2* asks the user for a new pattern.

The rules are ordered in the list as following: the more specific first and the more general last. Every new rule is compared with the other rules for subsumption. When the new rule is recognised as a more general form of another one, the older and specific one is deleted. If the new rule is a more specific form of an older rule,

it is deleted and the older one comes at the beginning. So the rules are ordered and the coincidences are removed.

## Learning Patterns

At the beginning, *Pal-2* has only general purpose chess knowledge, it contains no playing rule and knows the only pattern “being in check”. The main problem of learning a new rule is to know if a rule or a pattern is lacking. *Pal-2* cannot find it out alone, that is why the user has to choose if *Pal-2* needs a new pattern.

Patterns are learned with *Pal*. *Pal* is an inductive logic programming (ILP) system, which uses a restricted *least general generalisation* algorithm to guide its learning strategy. It has to be provided with a set of board positions containing the new pattern, and applies a new constraint which identifies the role of the pieces in the different examples to reduce the complexity of the generalisation algorithm.

primary name	our designation	description
ThreatkR	Adjacent WR-BK	The opposite king threatens our rook
Rook_divs	WR Between WK-BK	Rook divides both Kings either vertically or horizontally.
Alm_oppos	Kings almost in opp.	King is “almost” in opposition with the opposite King.
L_patt	L Pattern	The 3 pieces form a L shaped pattern with the opposite King in check by the rook.
rkK	WK Between WR-BK	The rank / file of Rook is $< / >$ rank / file of King which is $< / >$ rank / file of opponent’s King.

Table 2.4: [Mor97]: Patterns from *Pal-2*

### 2.2.2 Results

In the experiments, the user followed a very simple strategy where the opponent’s king was always moving away from the other king, the rook was dividing both kings either vertically or horizontally and the opponent’s king was not allowed

to move more than one file or rank away from the rook. The patterns learned by *Pal-2* are listed in table 2.4.

### 2.2.3 Benefit for our Work

The system described in this paper has found some patterns (listed in table 2.4) which it has learned while playing the KRK ending. It used these patterns to describe the board and find when it should apply some particular rules. Maybe the patterns it found while playing its particular strategy can help us describe the chess board in a more general context.

## 2.3 Learning Long-Term Chess Strategies From Databases, by A. Sadikov ([Sad06])

In his paper Sadikov investigates the learning of long-term strategies from computer-generated databases. The method consists in splitting the whole game in stages, and to achieve local goals in each stage. In the last stage, the goal is to mate the black king. To identify the stages, a list of known attributes was used, and a big change of their values defines the border between two stages. Sadikov experimented his approach in the KRK endgames and in the more difficult KQKR endgame.

### 2.3.1 Method

The work is based on chess endgames for which a complete database exists. The basic idea is to break down the endgame into stages to build a strategy with subgoals to play the endgame. In each stage, certain features are important and others are not. The stages have to be automatically extracted from the database given a set of position attributes specific for the endgame.

To detect borders between the stages, the changes of the values of the attributes are measured during the play. Sadikov assumed that large changes indicate borders between stages. So the computing of the stages starts with the calculation of the values  $a_{i,j}$  of the attributes for a depth of play  $i$ , for all  $i$  in the database (ie  $i \in [0, 16]$  for the KRK endgame) and for all attributes  $j$ . Then the information gain ratio of these values between the levels  $i$  and  $i+1$ :  $g(i, j)$  are calculated, and regrouped in a vector  $G(i) = (g(i,1), g(i,2), \dots)$ . The assumption is that the

behaviour of  $G(i)$ , when level  $i$  changes, is indicative for the stage of play: If  $G(i)$  stays about the same, then both levels belong to the same stage, otherwise they are in different stages. To test the evolution of  $G(i)$ , a function  $\text{Corr}(i)$  is used:

$$\text{Corr}(i) = \text{Corr}(G(i), G(i+1)) = \frac{E(G(i) \cdot G(i+1)) - E(G(i)) \cdot E(G(i+1))}{\sqrt{\text{Var}(G(i)) \cdot \text{Var}(G(i+1))}}$$

Here,  $E(X)$  is the average value of the values in vector  $X$  and  $\text{Var}(X)$  the variance of values in  $X$ .

kdist	the distance between the two kings
rsafety	rook safety: defined as the distance between the rook and the enemy king
oppL	true if the kings are in opposition
edist	black king's distance to the closest edge
fspace	the space the black king has, as limited by the rook
rdiv	true if the rook divides the two kings
srdiv	true if the rook divides the two kings in direction towards the closest edge
grook	true if the rook holds the black king towards the closest edge
soppL	true if the kings are in opposition that forces the black king towards the closest edge
squeeze	true if both 'grook' and 'srdiv' hold
cdist	the distance of the black king from the closest edge
wkcdist	the distance of the white king from the central cross

Table 2.5: [Sad06]: Attributes from Sadikov

To segment the database, first all the  $G(i)$  are calculated from the database. Second  $\text{Corr}(i)$  is calculated from  $G(i)$ . And third:  $\text{Corr}(i)$  is plotted versus  $i$ , and the local minima are candidate points for borders between stages of the play. When the database is broken down in stages, a classifier can be build, that returns the stage of play for a given position. The intention is that such classifiers for each stage of play are comprehensible, so that they can characterise the stage such that human players can follow the long-term strategy for playing this endgame.

To play the endgame, there are two possible ways: On the one hand the classifiers can be used to define subgoals for each stage (i.e. the conditions to attain the next stage). On the other hand, the classifiers can return an estimation of the distance-to-win. This estimation can be used as a heuristic evaluation function for minimax search up to a chosen depth. The move to play is decided by this minimax search. More precisely, from a given position, the classifier returns the stage of play the position was estimated to be in. Each stage of play has a particular regression function that returns for any position in this stage an estimation of the distance-to-win.

### 2.3.2 Application to KRK

For the KRK endgame the attributes listed in the table 2.5 were used. These attributes are based on a set used in some previous studies, for example in [Bra01].

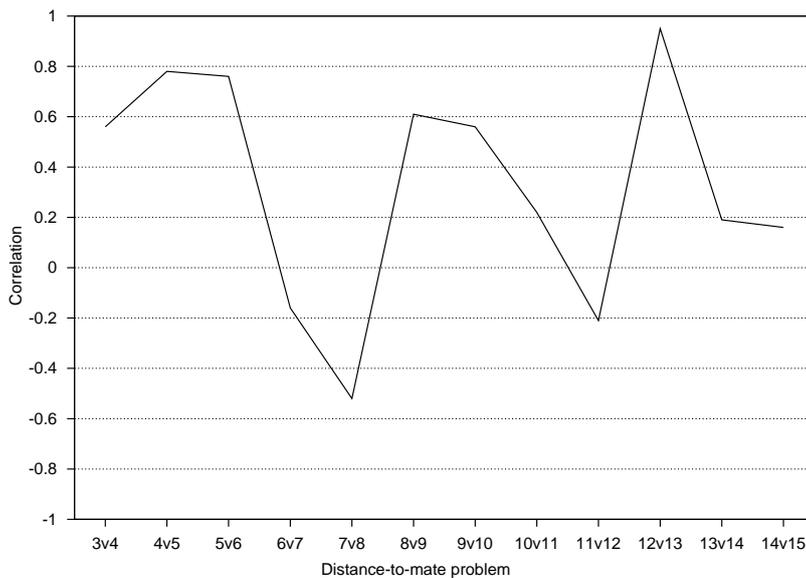


Figure 2.2: [Sad06]: Phase Separation for KRK Endgame

From this list, the value vectors can be calculated and thus the adjacent gain vectors. The figure 2.2 shows the correlation plot measuring the similarity between these vectors. There are two distinct local minima: between levels 7 and 8 and between levels 11 and 12, so the KRK endgame was divided into three phases: Phase CLOSE (levels 0 to 7), phase MEDIUM (levels 8 to 11) and phase FAR

(levels 12 to 16). The levels 0 to 2 don't appear on the plot because they are less numerous in the database, being so a kind of exception.

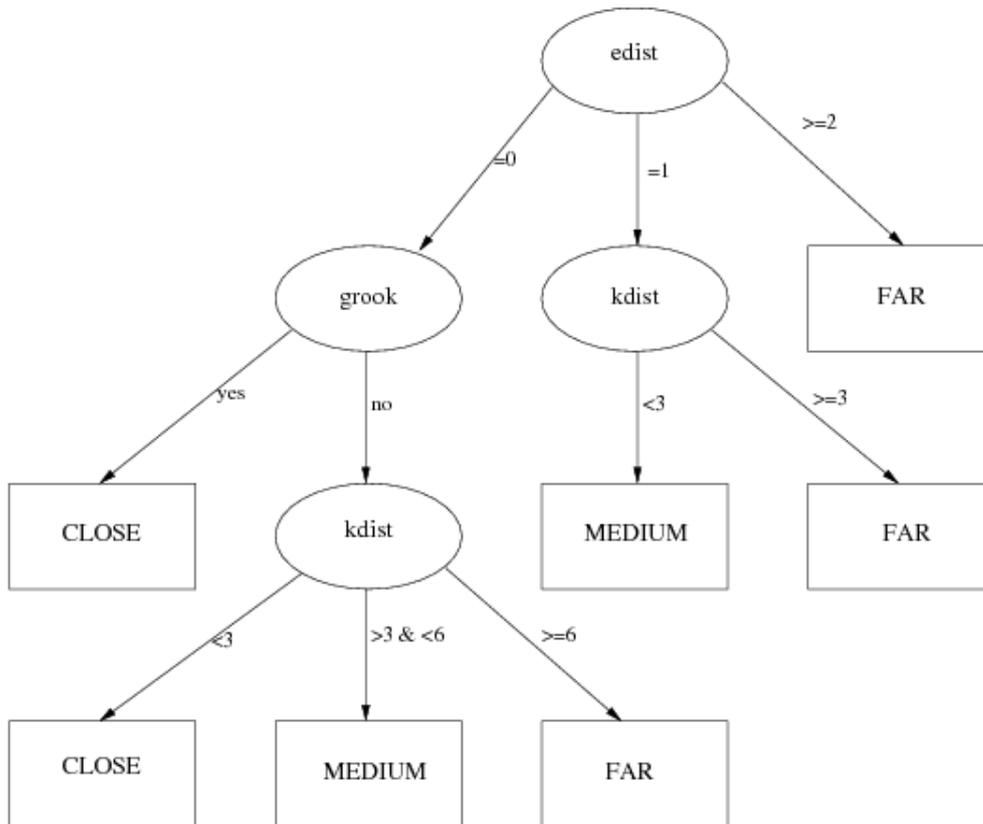


Figure 2.3: [Sad06]: Decision tree for classification into KRK phases

The induced decision tree for classification of positions into the three phases is shown in figure 2.3. It uses only three attributes and is simple to understand. The differences between the neighbored phases give the local targets to achieve next phase. Phase FAR consists mostly of positions where the black king is not on the edge of the board. The objective in this phase is thus to put the black king to an edge. Phase MEDIUM consists of positions where the black king is on the edge of the board, but the white king is far from it. The objective here is hence to bring the white king near the black one. Phase CLOSE regroups the other positions.

Given these stages, different types of evaluation functions were built: Some global functions, based on all the positions of the endgame and varying on the number of attributes they use; some local functions, based only on the positions of a particular stage and varying on the way they are built together to return a global function; and the same local functions used globally on any position. All the obtained regression functions are represented as coefficients for each used attribute.

Regression function	Success Rate	Suboptimality level (in plies)
global with full set of attributes	58.43%	+59.42
global w/o boolean attributes	100%	+13.64
global w/o boolean attributes and w/o ‘wkcdist’	100%	+7.64
local with tree classifier	100%	+8.44
local with perfect classifier (“cheat”)	100%	+12.96
local learned on tree classified data with tree classifier	100%	+16.78
CLOSE local used globally	100%	+8.52
MEDIUM local used globally	100%	+7.42
FAR local used globally	100%	+8.58

Table 2.6: [Sad06]: The performance of KRK evaluation functions

The calculated regression functions are used to play the endgame. The results are shown in table 2.6. The success rate tells the proportion of the positions in which a 6-ply minimax search using a given function was able to win. The suboptimality level tells how many more plies above optimal the player needs in average. The expectation was that local regression functions are better than global functions and than local functions used globally. This is surprisingly not true. The local functions used globally are the best and the global function without boolean attributes and without ‘wkcdist’ is also better than expected. These results indicate that the endgame decomposition into stages did not contribute to the success of play implemented as minimax with distance predictor. Until better ways of using it are found, the decomposition into stages only provided general hints about the endgame to a human player. A better comprehension of the proceeding details can be achieved by the observation of examples of plays of the player compared to the ideal player (which uses the whole database).

### 2.3.3 Experiments in the KQKR Endgame

Sadikov has also tested his methods on the King & Queen vs King & Rook (KQKR) endgame. As the results he obtained there are not related to our study, we will not detail them any further.

### 2.3.4 Benefit for our Work

The attributes Sadikov used are good patterns for the KRK endgame which are often used. They should allow us to distinguish the different depths of win, since Sadikov used them to separate the game in different phases. In table 2.7 we renamed the list shown in table 2.5 so that the names correspond to our designation.

Primary name	Our designation	Description
kdist	Distance WK-BK	the distance between the two kings
rsafety	Distance WR-BK	rook safety: defined as the distance between the rook and the enemy king
oppL	Kings in opposition	true if the kings are in opposition
edist	Dist. BK-Closest Edge	black king's distance to the closest edge
fspace	BK's free space	the space the black king has as limited by the rook
rdiv	WR Between WK-BK	true if the rook divides the two kings
srdiv	WR divides K. tow. CE	true if the rook divides the two kings in direction towards the closest edge
groom	WR holds BK tow. CE	true if the rook holds the black king towards the closest edge
soppL	Kings in opp. tow. CE	true if the kings are in opposition that forces the black king towards the closest edge
squeeze	WR squeeze BK	true if both 'groom' and 'srdiv' hold
cdist	Dist. BK-Closest Corner	the distance of the black king from the closest corner
wkcdist	Dist. WK-Central Cross	the distance of the white king from the central cross

Table 2.7: [Sad06]: Attributes from Sadikov, renamed

## 2.4 Leonardo Torrès y Quevedo's KRK Machine

The description of a chess machine was described first in French in [LaN14], and then in English in [Lev88]. Mr. Torrès y Quevedo, a Spanish engineer, built around 1910 a chess-playing machine that could play without help the KRK endgame against a human player. The operating mode of this machine is very interesting to investigate.

### 2.4.1 Operating Mode

The machine can not start from any starting position. A certain order in the positions of the pieces has to be verified at the start: A vertical direction has to be defined, and the black king should be beneath the white king and the rook. The white rook has to be either on the a-file or on the h-file.

in_zone_ac/1	tests if the rook and the black king are both in 'a', 'b' or 'c'
in_zone_fh/1	tests if the rook and the black king are both in 'f', 'g' or 'h'
vertical_distance/3	returns the vertical distance between two figures (WR and BK or WK and BK)
horizontal_distance/3	returns the horizontal distance between two figures (WK and BK)
odd/1	tests if a number is odd
even/1	tests if a number is even
zero/1	tests if a number is equal to zero
reduce_distance_by_one/3	subtracts a distance by one
>/2	tests if a number is larger than another

Table 2.8: List of the attributes used by Leonardo Torrès y Quevedo's machine

If the opponent plays an illegal move, the machine detects it and refuses to play, and a light gets on. Once three such illegal moves have been made, the robot ceases to play altogether. If on the contrary the defence plays correctly, the robot will carry out one of 6 operations, depending upon the position of the black king compared to the positions of the white king and the rook. The possible moves executed by the machine are: The rook moves to the a-file; the rook moves to the h-file; the rook moves one square down; the king moves one square down; the king moves one square to the left; the king moves one square to the right. One of

these moves is chosen by the machine with the help of a rules system, depending on the position of the black king. The 6 rules used by the machine are listed here:

If the black king is in the same zone as the rook, then the rook moves away horizontally. Otherwise look at the vertical distance between the black king and the rook. If it is more than one square, the rook moves down one square, but if it is just one square, look at the vertical distance between the two kings. If it is more than two squares, the king moves down one square, and if it is only two squares, look at the number of squares representing their horizontal distance apart. If it is zero the rook moves down one square; if it is even but not zero, the white king moves one square towards the black king; if it is odd, the rook moves one square horizontally. The predicates used to to apply these rules are summarised in table 2.8.

### 2.4.2 Benefit for our Work

From the patterns used by the chess machine, we extract some attributes which could help us to describe the chess board. They are listed in table 2.9.

Same Zone WR-BK
Vertical Distance WR-BK
Vertical Distance WK-BK
Horizontal Distance WK-BK

Table 2.9: List of the attributes originated from Leonardo Torrès y Quevedo's machine

Since these attributes allow the machine to play the endgame automatically, they may help us to build our chess knowledge. As they are direction dependent, we will have to find a way to adapt them for all directions so that they can give us a good information for all positions. This adaptation will be the key of our success in using the attributes, and will be described in the next chapter.

# Chapter 3

## Knowledge

This chapter contains the descriptions of the attributes we used to build different new databases. Our idea is that a good combination of these attributes could define an interesting knowledge that can help to compress the database without losing much information.

### 3.1 Preliminaries

#### 3.1.1 Attributes Classification

The attributes come mainly from the four sources we described in chapter 2, but some other were added since they seemed important to us. They are all listed with the set they belong to in the table 3.1. The first four sets, named *Bain*, *Morales*, *Sadikov* and *Torres* in reference of the main author of the papers they come from, represent also the origin of the attributes, in contrary to the fifth set, called *My Selection*, which regroups new attributes that we found interesting and a selection of some attributes from the source papers. These five sets and a sixth one containing all attributes will be used in the next chapters to build new databases.

To keep a clear overview, we classified the patterns in 7 categories depending on their nature: Some attributes just refer to figures distances, some other on the position on the board. . . All the attributes from each category will be explained with examples. Their properties will be detailed as well as the reasons why they are interesting and in which situation. Moreover the implementation of the attributes in the script will be also described.

Name	Bain	Morales	Sadikov	Torres	My Selection
<b>Figure Distance Patterns</b>					
1. Distance WK-BK			×		×
2. Distance WR-BK			×	×	×
3. Distance WK-WR					×
<b>Figure File and Rank Distance Patterns</b>					
4. File distance WR-BK	×				
5. Rank distance WR-BK	×				
6. File distance WK-WR	×				
7. Rank distance WK-WR	×				
8. File distance WK-BK	×				
9. Rank distance WK-BK	×				
10. Alignment dist. WR BK					×
<b>Adjacent Figure Patterns</b>					
11. Adjacent WK-WR	×				
12. Adjacent WK-BK	×				
13. Adjacent WR-BK	×	×			
<b>Between Figure Patterns</b>					
14. WR Between WK & BK	×	×	×		
15. WK Between WR & BK	×	×			
16. BK Between WK & WR	×				
<b>Board Distance Patterns</b>					
17. Dist. BK-Closest Edge			×		
18. Dist. BK-Closest Corner			×		×
19. Dist. WK-Central Cross			×		×
20. Dist. WK-Closest Corner					×
<b>Orientation-based Patterns</b>					
21. Vertical distance WR-BK				×	
22. Vertical distance WK-BK				×	
23. Horizontal dist. WK-BK				×	
24. Same Zone WR-BK				×	
<b>Figure Relation Patterns</b>					
25. Kings in opposition			×		
26. Kings almost in opp.		×			×
27. L pattern		×			
<b>Figure Relation Patterns Associated with the Board</b>					
28. WR divides K tow. CE			×		
29. WR holds BK tow. CE			×		
30. Kings in opp. tow. CE			×		×
31. WR squeeze BK			×		
32. BK's free space			×		×
33. BK's available squares					×

Table 3.1: Attributes with their category and the set their belong to

### 3.1.2 Basis Concepts

#### Figures Distance

The notion of distance that is used here is particular for the chess board: the distance between two figures is the maximal value of the difference between the files of both figures and of the difference of their ranks. In other words, it is the smallest number of moves a king would need to go from the place of the first figure to the place of the other figure on a free board (i.e. without any interference with other figures). For example on the figure 3.1, the white king is in a2 and the black king in c5 so their distance is 3.

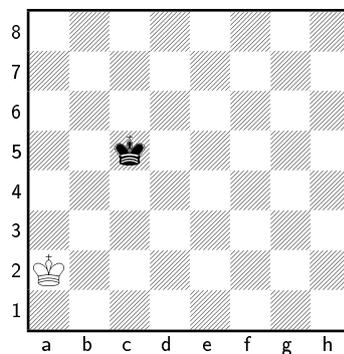


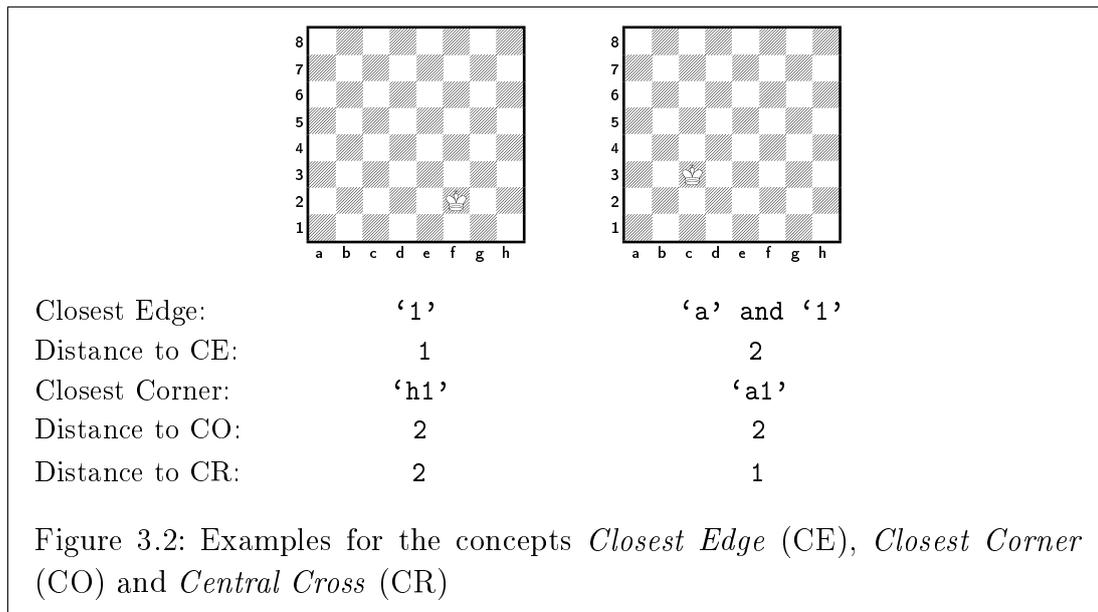
Figure 3.1: Distance example on the chess board:  $distance = 3$

#### Closest Edge

Some attributes are based on the relation between a figure and the border of the board. In this case we speak of the closest edge or the closest corner of the figure. In some cases there may be a problem to define which edge is the closest. The definition is done with a simple method: The distance from the figure to each edge is calculated (the edges are simply the 2 files a and h and the two ranks 1 and 8). Then we compare these distances with each other. If one distance is strictly smaller than the 3 other, like on figure 3.2 first board, then the closest edge is the corresponding one. If there are 2 distances which are the smallest, like on figure 3.2 second board, there are two closest edges, i.e. each one of the two corresponding edges can be taken as the closest edge. We then have to pay attention while implementing the attributes using this feature to determine which one of the two closest edges should be used. Except the attribute *Distance to the closest edge*, for which the value is the same even if there are two closest edges,

only boolean attributes use this feature. For such kind of attributes, if there are 2 closest edges, the returned value is `true` if the test is `true` for at least one of the two edges.

Some attributes use similarly the concept of closest corner. Such a corner is determined with the same method, but there can be only one closest corner.



## Notations

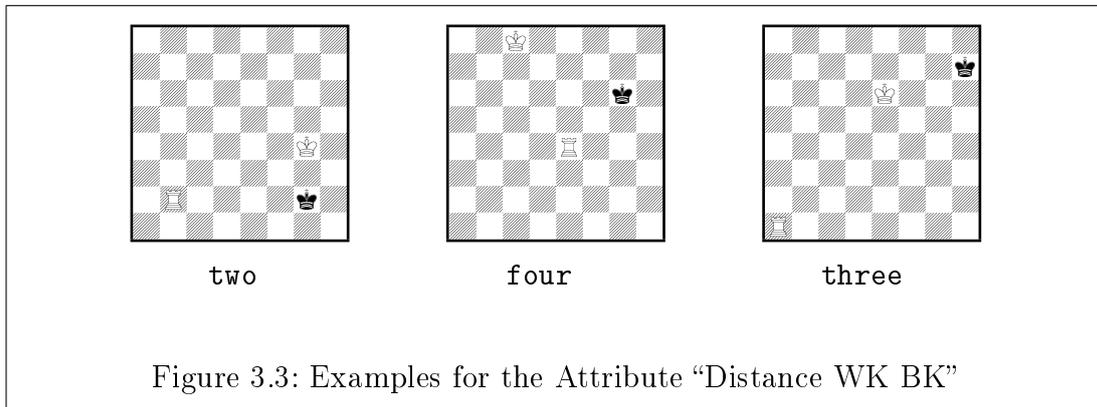
In this chapter, we use the standard notation *WK* for the white king, *BK* for the black king and *WR* for the white rook. Some other shortcuts may also appear, like *CE* for *Closest Edge*, *CO* for *Closest Corner* or *CR* for *Central Cross*.

## 3.2 Figure Distance Patterns

### 3.2.1 Distance WK BK

*How far from each other are the two kings?*

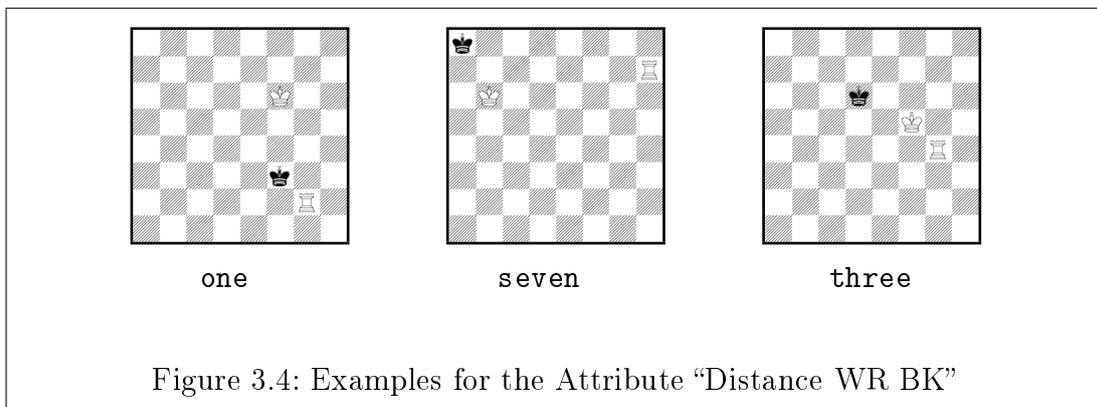
This attribute characterises the distance between the two kings. It can have any integer value between 2 and 7 (because the positions in the database are all legal, so a distance of 1 is not possible). For the winning side, a small value for this attribute is preferable, so that the enemy king is prevented to move as it wants



to. For example on the first board of the figure 3.3, the attribute is equal to 2 and the black king is forced to move one square down to the edge. In this case, this is also due to other reasons, but in general a large king distance (larger than 5 typically) means a long game (a distance to win of 6 or more, except the draws). But a small king distance does not imply anything on the depth of win.

### 3.2.2 Distance WR BK

*How far is the white rook from the black king?*

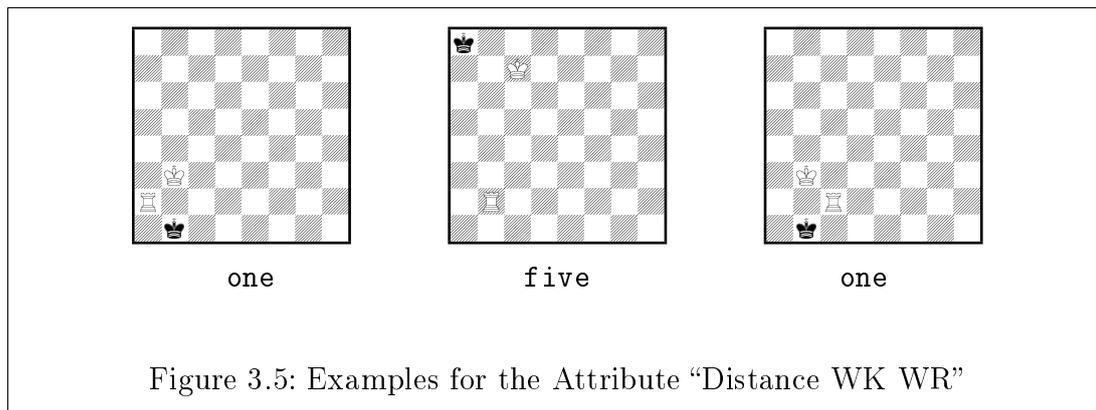


This attribute contains the distance on the board between the black king and the white rook. Its possible values are all the integers between 1 and 7. A value of 1 means that the attribute *Adjacent WR BK* is `true`. A small value here is dangerous for the white side, because the rook may be taken by the black king, and the game is then a draw. To avoid it, the white side has to react and to modify his strategy. Since the distance on the board makes no difference between file distance and rook distance, this attributes give no indication if the rook is on

a good position to hold the black king toward an edge or a corner. For example on figure 3.4, on the board in the middle, the white rook is far from the black king but is in a good position: The black side can be mated in 1 move.

### 3.2.3 Distance WK WR

*How far are the white king and the rook from each other?*



This attribute contains the distance on the board between the white king and the rook. So it can be called the protection distance. Its possible values are all the integers between 1 and 7. A small value here can be good for the white side, because then the rook may be protected by the king. But in the standard strategy this attribute is not particularly small at the end, because the rook can act from the other end of the file or the rank. As the two kings are often near from each other, the rook should better be far from both. This attribute was not used in the source papers, but it seemed interesting to me to test it because of his similarity to the two others of this subsection: *Distance WK BK* and *Distance WR BK*. On figure 3.5, the value of the attribute is one for the first and the third board, but the distance to win is 7 moves in the first case and only 1 in the second case. And for the board in the middle the distance WK - WR is five and the distance to win is one as well. So this attribute alone does not tell anything about the advancement of the game.

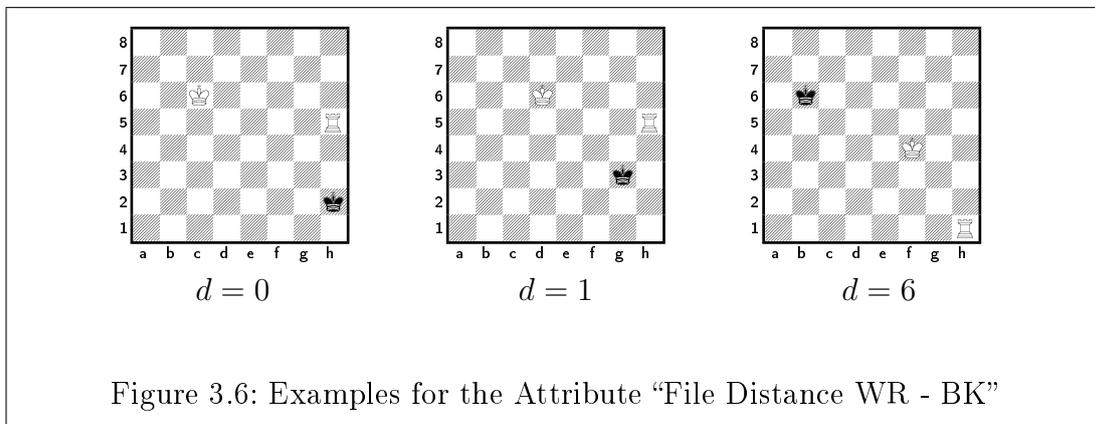
## 3.3 Figure File and Rank Distance Patterns

In this section, the file and rank distances between the figures are explained. The file distance between two figures is calculated by doing a simply difference between

the file of the first figure and the file of the second one. It is the same for the ranks. So the third figure is not taken into account for the calculation. Because of the symmetries of the board and the figures' moves, the rank and file distances have the same meaning taken separately, so only the file distance attributes will be explained, knowing that the rank distances have the same properties. The possible values for these seven attributes are the integers between 0 and 7.

### 3.3.1 File Distance WR - BK

*How far from each other are the file of the white rook and the file of the black king?*

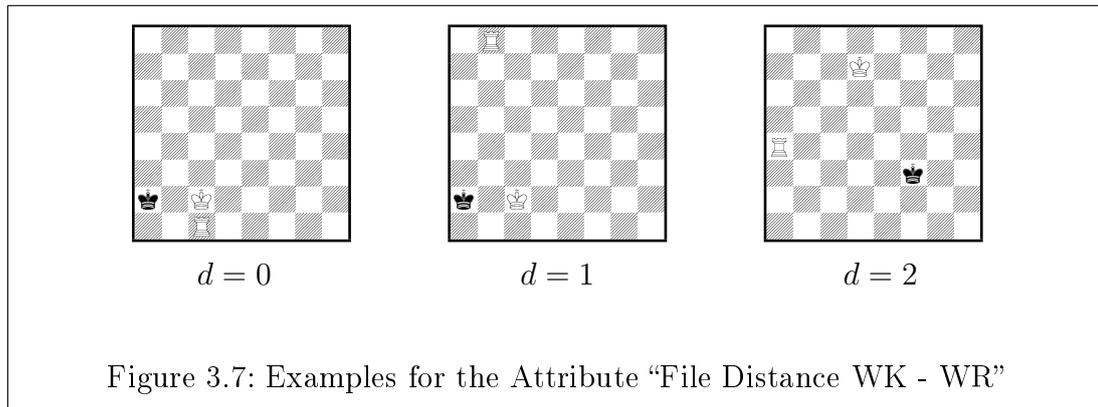


This attribute calculates the distance between the file of the white rook and the file of the black king. A value of zero means that the black king is in check, like on the first board of figure 3.6. If the value is 1, then the rook stands next to the black king, limiting thus its move possibilities. The other values are less interesting, they just tell how many place is left for the black king in the corresponding direction.

### 3.3.2 File Distance WK - WR

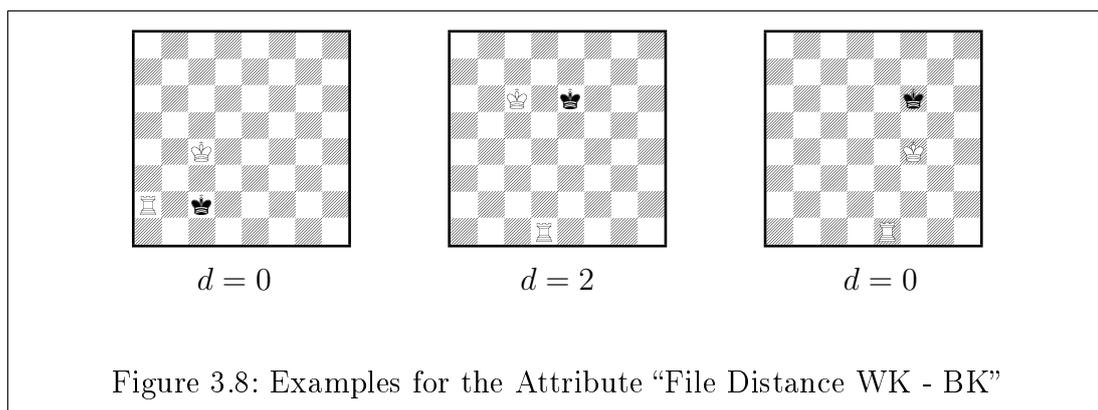
*How far from each other are the file of the white king and the one of the rook?*

This attribute gives the distance between the file of the white king and the file of the rook. If its value is 0, then the white king and the rook are on the same file, what can limit the move possibilities of the rook, like on the first board of figure 3.7 where the distance to win is 7 although the black king is near a corner and in opposition with the white king. A value of 1 can be good if the distance between the kings is not too large, like on the second board.



### 3.3.3 File Distance WK - BK

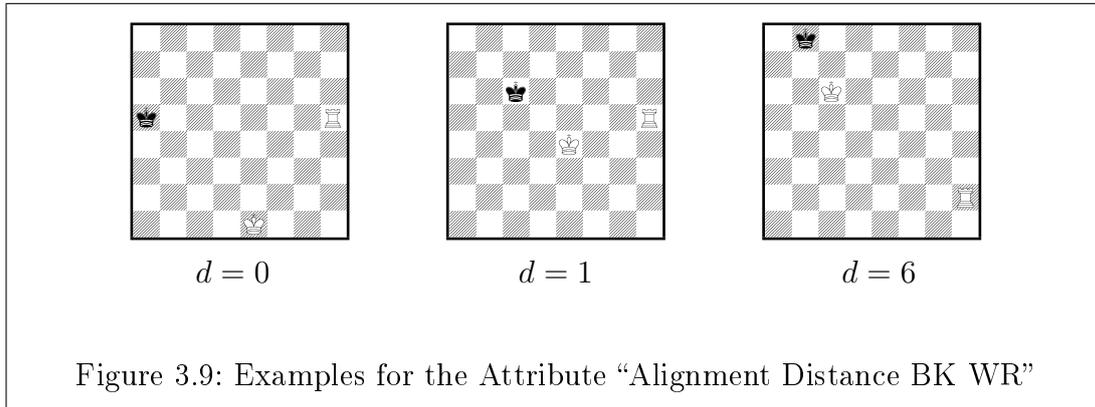
*How far from each other are the file of the white king and the file of the black king?*



This attribute tells us if the two kings are on nearby files or not. It is not as meaningful as the two previous attributes, because none of the kings can move far on a file in contrary to a rook. So the attribute *Distance WK - BK* is more important, but this one was added for completeness.

### 3.3.4 Alignment Distance WR BK

This attribute is different from the other of the section. It also deals with file and rank distance, but here the smaller one of the file and rank distance between the black king and the rook is calculated. It corresponds to the closest distance of the rook's line to the black king. It is more interesting than the other attributes, because it gives an information which does not depend on the direction we look



from. The possible values still are integers between 0 and 7. A value of 0 still means that the black king is in check. But a value of 1 is quite interesting, as it means that the black king is hold on one side by the rook, while being at least one square far from the rook in the other direction. As a matter of fact, this attribute selects what seems to be the most interesting direction between the files and the ranks and returns the distance along the chosen direction. Since the rook can act from far away (almost the other side of the board), this attribute gives more information than the *Distance WR - BK*.

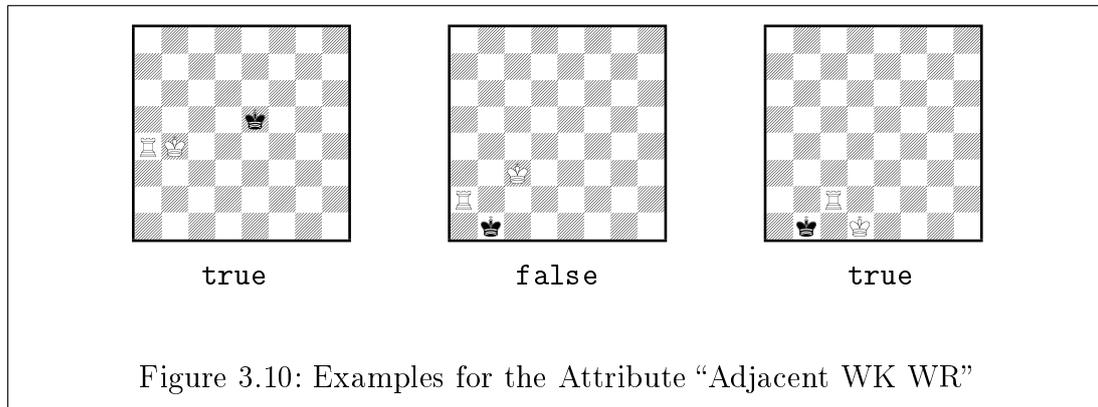
## 3.4 Adjacent Figure Patterns

The attributes of this section test if two of the three figures are on adjacent squares. The possible values are `true` and `false`. This property is important, because the kings can move one square in any direction. When such an attribute is true, it means that the two figures are able to interact (because two of the three figures of the endgame are kings). The importance and the consequences of the interaction depend on the nature of the two figures (mainly if they are on the same side or not). The values of these attributes are computed by testing if the distance between the two figures is one.

### 3.4.1 Adjacent WK WR

*Are the rook and the white king on adjacent squares?*

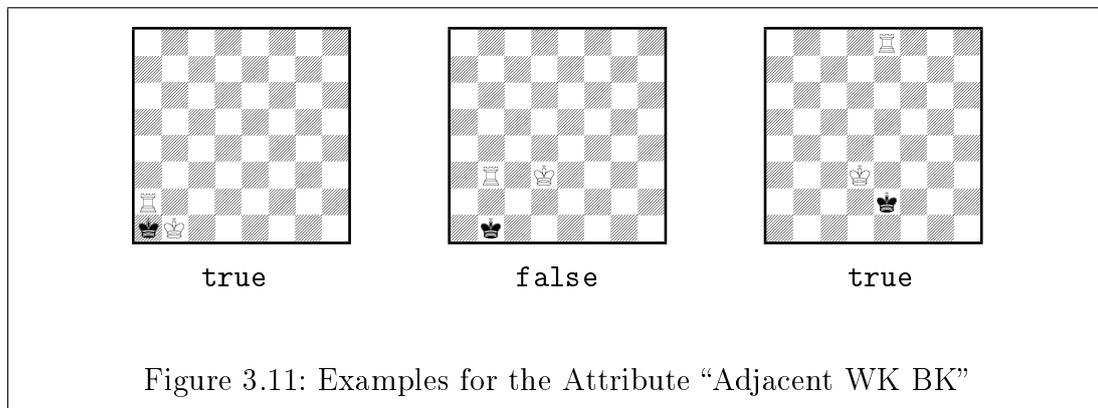
This attribute tests if the rook is protected by the white king or not. If true, the black king cannot take the rook, even if it stands next to it. In figure 3.10, the white king protects the rook on the first board, although it is not threatened. On



the second board the attribute is false, and the rook is threatened, so it can be taken by the black king, and the game will thus be draw. On the third board, the rook is threatened but the attribute is true, so it cannot be taken.

### 3.4.2 Adjacent WK BK

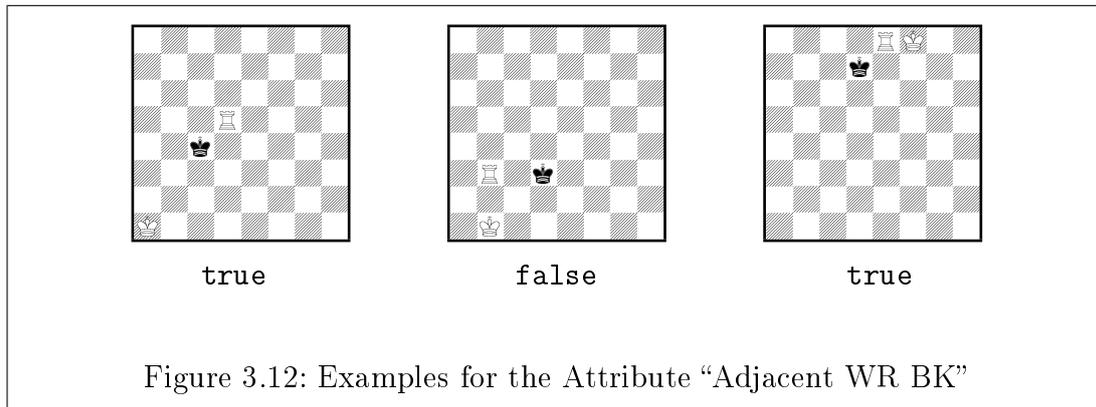
*Are the two kings on adjacent squares?*



This attribute tests if the distance between the two kings is one or more. If it is true, then the position is illegal. In fact, this attribute was used in [BMS95] to define and detect illegal positions. In our case, this attribute is not of any use if the database is built correctly, what was the case in the database we used. We only included this attribute for completeness.

### 3.4.3 Adjacent WR BK

*Are the rook and the black king on adjacent squares?*



This attribute tests if the distance between the white rook and the black king is one, or more. If the value is true, then it means that the black king threatens the rook. It is an important attribute, because then the white rook may be taken by the black king, if black has the move, what was the case in the database we used, unless the white king is protecting the rook. And if the rook is taken, then only the two kings stay on the board and the game is draw! This is the case in figure 3.12 first board. On the second board, the attribute is false so the rook cannot be taken. In order to know the gravity of the value, i.e. to know if the game will be drawn or not, we have to take care simultaneously of the value of the attribute *Adjacent WK WR*. On the third board, the white king protects the rook so that it cannot be taken although the attribute is true.

### 3.5 Between Figure Patterns

In this section we have grouped three attributes of the same kind: *WR between WK and BK*, *WK between WR and BK* and *BK between WK and WR*. These three attributes test if one of the three figures can be considered as standing between the two others on the board. The sense of “between” here is either concerning the ranks or the files. That is to say that a figure is between two others if its rank is between the ranks of the others *or* if its file is between the files of the others. This definition is compatible with the symmetries and rotations of the board and the figures possible moves. These attributes can take the values **true** or **false**. They are all implemented the same way: The four following possibilities are tested (the middle figure is the one which should be between the two others). Is the file of the middle figure larger than the file of the first other and smaller than the file of the second other figure. Is the rank of the middle figure larger than the rank of the first other and smaller than the rank of the second other.

Is the file of the middle figure smaller than the file of the first other and larger than the file of the second other. Is the rank of the middle figure smaller than the rank of the first other and larger than the rank of the second other. If one of the possibilities is true, then the attribute is true, otherwise false.

### 3.5.1 WR Between WK and BK

*Does the rook divide the two kings?*

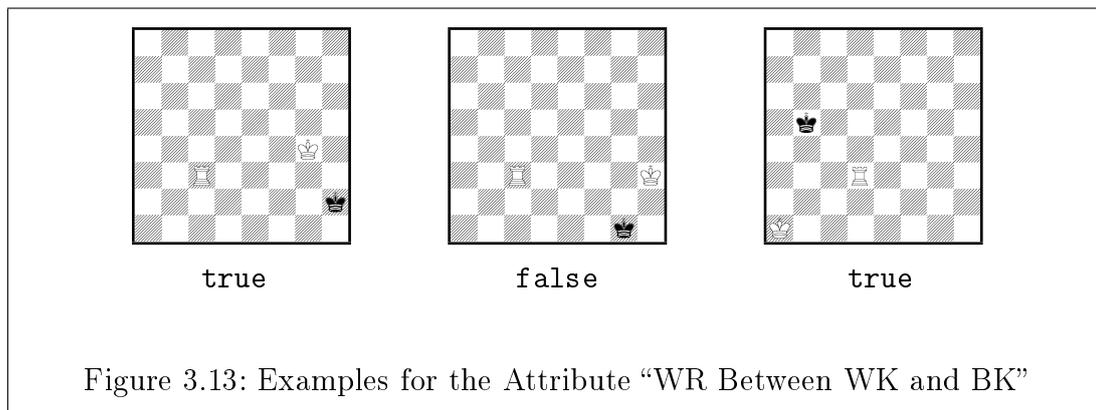


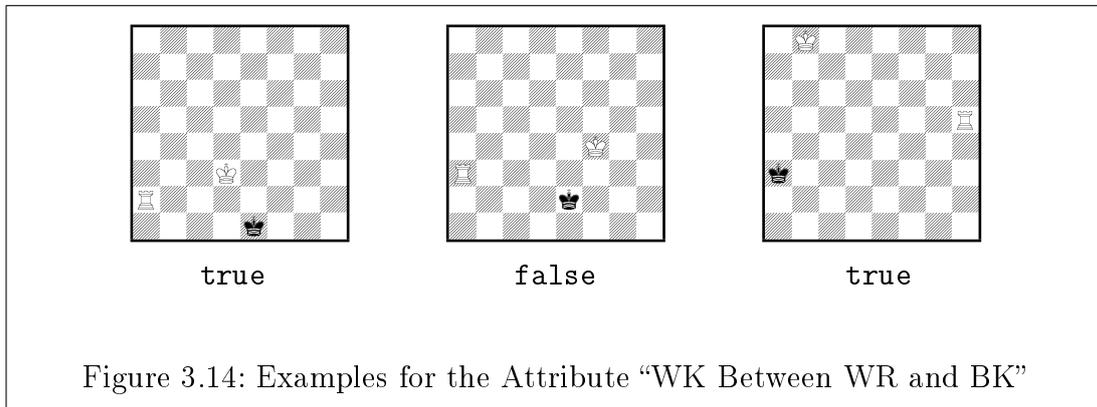
Figure 3.13: Examples for the Attribute “WR Between WK and BK”

This attribute tells if the rook is between the two kings. In the standard endgame strategy, it has to be **true** before the two kings get in opposition, like on the first board of figure 3.13 where the depth of win is 5. Then the rook can go “down” one square to force the black king to go towards an edge (or to mate if the black king is already in an edge). On the second board the position is slightly different and the attribute is no more true, thus the depth of win is 11. But taken alone in a general position on the board, this attribute does not give very significant information, like on the third board, where the attribute is true but the distance to mate is 13.

### 3.5.2 WK Between WR and BK

*Is the white king between the rook and the black king?*

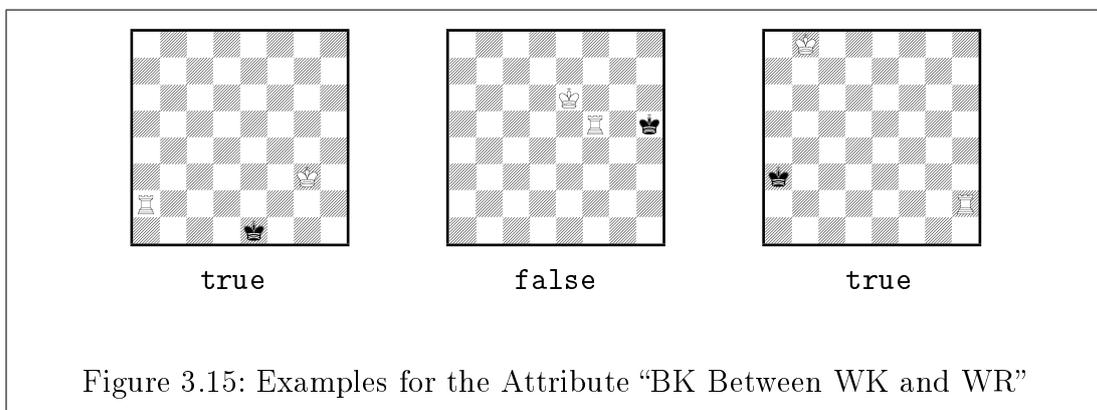
This attribute tests if the white king separates the rook from the black king. In the standard strategy, it should happen to be **true** when the rook divides the two kings so that the black king cannot approach the rook without getting in opposition with the white king, in which case the rook can move “down” to force the king to the edge, like on the first board of figure 3.14: If the black king goes



nearer to the white rook, it gets in opposition with the white king and can be mated. On the second board the position is almost the same, but the attribute is false. The black king can then threaten the white rook without getting in opposition to the white king, so to avoid this the rook has to move. Like the previous attribute and as we can see on the third board, this attribute does not mean much taken alone in a general position.

### 3.5.3 BK Between WK and WR

*Is the black king between the white king and the rook?*

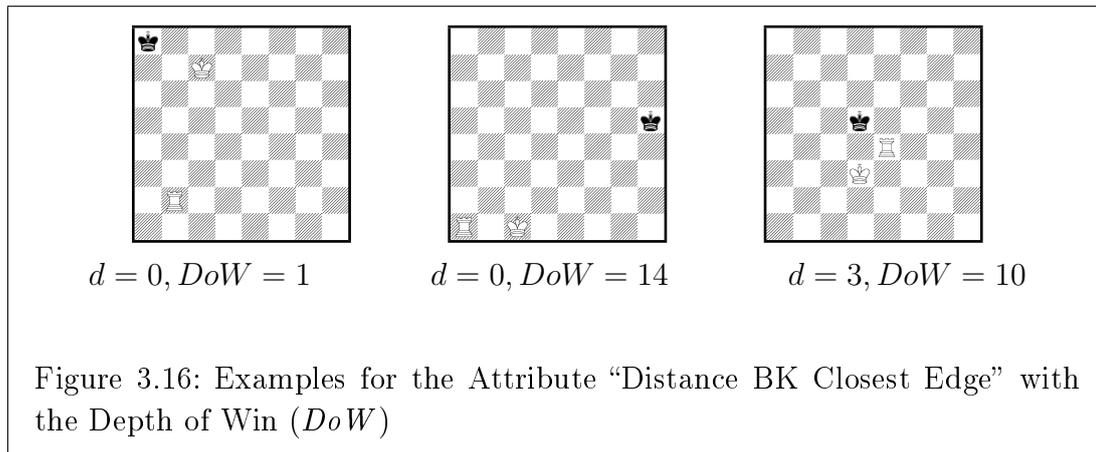


This attribute tests if the black king stands between the white king and the rook. Such a pattern is not a target pattern in the standard strategy, but it can show that the white side is not in an optimal position, because the white figures do not force the black king to an edge. On the first board of figure 3.15, the position is like the one of the second board of the figure 3.14, so in this case this attribute can be taken as the contrary of *WK Between WR and BK*. But in general, looking at this attribute isolated does not give us any useful information.

## 3.6 Board Distance Patterns

### 3.6.1 Distance BK - Closest Edge

*How far is the black king from the closest edge?*

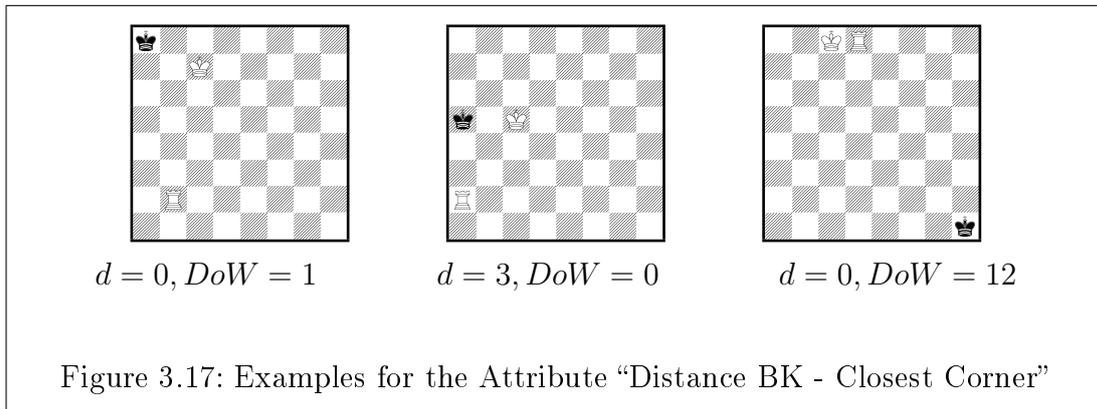


This attribute tells us if the black king is near an edge or not. Its possible values are the integers 0, 1, 2 and 3. It is important since it can give an idea of the number of moves left until the end of the game. If the black king is far away from the edge, it cannot be mated quickly: All the positions for which the attribute has a value of three have their distance to win larger than or equal to ten (except the draws). But the black king may be near an edge without having the game near its end. For example on the first board of the figure 3.16, the attribute has a value of zero and the distance to win is one: We have a typical case of small attribute value and small distance to win; but on the second board, the value of the attribute is also zero and the distance to win fourteen! On the third board, the attribute has a value of three with the smallest depth of win possible, i.e. ten. The value is computed like that: The distance between the king and each edge is calculated, and the the smallest value is returned. If the black king is on a diagonal, there are two closest edges.

### 3.6.2 Distance BK - Closest Corner

*How far is the black king from the closest corner?*

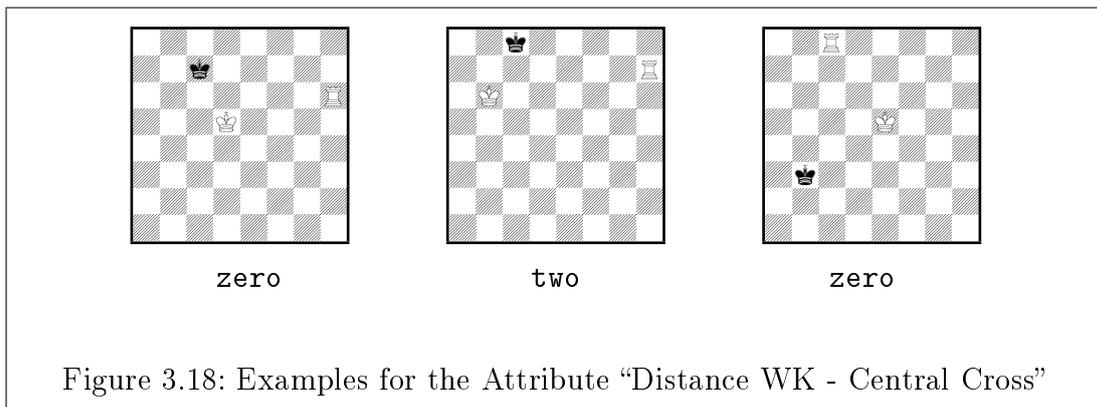
This attribute evaluates the distance from the black king to the closest corner. It is useful because it is simpler to mate when the black king is in a corner. The possible values of the attribute are all the integers between 0 and 3. Nevertheless



this attribute alone tells us less about the advancement of the game than the previous one (*Distance BK - Closest Edge*), since the black king can be mated in the middle of an edge, i.e. without being near to a corner (like on the second board of figure 3.17). And as well the game may still last long even if the black king is near a corner (like on the third board of figure 3.17 where the distance to win is twelve). The value of the attribute is computed that way: The distance from the black king to each corner is calculated and the smallest value is chosen.

### 3.6.3 Distance WK - Central Cross

*How far is the white king from the middle of the board?*

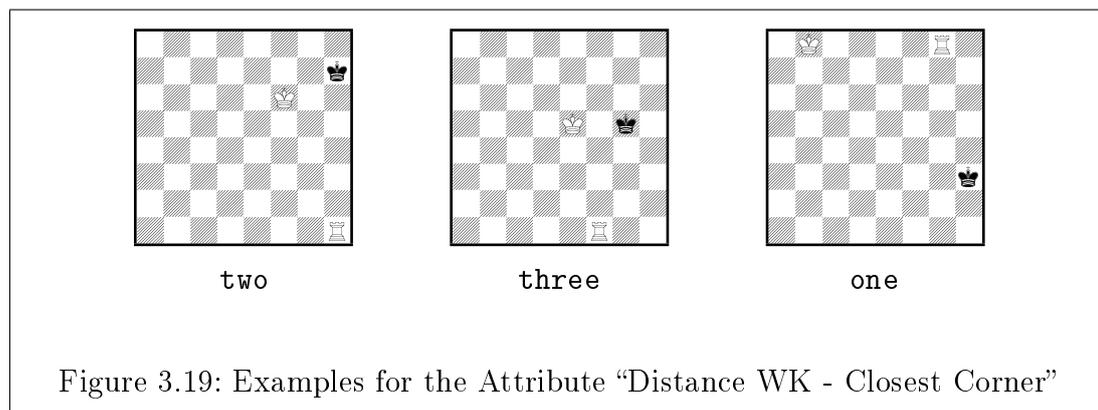


This attribute cares about the position of the white king. It tells us how far is the white king from the closest of the four fields d4, d5, e4, e5. Any integer between 0 and 3 is valid value. This attribute is the opposite of the distance to an edge: The sum of the value of this attribute and of the distance between the white king and the closest edge is always three. It does not give a direct hint on

the distance to win, since the position of the black king can be anything, but rather an indication on the state of the endgame: When the white king is in the middle of the board (i.e. when the attribute has a value of zero), it is in a good position to start forcing the black king to an edge. For example on the first board of figure 3.18, the white king is on the central cross with the smallest distance to win possible three (except the draws). On the second board, it is also on the central cross but with the largest distance to win possible, twelve. In contrary, when the white king is far from the central cross, the attribute does not give us any information about the state of the endgame.

### 3.6.4 Distance WK - Closest Corner

*How far is the white king from the closest corner?*

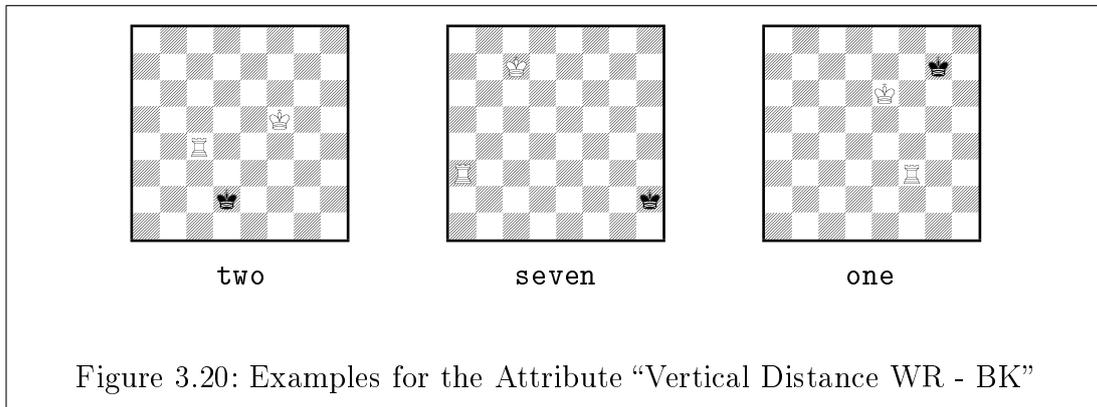


In completion to the previous one, this attribute also deals with the position of the white king on the board. It tells us how far the white king is from the closest corner. Any integer between 0 and 3 is a valid value. Since it is easier to mate the black king in a corner, this attribute is pertinent because it brings more information than the distance to the central cross (or to the closest edge).

## 3.7 Orientation-based Patterns

### 3.7.1 Vertical Distance WR - BK

*How far from each other are the white rook and the black king in the "vertical" direction?*



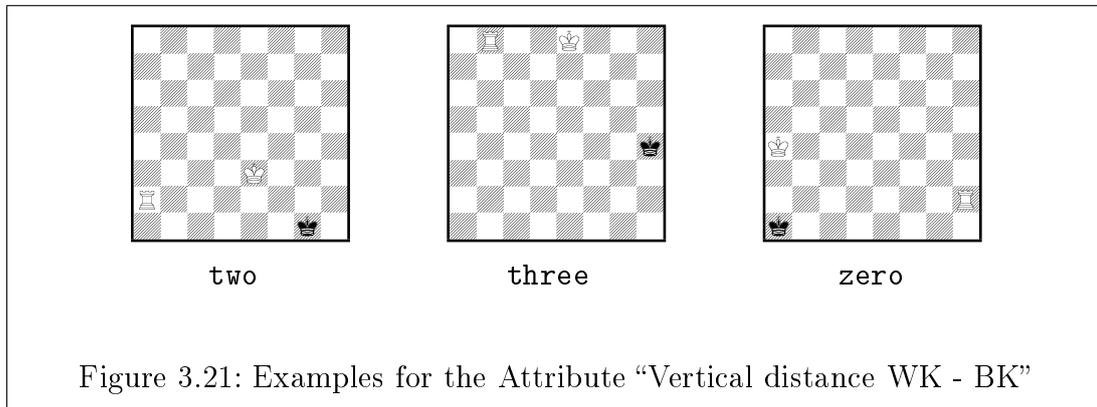
This attribute tells us how much liberty the black king has, as let free by the rook. Its possible values are `more_than_one_square` and `one_square`. If it equals `one_square`, then the rook is just “above” the black king and this one cannot go upwards. We are therefore in the same situation as in the simple strategy of Morales with the black king been held against the bottom edge. The problem here is to define the vertical direction so that the attribute has a reliable value. This attribute makes no sense if the vertical direction is defined so that the rook is on the same grade as the black king. The best definition of the verticality is for us: The black king should be as low as possible, i.e. the edge representing the bottom is the edge closest to the black king. If it is not enough to define one direction, then the white king has to be as high as possible, i.e. the edge representing the bottom is the one furthest from the white king. And if there still exist two possibilities, then one of the two can be chosen randomly.

This attribute was used by the machine of Leonardo Torrès y Quevedo: If its value is `more_than_one_square`, then the machine moved the rook one square towards the black king.

### 3.7.2 Vertical Distance WK - BK

*How far from each other are the white king and the black king in the “vertical” direction?*

This attribute tells us how much liberty the white king let to the black king. Its possible values are `more_than_two_squares` and `two_squares`. The aim of this algorithm is to detect if the white king is as near as it should be to the black king and to bring it nearer if not. The role of the white king is to block the black king in a direction (the here so called upper direction). As for the attribute *Vertical distance WR - BK*, the problem is to define what is vertical, but with

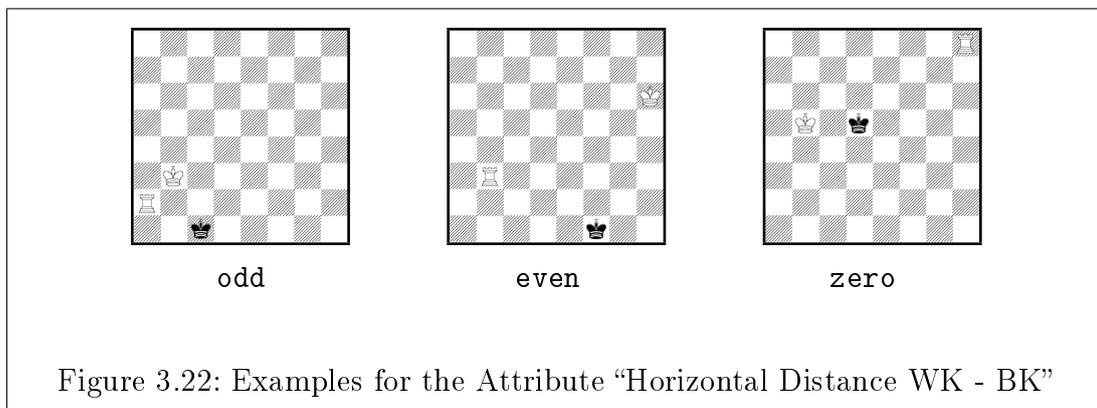


the definition given for the last attribute it is well defined.

This attribute was used by the machine of Leonardo Torrès y Quevedo. If the value is **more than two square**, the machine moved the white king one square down.

### 3.7.3 Horizontal Distance WK - BK

*How far from each other are the white king and the black king in the “horizontal” direction?*



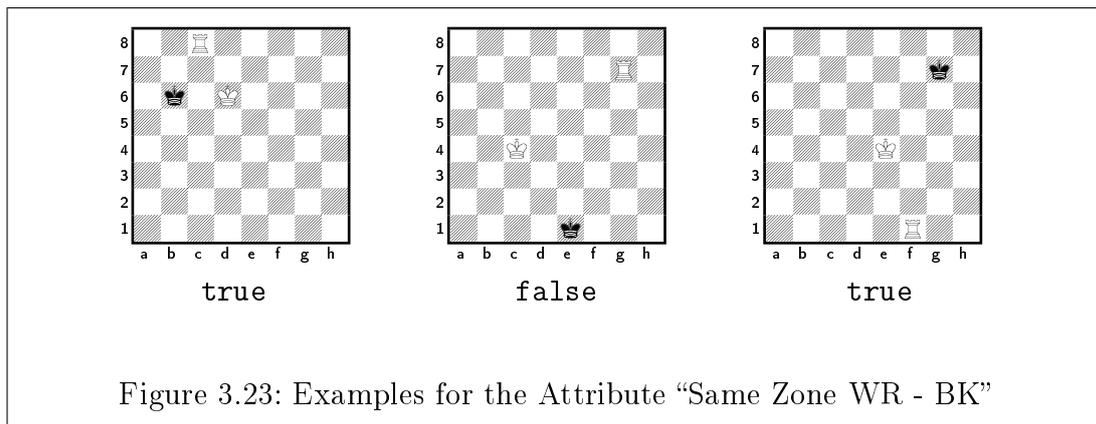
This attribute compares the position of the white king with the position of the black king. Its possible values are **odd**, **even** and **zero**. It should help detecting the right move to do to bring the two kings in opposition so that the black king has to go “down” one square.

This attribute was used by the machine of Leonardo Torrès y Quevedo. If the value was **odd**, the machine moved the rook one square horizontally. If the value

was **even**, the machine moved the white king one square horizontally towards the black king. If the value was **zero**, the machine moved the rook down one square.

### 3.7.4 Same Zone WR BK

*Are the white rook and the black king in the same zone?*



This attribute was used by the machine of Leonardo Torrès y Quevedo to check if the white rook could come near the black king if it moves down on the board. The possible values are **true** and **false**. You need to have defined the vertical and horizontal directions to use this attribute. Its value is computed by testing if the rook and the black king are in neighbour columns (where columns are vertical). It is **true** if the both figures are in the three first columns, or if they both are between the third and the sixth column, or in the three last columns.

You can see three examples of this attribute in figure 3.23. On the first board, the nearest edge from the black king is on the left, so the vertical areas are in fact horizontal. As the rook is on rank 8 and the black king on rank 6, they are in the same area: The attribute is **true**.

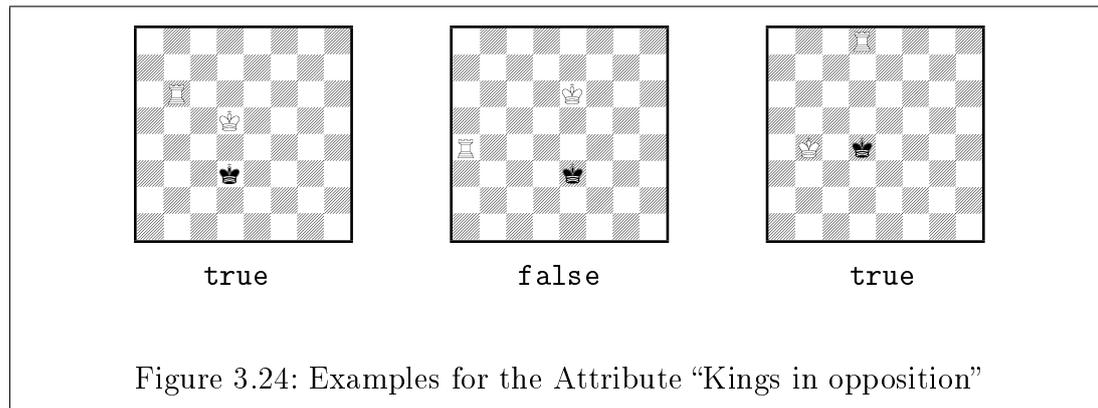
In the second example, the nearest edge is on the bottom of the board. The files e and g of the black king and the rook respectively are not in the same area: The attribute is thus **false**.

In the third example, the nearest edge is the upper one (because the black king is as far from the upper one as from the right one, but the white king is further from the upper one), so the vertical zones are the files. That is why the rook and the black king are considered in the same area: The attribute is here **true**.

## 3.8 Figure Relation Patterns

### 3.8.1 Kings in Opposition

*Are the both kings in opposition?*



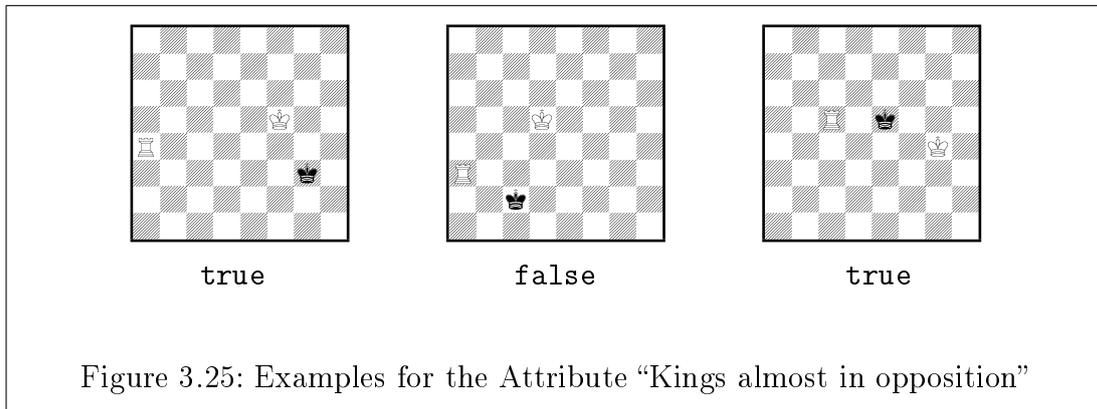
This attribute detects if the both kings are in a position on the board such that the next king that moves has to move away from the other one. That happens when the two kings are on the same file or the same rank and are separated by just one free field. The attribute can take the values **true** or **false**. It is true in one of the following two cases: The two kings are on the same file and their rank distance is two, or the two kings are on the same rank and their file distance is two. In the standard strategy to play the endgame, this attribute tells when you can move the rook to put the black king in check so that it has to move away from the white king.

Some example positions with the values of this attribute are exposed in figure [3.24](#).

### 3.8.2 Kings Almost in Opposition

*Are the both kings almost in opposition?*

This attribute is related to the previous one. If the both kings are not in opposition but only “almost” in opposition, they are on the right way to get in opposition. They are called almost in opposition if they are just one move away from being in opposition without being on the same file or on the same rank. The possible values of this attribute are **true** or **false**. It is true if the file distance between the both king is one and the rank distance is two, or if the file distance is two and

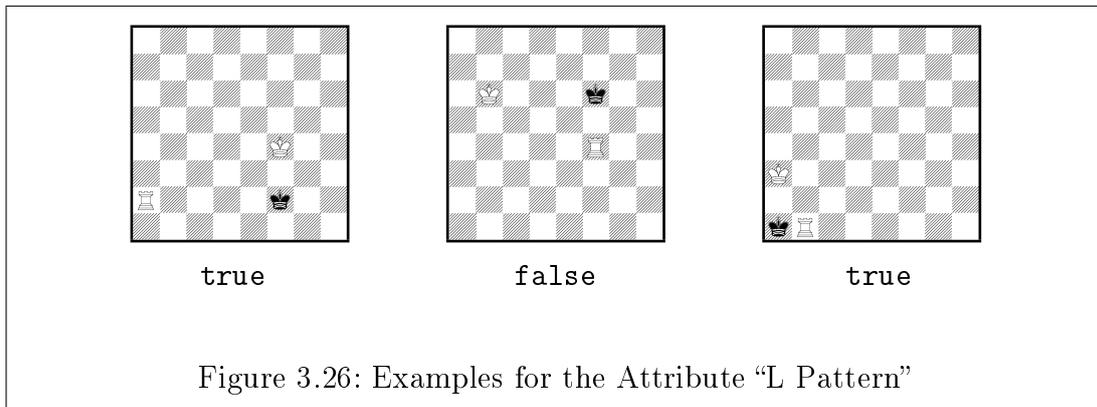


the rank distance one. It can be summarised by: The product of the file and the rank distance is two. To obtain that the kings are in opposition, this attribute has to be first fulfilled.

Some example positions with the values of this attribute are exposed in figure 3.25.

### 3.8.3 L Pattern

*Do the three figures on the board form a L shaped pattern with the black king in check by the rook?*



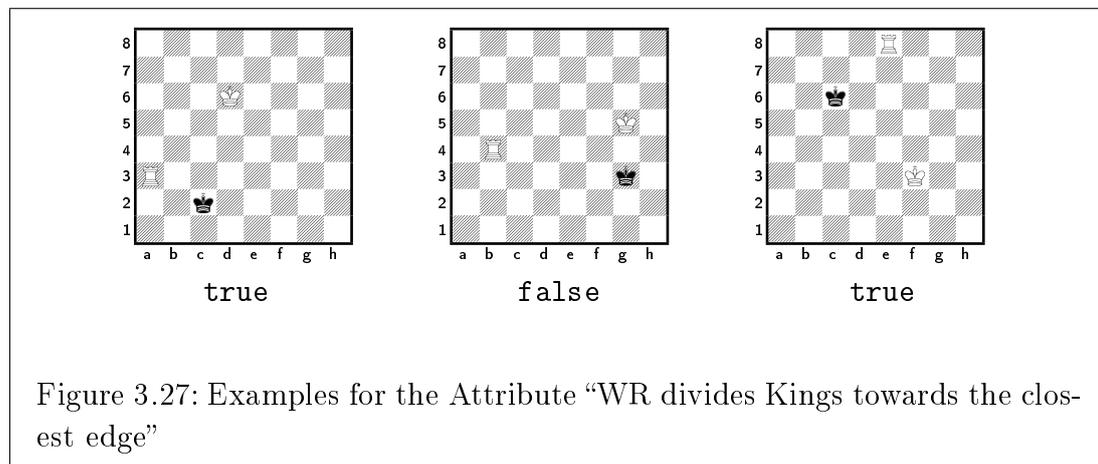
This attribute characterises the next step after having the two kings in opposition in the standard strategy. The rook goes “down” (i.e. in the direction of the black king) to check the black king, and as the white king blocks a whole side of the possible moves of the black king, this one also has to move “down”. The possible values of this attribute are **true** or **false**. It is true in one of the following cases: The two kings are on the same file with a rank distance of two and the rook is

on the same rank as the black king, or the two kings are on the same rank with a file distance of two and the rook is on the same file as the black king.

## 3.9 Figure Relation Patterns Associated with the Board

### 3.9.1 WR Divides Kings towards the Closest Edge

*Does the white rook divide the two kings in direction towards the closest edge?*

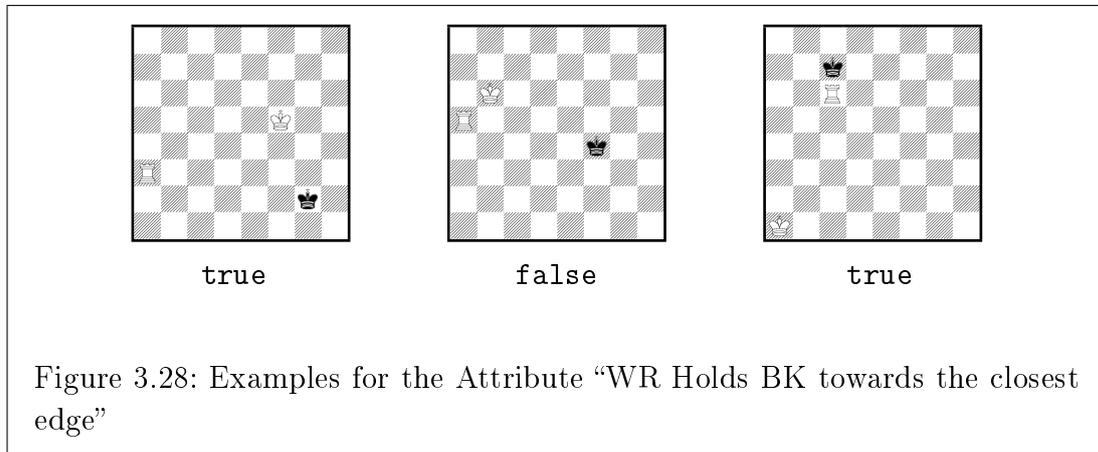


This attribute tells us if the white rook separates the two kings in a judicious manner. The possible values are **true** or **false**. The value is **true** if the rook is between the two kings, so that the black king is on the side of its closest edge. If there are two closest edge, any of the two can do.

In the first example of figure 3.27, the closest edge is the lower one. The rook on rank 3 is then between the white king on rank 6 and the black one on rank 1, so that the black king is on the side of the edge closest to him: The attribute value is **true**. In the second example, the rook separates the two kings, but not towards the closest edge, because the closest edge here is the right one: The attribute value is thus **false**. In the third example, we cannot determine the closest edge (because the two kings are on the diagonal a8 - h1). As the rook divides the two kings towards the left edge and the left edge is one of the closest edge, then the attribute value is **true**.

### 3.9.2 WR Holds BK towards the Closest Edge

*Does the white rook prevent the black king from going away from the closest edge?*



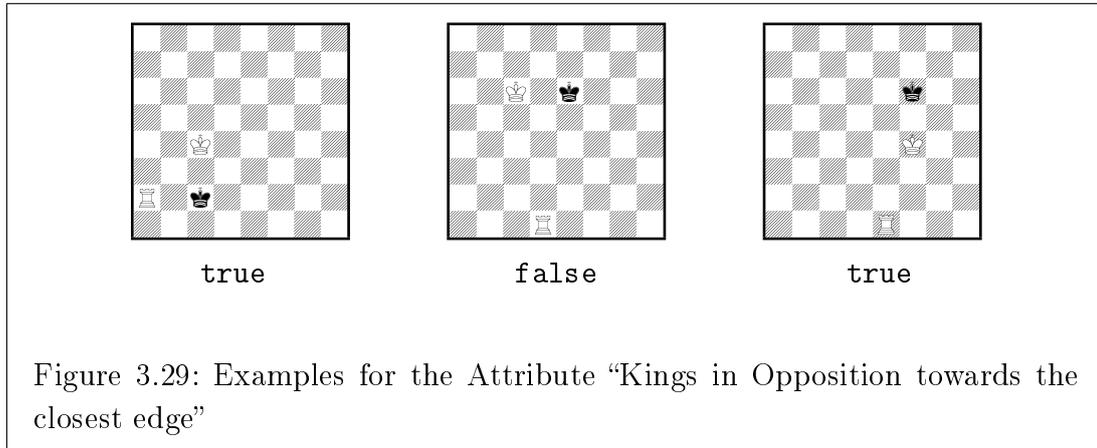
This attribute characterises the position of the rook compared to the position of the black king. It has to be just a field beyond the black king as seen from the BK’s closest edge to be classified as good by this attribute. The possible values are **true** or **false**. To compute the value, the BK’s closest edge is detected. If the rook’s distance to this edge is just one point larger than the BK’s distance, then the attribute is true. If there are 2 closest edges, the rook has to fulfil the condition for only one of the two edges.

### 3.9.3 Kings in Opposition towards the Closest Edge

*Are the both kings in opposition, so that the black king is hold towards the closest edge?*

This attribute is based on the one called Kings in Opposition, with an additional condition: the black king has to stand near the closest edge. The possible values of this attribute are **true** or **false**. It is **true** if the two kings are in opposition (same file and two ranks away or same rank and two files away from each other) and if the edge behind the black king when he “looks” at the white king is a or the closest edge.

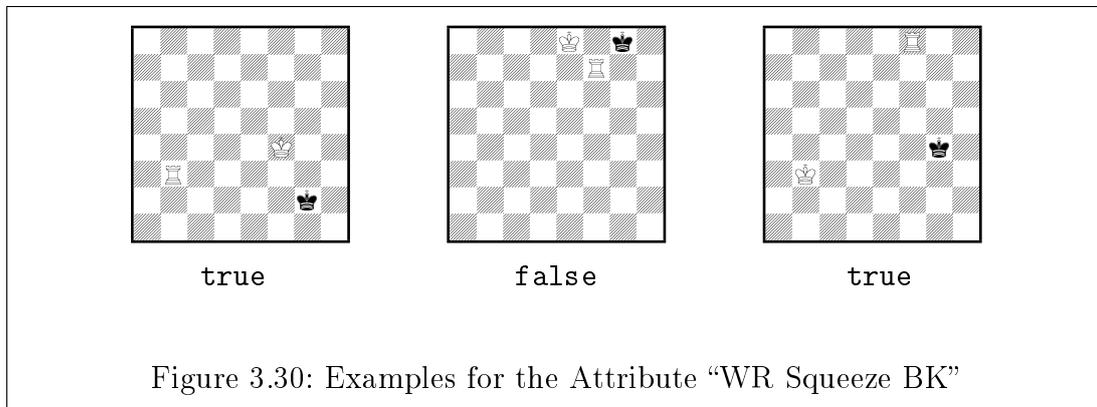
Some example positions with the values of this attribute are exposed in figure 3.29. In the first example, we are in the standard strategy: The black king is threaten by the rook and cannot go upwards because it is in opposition with the white king, so it has to go to the bottom edge. In the second example, the kings are in opposition but not towards the closest edge, which is the upper one. In



the third example, the closest edge is also the upper one, so the attribute value is **true**.

### 3.9.4 WR Squeeze BK

*Does the white rook “squeeze” the black king against the closest edge?*

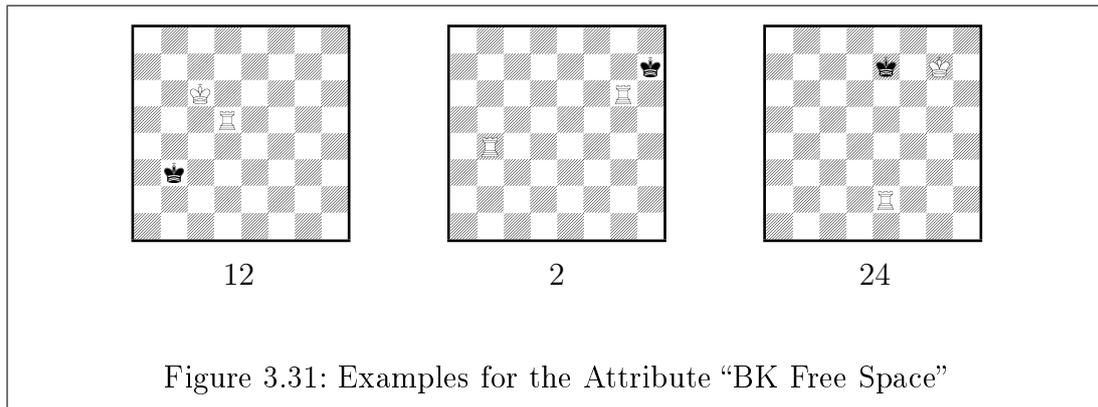


This attribute combines the two following patterns: *WR divides Kings towards the Closest Edge* and *WR holds BK towards the Closest Edge*. If these two attributes are true, then this one is true as well. The possible values are **true** or **false**. The value is computed as a simple logical **AND** between the two attributes. It means that this attribute is true if the rook divides the two kings in the direction towards the closest edge, so that the black king is held towards that edge. On the first board of the figure 3.30, we have a typical position from the standard strategy where the attribute is true because the whites are going to force the black king the lower edge. On the second board, it is false because the closest

edge is the upper one, and not the one on the right. In the third example, it is true although the white king is quite far from the black king.

### 3.9.5 BK's Free Space

*How many fields on the board are left free to the black king by the white rook?*



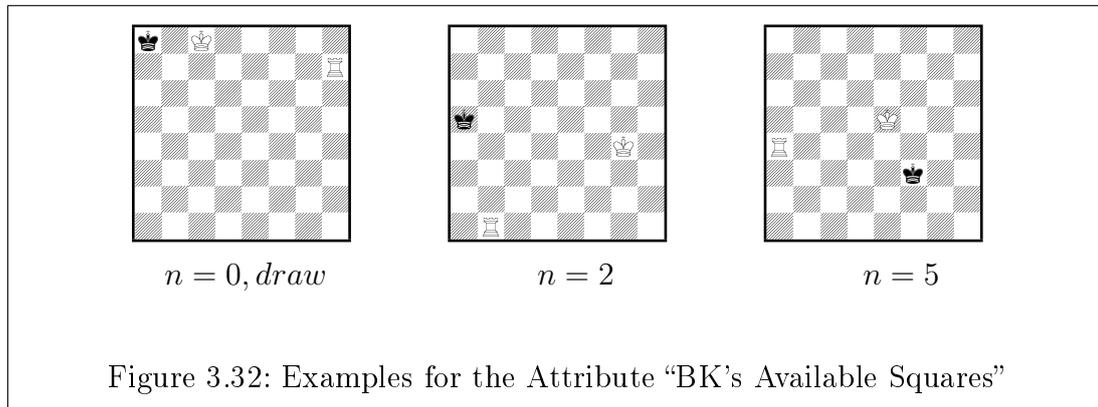
This attribute characterises the number of free squares on the board where the black king can go without being in check by the rook. The position of the white king plays no role here: Only the rook's and the black king's position are taken into account. This attribute can take some integer values (but not all) between 1 and 49. Theoretically the value could be zero, if the black king is mate, but the rook alone cannot achieve that. If the black king is on the same file or rank as the rook, the side with the biggest free space is used to calculate the attribute. Another interesting related attribute would be the black king's free space in general, i.e. its free space computed by taking the rook's and the white king's position into account, like the attribute *Black King's Available Squares*.

In figure 3.31, the attribute has a value of 12 in the first example because the black king has a free space of 3 squares per 4 squares. In the second example, the value is 2, and it does not matter if the rook can be taken or not. In the third example, as the rook and the black king are on the same file, the biggest side (i.e. the left one) is chosen for the calculation, which yields 24 free squares.

### 3.9.6 BK's Available Squares

*How many squares are available to the black king for the next move?*

This attribute counts the number of fields where the black king is free to go in the next move. If its value is 0, then the game is over because the black king has



the move but has no field where he can go. It is then a draw or a lose for the black, like on the first board of figure 3.32. Otherwise and in general, the less free squares the black king has available, the better it is for the winning side. There may though be an exception, as a value of zero may mean a draw, which is a bad result for the white side. The value of the attribute is computed like in the following: From the maximal value of eight free squares around the black king, we substract the number of squares which are either not on the board (if the black king is on an edge or in a corner), or too near too the white king (if the black king is only 2 squares away from the white king), or threatened by the rook (i.e. on the same file or the same rank as the rook). All the remaining squares are free squares for the black king.

# Chapter 4

## Knowledge Pertinence

Once we have defined the attributes, grouped them into 6 sets and calculated their values for all the positions of the endgame, we have to look at the 6 new formed databases to see how useful they are.

### 4.1 Purpose

The goal of the new database is to represent the same informations of the original database in a different way.

To achieve that, we use the higher level knowledge defined in the last chapter. Instead of storing the position of the pieces on the board with the corresponding distance to win, we store the values of chosen attributes, which need less storage place, hoping that we will be able to recover from them the original distance to win. Our hope is to find a combination of attributes that gives us a new database as small as possible that can then be compressed as good as possible, in which the depth of win is as well-defined as in the original one.

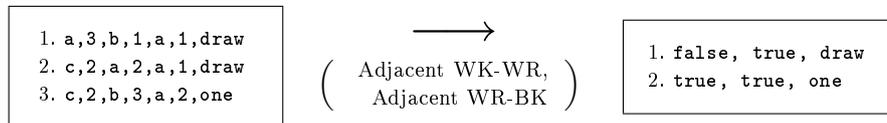
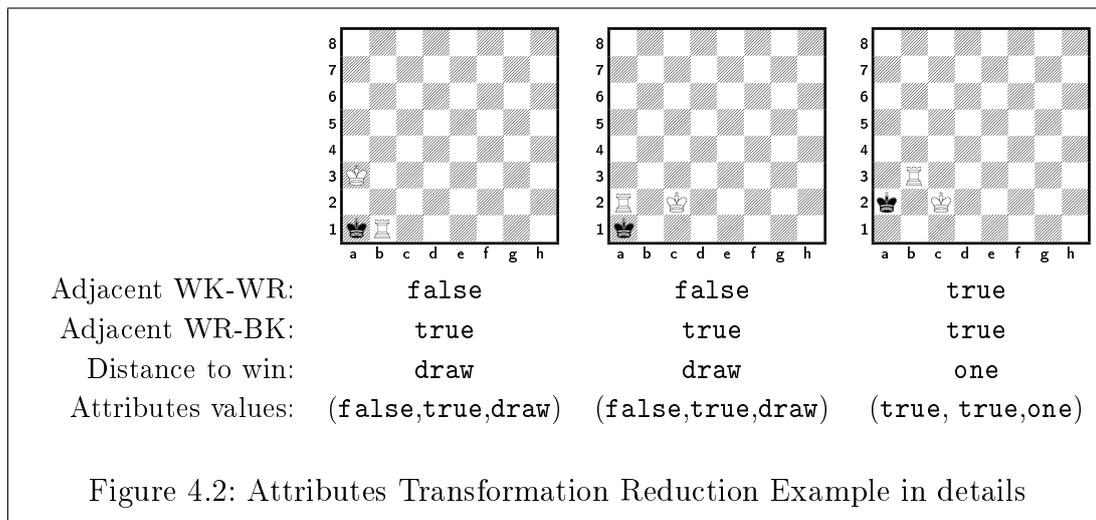


Figure 4.1: Attributes Transformation Reduction Example

The transformation of the position data in attribute values brings a reduction of the number of lines in the database, because several positions can be described by the same attributes values. An example is shown on figure 4.1 and explained on figure 4.2: We consider a simple set with only the 2 attributes *Adjacent WK*

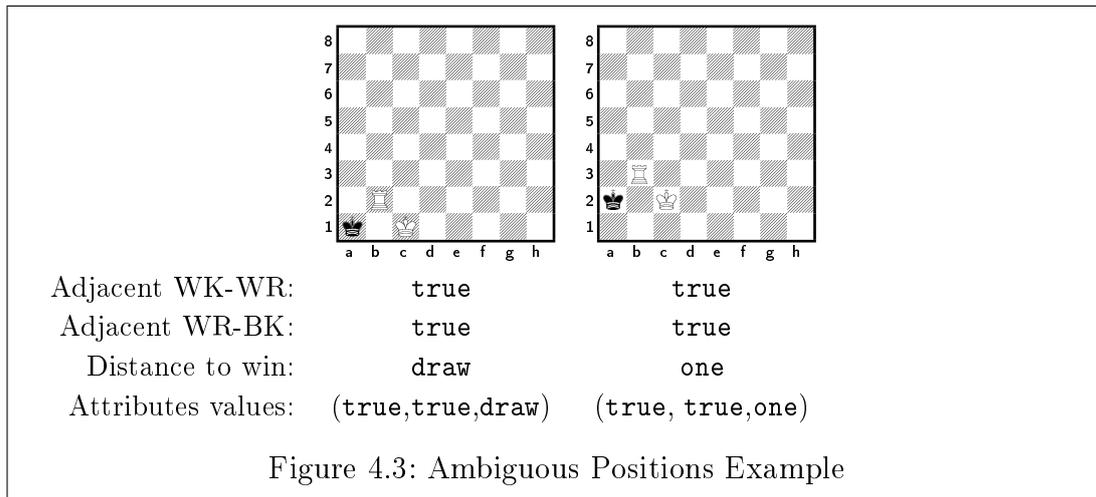
- *WR* and *Adjacent WR - BK*. The position of the figures on the first board is then represented by  $(false, true)$ , as well as on the second board, and they both are drawn. The position on the third board is represented by  $(true, true)$  and is mate in one move. If we consider only these 3 positions, we can conclude that all positions with the values  $(false, true)$  are drawn, and all positions with the values  $(true, true)$  are mate in one. In other words, such a 3-line primary database would be transformed with the attribute set  $(Adjacent WK - WR, Adjacent WR - BK)$  to a new database containing only two lines. Therefore we have reduced the number of lines from 3 to 2 while transforming the database.



Since we do not want to lose information from the database, we cannot let ambiguous combinations appear. Ambiguous positions, as explained in section 1.4, are positions for which the same attribute values correspond to different distances to win. We illustrate that in figure 4.3 with the same attribute set as in the previous example. As you can see, the two shown positions have the same attribute values but a different depth of win. Thus the combination  $(true, true)$  cannot be classified as “draw” or as “mate in one” without misclassifying a position, therefore we say that the positions are ambiguous for the used attribute set. Thus we have to store in a special file all the positions which can lead to ambiguous combinations. This file should be as small as possible, because it cannot be compressed in the same way as the new database. So it represents a gap in our compression process, unless it is empty or almost empty, what is the case when the attributes distinguish the depth of win efficiently. Nevertheless, it is interesting to look at the structure of this error file, to see which depths of win raise problems for a given attribute combination. If the file is empty (perfect case), the attributes are well chosen, then we have not lost information by changing the database and all

the positions have an image in the new database.

In this chapter, we will look at the database generated of each attribute set, and compare the properties of each of them. For the ones which are interesting, we will then look at the repartition of the errors on the depth of win, to see which depths of win present a problem.



## 4.2 General Statistics

Six different databases were created from the initial database. To know the accuracy of the attribute sets used to generate them, we look first at general properties., which we describe in the following: First the size of the database, measured as the number of lines in the new database; this amount is smaller than or equal to the number of lines in the primary database, because the attribute transformation reduces the number of lines, as explained in the previous section. We measure the size of the database with the number of lines it contains, because the size of a line is not important: This is indeed about proportional to the number of attributes, and some attributes may be left aside by the next step of the compression, which relies on knowledge based analysis. Second the number of errors, which tells how many positions from the initial database are transformed in a combination of attributes values with an ambiguous depth of win (as explained in the previous section, particularly on figure 4.3); the smaller this number is, the better the set describes the different positions. Third the correctness, which is the proportion of positions in the primary database which will not be ambiguous after the transformation; this number is almost the inverse of the number of

errors. Fourth the compression, which tells how many times smaller is the new database; this number takes into account the number of errors; since the errors are not compressed, the compression can just come from the transformed part of the database. Please keep in mind that the compression here does not directly imply anything on the disc space which will be needed to store the new database, since we only consider the number of lines of each file, and not the size of each line. These properties for all databases are summarised in the table 4.1.

Attr. set	DB size	Errors	Correctness	Compression
Bain	1864	23 585	15.94 %	0.91
Morales	3	26 736	4.70 %	0.95
Sadikov	5 567	17 442	37.83 %	0.82
Torres	0	28 056	0 %	1
My Selection	22 531	2 272	91.90 %	0.88
All Attributes	28 056	0	100 %	1

Table 4.1: Properties of the six new databases

The results shown are various. For the first set, i.e. the attributes from [BMS95], the new database is small, what could be good if it would be caused by a high compression, but the correctness is quite low. So there are a lot of ambiguous positions, but despite that the compression is not totally bad. We will have a look on the repartition of the errors in the next section. For the second set, [Mor97], only three combinations are unambiguous! That is very few, and they are in fact only for draw positions. That may be because the system which discovered the attributes was trained with a simple non-optimal strategy. And the small number of attributes makes it also complicated. The third set, [Sad06], is much better. The correctness is still smaller than 50 %, but the compression is the best one. Here also, it is interesting to analyse the repartition of the errors. The fourth set, with the attributes from the chess machine, gives very bad results. No position at all can be clearly recognised. This result is surely because the machine does not take care of the exact position of the figures, but just tests if their distances respect some criteria. Moreover the machine cannot play from any start position, and our adaptation of the attributes so that every position can be computed may introduce ambiguities. The fifth set, with special attributes selected to distinguish the small depth of win, is quite efficient. The number of errors is small, what leads to a big database. The compression is smaller than the one of the set of Sadikov, but since the errors are fewer, we will be able to compress better in the next step. The last set, which regroups all the attributes, goes further in the same

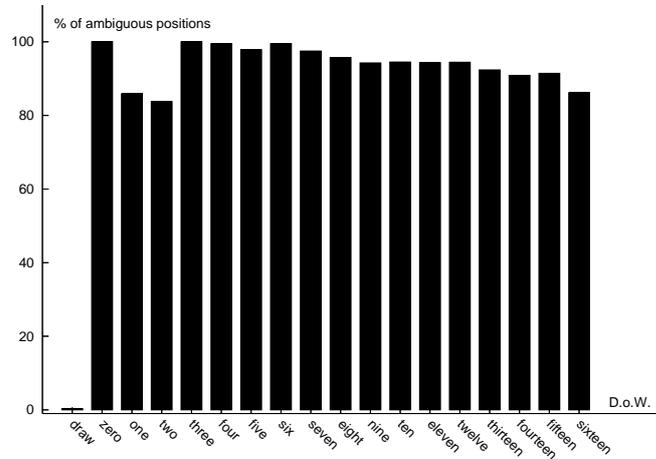


Figure 4.4: Part of ambiguous positions for each depth of win for Bain's attributes

direction: There is no ambiguous position (zero error) but also no compression: Each position of the initial database is represented by one particular combination.

In the next section, we will analyse the error repartition for the databases for which it may be interesting, i.e. for Bain, Sadikov, and my selection. Since all the attributes together do not generate any error, there is nothing to analyse there.

### 4.3 Differentiation Power of the Attributes Combinations

The graphs shown in this section are built that way: In the building process of each database, if a combination of attributes values happens to appear more than one time for different depth of win, all the positions that pointed to this combination are marked as ambiguous and stored in the error file. Then we counted simply the number of position for each depth of win, and the division by the number of positions for this distance to win in the original database gives us the proportions that are shown on the graphs.

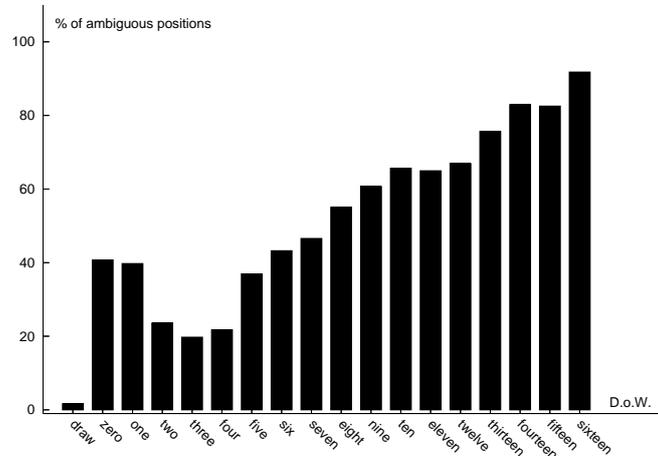


Figure 4.5: Part of ambiguous positions for each depth of win for Sadikov's attributes

### 4.3.1 Bain

As we can see on the figure 4.4, the attributes from Bain can perfectly distinguish the draw positions: No error appears. But for the other positions, the attributes are quite bad: more than 80% of the not draw positions are ambiguous. Some depth of win are even totally unclear (100 % ambiguous). We can conclude from this result that the attributes of this set are only able to identify surely positions facts (like illegal positions, mate positions, pat positions, rook takeable ...) but cannot distinguish well more subtle patterns which could give informations about the depth of win, if it is not draw. With such a proportion of errors, this set is not interesting for proceeding further analysis with WEKA.

### 4.3.2 Sadikov

Sadikov's attributes compose the best set from the four litterature sources. As seen in figure 4.5, the draw positions are almost all distinct after the transformation. For larger distances to win, the part of univocal positions decreases slowly, what is logic, since it is more difficult to take two positions of a large distance to win apart. Until a distance to win of seven, the proportion stays under 50%, which is good. But then the error proportions become larger and since the amount of

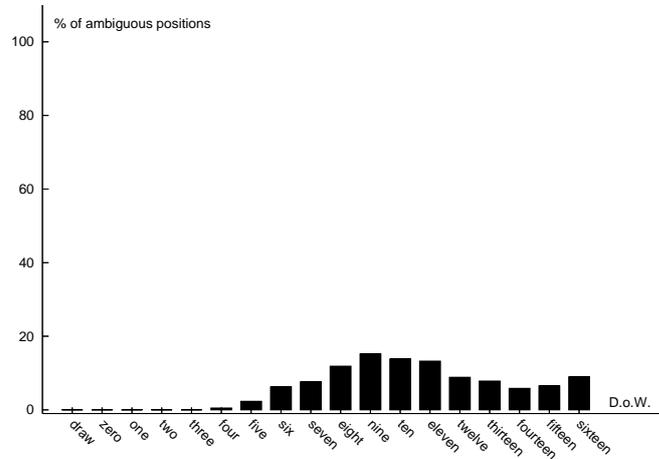


Figure 4.6: Part of ambiguous positions for each depth of win for the attributes of My Selection

positions for high depths of win is also large, the whole correctness of the database is low (37.83%). Still, it is a good set of attributes, because the compression is the best one.

### 4.3.3 My Attributes

The attributes I selected were chosen to detect at best the lower distances to win. I proceeded this way: starting from a small amount of attributes selected intuitively, I added some other by looking at the positions that the current selection could not detect. The results in figure 4.6 are really good: For every depth of win, less than 20% of the positions are ambiguous! And as expected, the lower distances to win (draw to three) are clearly set aside, and four and five almost perfectly detected.



# Chapter 5

## Knowledge-Based Compression

Now that we have generated three useable databases with higher level knowledge, we build a decision tree of each of them with variable properties to obtain a database representation as small as possible. After the analyses of last chapter, we only consider the new databases from *Sadikov*, *My Selection*, and *All Attributes*.

This last step of the compression relies on knowledge-based algorithms. Such algorithms allow us to find the most important attributes in the database, i.e. the ones which can best differentiate the depth of win. Thus the size of the compressed database will not depend directly on the number of attributes it contains, but rather on the quality of its attributes. The number of lines of the database has on the contrary much impact on the final size because it represents the amount of instances which have to be classified. With a lot of instances, the knowledge algorithms have less chances to find a simple representation of the database, i.e. in our case, the decision trees which we build would probably be more complex.

### 5.1 Perfect Tree

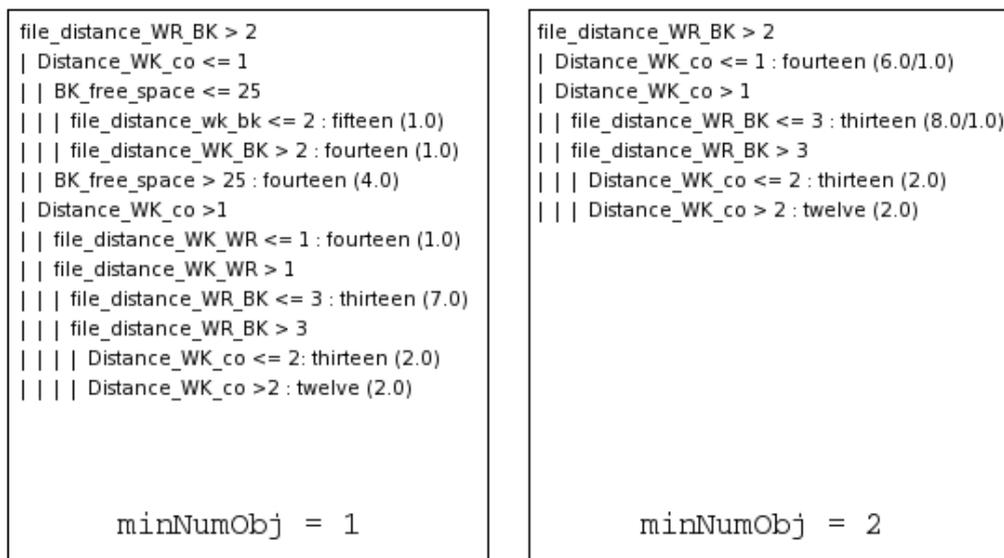
#### 5.1.1 Preliminaries

First we want to build a perfect tree from each database, to compare generally the complexity of the different databases. To do that, we use the software WEKA[WF05]. This software implements amongst other algorithms a c4.5 decision tree classifier<sup>1</sup>, which allows us to build a decision tree with adapted options.

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<sup>1</sup>`weka.classifiers.trees.J48`

This classifier accepts two main options: We can set the minimum number of instance per leaf (short `minNumObj`), and if the tree should be pruned or not. We chose to build an unpruned tree with a `minNumObj` of 1, to obtain a classification as perfect as possible. The pruning of a decision tree reduces its complexity and the time needed to build it, of course at the price of some information loss. The `minNumObj` defines the level of detail of the tree.



*Beneath the main node `file_distance_WR_BK > 2` there are 18 instances: 2× twelve, 9× thirteen, 6× fourteen and 1× fifteen. Since there are different depths of win with more than 2 instances, the algorithm has to look further for another division. Beneath the second node `distance_WK_co ≤ 1`, there are 6 instances: 5× fourteen and 1× fifteen. For a `minNumObj` of 1, the algorithm has to look further to distinguish fourteen from fifteen. But for a `minNumObj` of 2, the one instance with fifteen is negligible; in that case, the algorithm does not search further, it classifies the 6 instances as fourteen, specifies that one error has occurred and continues then with the 12 other instances. The same procedure is applied for the rest of the tree: each time an instance is alone on a leaf, it is neglected if the `minNumObj` is 2.*

Figure 5.1: Decision tree Example for 2 values of `minNumObj`

In figure 5.1, you can see on the left side a part of a c4.5 unpruned decision tree with a `minNumObj` of 1. On the right side, the corresponding part of the tree with a `minNumObj` of 2 is shown. You can notice that each time that a leaf contains only one object on the left, this part of the tree is cut off on the right.

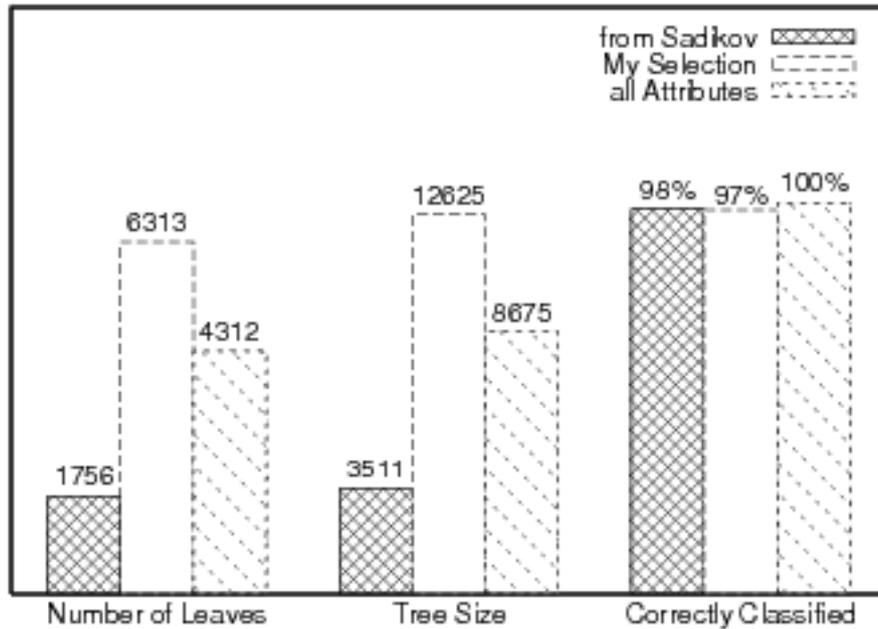


Figure 5.2: Properties of the Decision Trees

### 5.1.2 Perfect Tree Properties of the New Databases

With the options we chose (unpruned and a `minNumObj` of 1), WEKA should generate perfect trees. The general properties of the different generated trees can be seen in figure 5.2.

As you can see, the trees are not perfect, but almost. Less than 3% of the instances are incorrectly classified. To still be able to recover precise information from the database, we have to save the errors separately. This can be achieved with another tool from WEKA, the `weka.unsupervised.filter.RemoveMisclassified`<sup>2</sup>. This is a filter that extracts or removes instances which are incorrectly classified by a specified classifier, the J48 Decision Tree in our case. Thus we store the incorrectly classified instances in a special file, and then the decision tree does not contain any error. The properties of the perfect trees show also that the database from Sadikov gives a much simpler tree than the other (smaller tree and less leaves). This is what we expected, since the Sadikov's database is much smaller than the two others. Another interesting result is that the database containing all attributes gives a tree which is less complex than the one of the attributes from my selection, although the database from all attributes is slightly bigger. This

<sup>2</sup>`weka.filters.unsupervised.instances.RemoveMisclassified`

can be explained by the fact that the algorithm from WEKA can select better attributes amongst all attributes to build the tree than amongst my selection, because more attributes are available, so that the small size difference can be compensated. But the scale of this phenomena is too small to bring the tree complexity of all attributes to the level of the one of Sadikov.

Except for Sadikov, you can see that the trees are quite complex. For this reason we tried to build decision trees with a larger `minNumObj` (and also more errors), to see if the space gain due to the tree being less complex brings more than the loss due to the error file being larger.

## 5.2 Variation of the Tree Complexity

We tried different values for the *Least Instance Number* per leaf, from 1 to 128. We could have gone further if the graph would have shown no clear tendance. The evolution of the complexity of the trees is shown in the graph 5.3. You can find in Appendix A the detail of the output returned by WEKA while computing the decision trees.

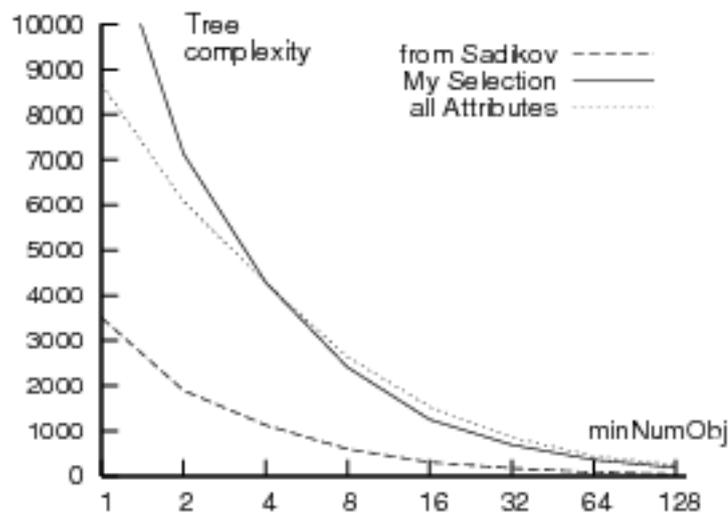


Figure 5.3: Complexity of the Decision Trees depending on the `minNumObj`

The complexity of the tree from Sadikov's attributes set is always simpler than the other two. But the tree from my selection becomes as simple as the one from all attributes when `minNumObj` grows. The reason may be the difficulty to build an (almost) perfect tree with less attributes, implying a much larger complexity at

the beginning, which then decreases quickly. To complete this graph, we also have to look at the accuracy of the tree. The evolution of the proportion of correctly classified instances is shown in figure 5.4.

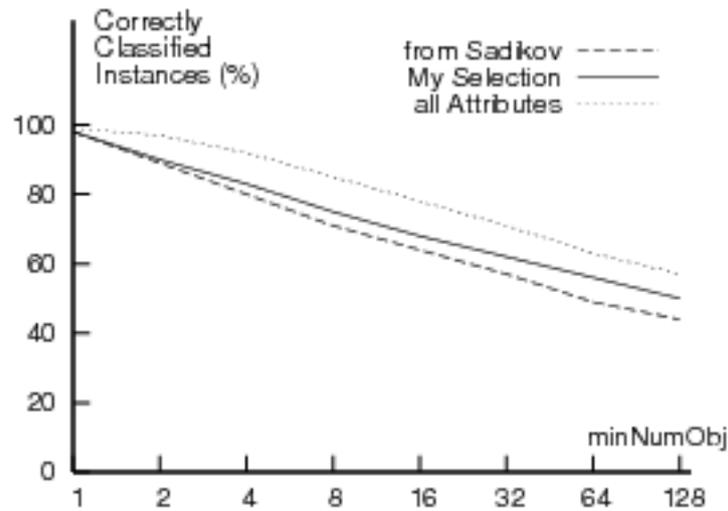


Figure 5.4: Accuracy of the Decision Trees depending on the minNumObj

As seen in the figures, the classification errors grow linearly while the complexity of the trees decreases exponentially. There are slightly less errors made by the set "my selection" in comparison to the set from Sadikov's paper, but the behaviour of the three lines is the same in general. The analysis of the results shown on the two graphs lets us expect that a particular value of the minNumObj should give a good combination of a simplified decision tree with few classification errors. The complexity of the trees reflects the size needed to store them, but does not take into account the space needed to save the errors. Depending on the size of the misclassified instances, it is better to keep a complex tree with very few errors or to simplify the tree by letting instances be misclassified. To know exactly in which case we are, we have to analyse the space needed to store the whole new databases.

### 5.3 Memory Space Needed by the New Databases

What we are interested in is to load the whole database in the computer memory, to be able to use it at high speed. The important measure of the quality of our compression is thus the disc space required to store the new databases.

Unlike the initial database, the new databases do not consist in only one file. They are composed of three files:

1. The position error file, which contains the ambiguous positions and was obtained while calculating the attribute values;
2. The classification error file, which contains the combinations of attribute values misclassified by the decision tree, obtained while building the decision tree with WEKA.
3. The tree file, which is the most important part of the database, which consists of a text file containing a representation of a the tree as it was shown in the figure 5.1.

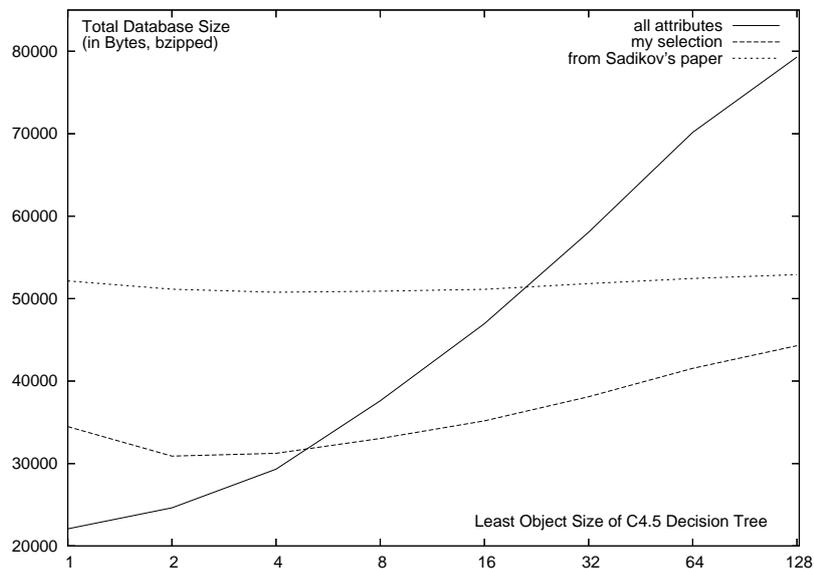
This representation is not space-optimised because lots of characters are used to build a human understandable design, but these character repetitions are compressed very efficiently by the compression algorithm so that this representation is really effective. The first file does not depend on the properties of the tree, since the ambiguous positions are already calculated and cannot be influenced by the `minNumObj` of the tree. On the contrary, the size of the two other files directly depend on the tree properties. The second file should have its size getting larger as the complexity of the tree decreases, since more errors are made. The third file should on the contrary become smaller, as less place is needed to store a simple tree. Like in the previous section, we let vary the `minNumObj` because it is a simple way of controlling the complexity of the tree and also the number of errors made by the classifier. The results are shown on figure 5.5.

The size of the database is measured as follows: The three files are grouped together in an archive and compressed with the algorithm `bzip2`. `bzip2` uses the Burrows-Wheeler transform to convert frequently recurring character sequences into strings of identical letters. This method allows us to store the decision tree directly as a graphical output from WEKA in a text file, since the character repetitions used to draw the tree can be efficiently compressed, so that they do not have much influence on the compressed file size. We used the Unix tar utility<sup>3</sup>. The size in bytes of the bzip2 tar archive is used to measure the size of the database.

As you can see, the influence of the `minNumObj` parameter depends on the attributes set. This result could be foreseen, because the errors are written as attribute value combinations, therefore the number of attributes directly influences

---

<sup>3</sup>GNU tar version 1.15.1, with the `-bzip2` option, see <http://www.gnu.org/software/tar/>

Figure 5.5: Database sizes for different `minNumObj`

the disc space needed to save each error made while building the tree (lots of attributes mean more values to store i.e. more disc space needed). We could save the chess positions corresponding to the misclassified combinations, but it would not be necessarily smaller, as one combination may represent more than one position, what implies that the expense to retrieve the corresponding positions is not worth it.

As we said, the first file does not depend on the number of instances per leaf, therefore the number of position errors just displace the curves upwards or downwards on the graph. Since the set from Sadikov produced the most position errors, it is not surprising that the corresponding curve is the upper one at the beginning. It then stays about at the same level: That is due to the small complexity of the tree, which cannot get much simpler, so there is no big change. The smallest size is obtained for a `minNumObj` of 4. For the two other on the contrary the evolution is quite interesting: The set *My Selection* has a minimum for a `minNumObj` of two, and then goes slowly upwards, so the best condition to store the database with this set is with a tree with two instances per leaf. The set *All Attributes* on the contrary always grows, because of the big space needed to store the errors. But it starts at the lowest level. The smallest possible size of the new database is thus

for a `minNumObj` of one by the set *All Attributes*, and consists of 22087 Bytes. We will discuss this value in the next section.

## 5.4 Compression Results

The results obtained in the previous section should be compared to the size of the other representations of the database to know their quality. This comparison is shown in figure 5.6. For each database, we proceeded identically to measure the size: we grouped first the files (or just one for the primary database) together in a tar-archive, and then compressed it with bzip.

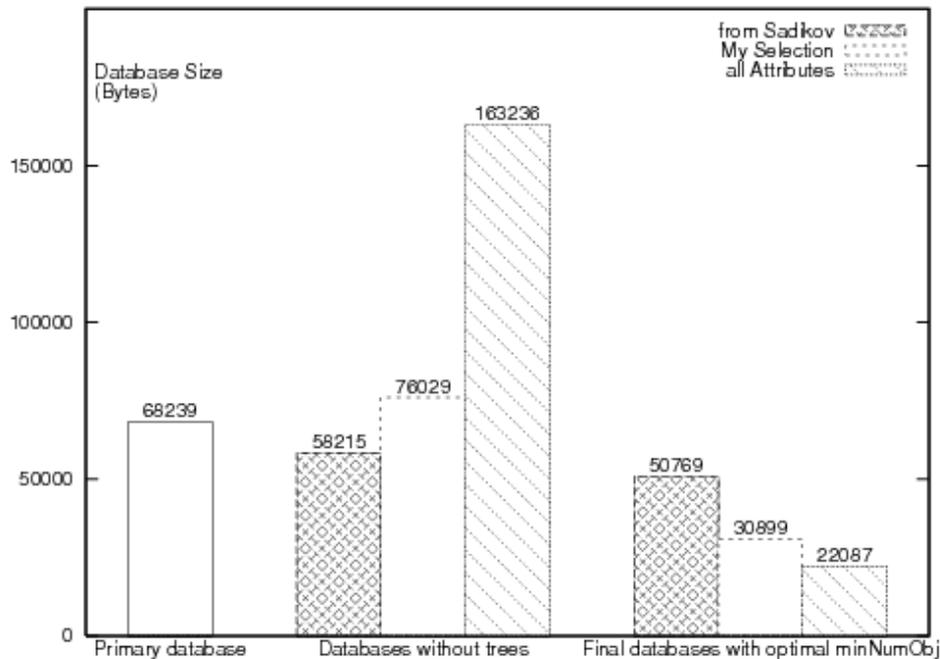


Figure 5.6: Size of the different Databases

As you can see, the primary database was 68239 Bytes big, and our best result (for all attributes and a `minNumObj` of 1) reduces the database to 22087 Bytes. So the best compression we achieved with our method is 32%, which means a compression rate of 3.09. Moreover, you can observe the efficiency of the knowledge-based compression for the set *All Attributes*: From the biggest database after the attributes transformation (due to the high number of attributes, which means a lot of values to store), it becomes the smallest at the end. In contrary, the database of the set *Sadikov* stays about the same; that should be caused by the small size

of the database and the big size of the position error file, letting few possibilities for the decision tree to compress it.

The compression we achieved could be surely enhanced with some methods adapted to decision trees. For example, some incorrectly classified combinations could be stored at low space cost if the position in the tree would be stored instead of the whole combination for each error (in case of several errors in the same area of the tree). Another possibility would be in some case to store the position of the three figures on the board for the misclassified combinations instead of storing directly the combinations of the attributes values. To do that, we would have to undo the attributes transformation, i.e. to retrieve the chess positions corresponding to these combinations. Depending on the number of positions for each combination, the disc space needed could become smaller, because a chess position has only six values, compared to up to 33 for the set *All Attributes*. But that was not the main goal of our work, it can just help to improve the efficiency while implementing the methods. We showed that the knowledge-based compression could help a lot to reduce the size of the database, but that it is in a great part depending on the quality of the attributes.



# Chapter 6

## Conclusion

Chess endgame databases allow to play perfectly all endgames with less than 6 figures, but their large size is a big handicap which cannot be solved with standard compression method. Knowledge based methods from the Artificial Intelligence domain are promising for this task. In this report, we detailed our efforts to obtain a compressed form of the 3 figures KRK endgame database relying on chess attributes. Basing our research on interesting works using KRK patterns, we have developed a knowledge consisting in 33 attributes. Thus we were able to transform the primary database in a bunch of new ones by using different attribute sets. Thanks to Data Mining decision trees, we could then save the new databases very optimally, obtaining in the best case a compression 3 times better than standard compression methods.

This result shows that it is surely possible to use Artificial Intelligence method to store and then to use chess endgames databases. Saved under this new form, the tablebases can be easier loaded in the main memory of computers and with an appropriate algorithm efficiently used to play perfectly chess endgames. The compression rate we achieved may be improved by adding other attributes, with which the database would be better described, although the patterns we used are known by chess grand masters as the most relevant ones for the KRK endgame. However it may be more efficient to rather split the database in subsets and to specify different attributes sets for each subset, since the attributes have different weight depending on the state of the endgame (mate in 2 moves or rather mate in 15 moves for example). The errors may also be stored more optimally, either with the help of appropriate attributes or, for the database lines misclassified in the tree, by specifying the branch of the decision tree where the errors happened. The same compression system should also work for bigger chess endgame

databases. It requires however that the database had been deeply studied, so that efficient attributes are available. Standard attributes like figures distances, board distances etc. . . are easy to define, but important figures relations on key positions on the board are specific for each endgame. Since our method rely on the quality of the attributes, the results which are to be expected depend also on the previous studies about the endgame in question and on the quality of the patterns they found. An efficient system which can detect which characteristics are the most important to measure the depth of win and deliver best quality attributes would enable the use of our method for any other endgame. Unfortunately, such a system does not exist yet, but some endgames have already been deeply studied. For example, the KQKR endgame database could surely be efficiently compressed with our method.

Some other general methods, which do not rely on specific chess patterns and use nonetheless knowledge based compression, may be a good idea to avoid the issue of finding adapted attributes. Such systems already exist for big and complex databases (like SPARTAN, [BGR01]), and some similar processes, maybe adapted to chess endgames, should bring good compression possibilities for larger chess tablebases.

# Appendix A

## Decision Trees Output

In this appendix, we show the whole output returned by WEKA while computing the decision trees. For each of the three databases *Sadikov*, *My Selection* and *All Attributes*, the properties of the trees as well as the confusion matrix are included for the `minNumObj` parameter taking the values 1, 2, 4, 8, 16, 32, 64 and 128.

### Sadikov

#### One Object per Leaf

Options: -U -M 1

J48 unpruned tree  
-----

Number of Leaves : 1756

Size of the tree : 3511

Time taken to build model: 11 seconds

Time taken to test model on training data: 0.73 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	5478	98.4013 %
Incorrectly Classified Instances	89	1.5987 %
Kappa statistic	0.9824	
Mean absolute error	0.0025	
Root mean squared error	0.0355	
Relative absolute error	2.49 %	
Root relative squared error	15.7808 %	
Total Number of Instances	5567	



```

1 0 28 6 0 0 0 1 1 0 0 0 0 0 0 0 0 | c = one
1 0 1 134 1 0 0 0 0 0 0 0 0 0 0 0 0 | d = two
0 0 0 3 54 2 1 1 0 0 0 0 0 0 0 0 0 | e = three
2 0 0 0 2 128 8 1 0 0 0 0 0 0 0 0 0 | f = four
1 0 0 0 4 6 222 9 10 2 0 1 0 0 0 0 0 | g = five
2 0 1 0 1 3 7 248 12 7 5 1 1 1 0 0 0 0 | h = six
2 0 0 1 0 1 5 11 270 16 2 1 1 1 1 0 0 0 0 | i = seven
2 0 0 0 0 0 4 8 8 451 15 13 5 0 0 1 0 0 0 | j = eight
1 0 0 0 0 1 1 4 6 17 372 11 8 2 2 0 0 0 0 | k = nine
0 0 1 0 0 0 2 6 7 20 15 317 14 10 0 0 1 0 0 | l = ten
1 0 4 0 0 0 2 2 0 0 9 11 15 412 14 4 0 0 0 0 | m = eleven
1 0 0 0 0 0 3 1 0 2 3 8 8 20 418 16 4 1 0 0 | n = twelve
1 0 0 0 0 0 0 0 1 0 1 2 10 5 39 320 14 2 0 0 | o = thirteen
1 0 0 0 0 0 0 0 0 0 1 0 4 3 12 29 279 13 0 0 | p = fourteen
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 6 17 170 1 0 | q = fifteen
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 8 17 0 | r = sixteen

```

## Four Objects per Leaf

Options: -U -M 4

J48 unpruned tree

-----

Number of Leaves : 567

Size of the tree : 1133

Time taken to build model: 7.19 seconds

Time taken to test model on training data: 1.29 seconds

=== Error on training (and test) data ===

```

Correctly Classified Instances      4434      79.6479 %
Incorrectly Classified Instances    1133      20.3521 %
Kappa statistic                    0.7761
Mean absolute error                 0.0311
Root mean squared error             0.1247
Relative absolute error             30.7596 %
Root relative squared error         55.4648 %
Total Number of Instances          5567

```

=== Confusion Matrix ===

```

  a  b  c  d  e  f  g  h  i  j  k  l  m  n  o  p  q  r  <-- classified as
1071 0  0  0  0  0  0  2  0  0  0  0  2  0  0  0  2  0 | a = draw
  0 11  0  5  0  0  0  0  0  0  0  0  0  0  0  0  0  0 | b = zero
  1  0 28  6  0  0  0  1  1  0  0  0  0  0  0  0  0  0 | c = one
  3  0  1 129  4  0  0  0  0  0  0  0  0  0  0  0  0  0 | d = two
  1  0  0  3 51  3  2  0  1  0  0  0  0  0  0  0  0  0 | e = three
  2  0  0  1  2 119 13  1  1  0  2  0  0  0  0  0  0  0 | f = four
  1  0  0  1  3  13 202 14 14  3  2  1  1  0  0  0  0  0 | g = five
  2  0  1  3  2  12  15 200 28 12  9  2  1  1  1  0  0  0 | h = six

```

4	0	0	2	0	2	19	19	225	24	7	5	2	2	1	0	0	0		i = seven
3	0	0	0	0	0	6	16	31	385	35	18	11	1	1	0	0	0		j = eight
2	0	0	0	0	1	3	10	9	37	320	21	11	9	1	0	1	0		k = nine
3	0	0	0	0	0	4	7	10	24	30	254	32	25	2	2	0	0		l = ten
3	0	4	0	0	2	2	2	2	14	21	26	363	29	3	3	0	0		m = eleven
1	0	0	0	0	3	2	0	2	5	15	14	42	348	38	11	4	0		n = twelve
5	0	0	0	0	0	1	2	0	1	4	9	16	41	291	23	2	0		o = thirteen
0	0	0	0	0	0	0	0	0	0	0	4	8	7	33	264	26	0		p = fourteen
0	0	0	0	0	0	0	0	0	0	0	0	0	1	6	28	158	2		q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	10	15		r = sixteen

## Eight Objects per Leaf

Options: -U -M 8

J48 unpruned tree

-----

Number of Leaves : 302

Size of the tree : 603

Time taken to build model: 6.27 seconds

Time taken to test model on training data: 0.72 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	3974	71.3849 %
Incorrectly Classified Instances	1593	28.6151 %
Kappa statistic	0.6851	
Mean absolute error	0.0427	
Root mean squared error	0.1462	
Relative absolute error	42.2893 %	
Root relative squared error	65.0342 %	
Total Number of Instances	5567	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
1066	0	0	6	0	0	0	0	0	1	0	0	0	0	2	0	0	2	0	a = draw
0	13	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	20	11	0	0	0	4	1	0	0	0	0	0	0	1	0	0	0	c = one
1	3	0	121	10	0	2	0	0	0	0	0	0	0	0	0	0	0	0	d = two
2	0	0	0	51	4	3	0	1	0	0	0	0	0	0	0	0	0	0	e = three
2	0	0	0	9	107	14	1	1	0	2	0	5	0	0	0	0	0	0	f = four
2	0	2	0	16	20	156	16	23	13	3	0	3	0	1	0	0	0	0	g = five
6	0	0	0	4	14	20	188	29	17	3	3	3	1	1	0	0	0	0	h = six
6	0	0	1	4	3	14	33	184	41	18	2	1	1	4	0	0	0	0	i = seven
3	0	0	0	1	0	10	17	34	356	40	16	21	8	1	0	0	0	0	j = eight
2	0	0	0	0	5	3	14	13	45	282	26	15	18	1	0	1	0	0	k = nine
3	0	1	0	3	0	1	11	17	44	62	171	43	27	8	2	0	0	0	l = ten
5	0	5	0	0	1	0	3	3	21	45	17	303	51	18	2	0	0	0	m = eleven
0	0	1	0	0	3	0	1	3	7	33	20	34	306	65	10	2	0	0	n = twelve



## Thirty two Objects per Leaf

Options: -U -M 32

J48 unpruned tree

-----  
Number of Leaves : 89

Size of the tree : 177

Time taken to build model: 3.87 seconds

Time taken to test model on training data: 0.59 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	3176	57.0505 %
Incorrectly Classified Instances	2391	42.9495 %
Kappa statistic	0.5261	
Mean absolute error	0.0602	
Root mean squared error	0.1735	
Relative absolute error	59.5527 %	
Root relative squared error	77.1752 %	
Total Number of Instances	5567	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
1062	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	a = draw
0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
1	0	0	25	0	0	0	1	0	3	0	0	7	0	0	0	0	0	0	c = one
1	0	0	132	0	0	3	0	0	1	0	0	0	0	0	0	0	0	0	d = two
3	0	0	16	0	3	31	0	0	8	0	0	0	0	0	0	0	0	0	e = three
4	0	0	4	0	61	36	25	4	1	1	0	5	0	0	0	0	0	0	f = four
7	0	0	8	0	13	132	36	15	32	0	0	9	1	2	0	0	0	0	g = five
10	0	0	6	0	2	27	140	28	58	5	0	10	1	2	0	0	0	0	h = six
11	0	0	6	0	0	48	48	85	66	23	1	18	3	2	1	0	0	0	i = seven
6	0	0	0	0	0	8	27	33	261	65	17	49	26	15	0	0	0	0	j = eight
7	0	0	0	0	0	25	10	9	68	198	8	68	22	9	1	0	0	0	k = nine
6	0	0	0	0	0	5	3	16	45	83	77	96	42	16	0	4	0	0	l = ten
8	0	0	0	0	0	0	0	3	22	52	18	278	46	38	9	0	0	0	m = eleven
4	0	0	0	0	0	0	0	8	9	42	27	77	178	91	45	4	0	0	n = twelve
2	0	0	0	0	0	0	0	1	0	12	6	26	50	227	66	5	0	0	o = thirteen
0	0	0	0	0	0	0	0	0	0	2	2	7	8	66	222	35	0	0	p = fourteen
0	0	0	0	0	0	0	0	0	0	1	0	0	2	12	57	123	0	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	23	0	0	r = sixteen

## Sixty four Objects per Leaf

Options: -U -M 64

J48 unpruned tree

-----

Number of Leaves : 45

Size of the tree : 89

Time taken to build model: 3.06 seconds

Time taken to test model on training data: 1.46 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	2744	49.2905 %
Incorrectly Classified Instances	2823	50.7095 %
Kappa statistic	0.4406	
Mean absolute error	0.0684	
Root mean squared error	0.185	
Relative absolute error	67.7427 %	
Root relative squared error	82.3111 %	
Total Number of Instances	5567	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
1071	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
1	0	0	6	0	25	0	1	3	0	0	0	1	0	0	0	0	0	0	c = one
1	0	0	104	22	10	0	0	0	0	0	0	0	0	0	0	0	0	0	d = two
3	0	0	6	23	19	5	0	5	0	0	0	0	0	0	0	0	0	0	e = three
4	0	0	4	8	56	31	26	6	5	0	0	0	0	1	0	0	0	0	f = four
7	0	0	8	22	41	62	59	33	16	0	1	3	1	2	0	0	0	0	g = five
10	0	0	6	5	4	12	138	44	55	2	1	7	1	4	0	0	0	0	h = six
12	0	0	6	20	3	6	56	88	80	10	14	9	4	4	0	0	0	0	i = seven
6	0	0	0	0	1	2	34	53	243	51	33	43	19	22	0	0	0	0	j = eight
8	0	0	0	0	12	3	12	45	85	129	22	76	16	17	0	0	0	0	k = nine
6	0	0	0	0	1	5	3	20	82	65	49	62	63	26	11	0	0	0	l = ten
9	0	0	0	0	5	12	1	26	37	97	30	146	37	72	2	0	0	0	m = eleven
8	0	0	0	0	1	10	0	6	34	30	14	62	168	92	60	0	0	0	n = twelve
7	0	0	0	0	0	1	0	1	15	23	9	23	50	187	79	0	0	0	o = thirteen
11	0	0	0	0	0	0	0	0	1	15	1	5	12	63	198	36	0	0	p = fourteen
8	0	0	0	0	0	0	0	0	0	1	0	0	21	6	77	82	0	0	q = fifteen
1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	4	20	0	0	r = sixteen

## Hundred twenty eight Objects per Leaf

Options: -U -M 128

J48 unpruned tree

-----

Number of Leaves : 23

Size of the tree : 45

Time taken to build model: 2.09 seconds

Time taken to test model on training data: 0.67 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	2476	44.4764 %
Incorrectly Classified Instances	3091	55.5236 %
Kappa statistic	0.3867	
Mean absolute error	0.073	
Root mean squared error	0.191	
Relative absolute error	72.2328 %	
Root relative squared error	84.9952 %	
Total Number of Instances	5567	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as	
1071	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
1	0	0	0	6	0	19	6	0	0	1	3	0	0	0	1	0	0	0	0	c = one
1	0	0	0	104	0	31	1	0	0	0	0	0	0	0	0	0	0	0	0	d = two
3	0	0	0	6	0	29	18	0	0	0	0	0	5	0	0	0	0	0	0	e = three
4	0	0	0	4	0	45	45	30	0	6	5	0	1	0	1	0	0	0	0	f = four
7	0	0	0	8	0	45	46	71	0	37	16	0	21	0	3	1	0	0	0	g = five
10	0	0	0	6	0	7	24	101	0	108	19	0	8	0	5	1	0	0	0	h = six
12	0	0	0	6	0	23	0	42	0	127	61	1	32	0	7	0	1	0	0	i = seven
6	0	0	0	0	0	27	17	0	257	121	4	45	4	24	1	1	0	0	0	j = eight
8	0	0	0	0	0	5	9	16	0	96	165	44	30	22	25	2	3	0	0	k = nine
6	0	0	0	0	0	0	6	0	0	93	80	67	49	39	30	17	6	0	0	l = ten
9	0	0	0	0	0	29	2	0	48	118	45	89	62	63	5	4	0	0	0	m = eleven
8	0	0	0	0	0	0	11	0	0	30	96	44	37	86	103	47	23	0	0	n = twelve
7	0	0	0	0	0	0	1	0	0	15	39	3	29	14	196	58	33	0	0	o = thirteen
11	0	0	0	0	0	0	0	0	0	1	19	0	8	0	100	128	75	0	0	p = fourteen
8	0	0	0	0	0	0	0	0	0	0	1	0	0	0	18	47	121	0	0	q = fifteen
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	22	0	0	r = sixteen

## My Selection

### One Object per Leaf

Options: -U -M 1

J48 unpruned tree

-----

Number of Leaves : 6313

Size of the tree : 12625

Time taken to build model: 18.31 seconds

Time taken to test model on training data: 0.47 seconds

=== Error on training (and test) data ===

```

Correctly Classified Instances      22100          98.0871 %
Incorrectly Classified Instances    431            1.9129 %
Kappa statistic                    0.9787
Mean absolute error                0.0027
Root mean squared error            0.0368
Relative absolute error            2.719 %
Root relative squared error        16.4899 %
Total Number of Instances          22531

```

```
=== Confusion Matrix ===
```

```

   a   b   c   d   e   f   g   h   i   j   k   l   m   n   o   p   q   r  <-- classified as
1927  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0 |  a = draw
   0  27   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0 |  b = zero
   0   0  68  10   0   0   0   0   0   0   0   0   0   0   0   0   0 |  c = one
   0   0   0 246   0   0   0   0   0   0   0   0   0   0   0   0   0 |  d = two
   0   0   0   0  81   0   0   0   0   0   0   0   0   0   0   0   0 |  e = three
   0   0   0   0   4 180   6   5   0   0   0   0   0   0   0   0   0 |  f = four
   0   0   0   0   0   6 441   4   0   0   0   0   0   0   0   0   0 |  g = five
   0   0   0   0   0   0  21 521   1   5   0   0   0   0   0   0   0 |  h = six
   0   0   1   0   0   0   6   6 578   3   9   0   2   0   0   0   0 |  i = seven
   0   0   0   0   0   0   0  22   0 1177   7   5   3   0   0   0   0 |  j = eight
   0   0   0   0   0   0   0   0   9   6 1331   1   3   1   0   0   0 |  k = nine
   0   0   0   0   0   0   0   0   0   2  19 1577   7   2   2   0   0 |  l = ten
   0   0   0   0   0   0   0   0   0   3   3  18  24 2184  12   6   0   0 |  m = eleven
   0   0   0   0   0   0   0   0   0   0   3   2  14  19 2868  15   2   0   0 |  n = twelve
   0   0   0   0   0   0   0   0   0   0   0   6   8  14 3317  22   2   1 |  o = thirteen
   0   0   0   0   0   0   0   0   0   0   0   0   0   4  31 3611  16   0 |  p = fourteen
   0   0   0   0   0   0   0   0   0   0   0   0   0   0  11  12 1672   1 |  q = fifteen
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   4 294 |  r = sixteen

```

## Two Objects per Leaf

```
Options: -U -M 2
```

```
J48 unpruned tree
```

```
-----
```

```
Number of Leaves : 3567
```

```
Size of the tree : 7133
```

```
Time taken to build model: 14.32 seconds
```

```
Time taken to test model on training data: 0.44 seconds
```

```
=== Error on training (and test) data ===
```

```

Correctly Classified Instances      20365          90.3866 %
Incorrectly Classified Instances    2166           9.6134 %
Kappa statistic                    0.8929
Mean absolute error                0.015
Root mean squared error            0.0867
Relative absolute error            15.0691 %

```

```

Root relative squared error      38.8198 %
Total Number of Instances      22531

```

```

=== Confusion Matrix ===

```

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
1927	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	77	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	c = one
0	0	8	234	1	2	0	0	1	0	0	0	0	0	0	0	0	0	0	d = two
0	0	0	3	73	3	1	0	1	0	0	0	0	0	0	0	0	0	0	e = three
0	0	0	1	2	173	10	7	1	1	0	0	0	0	0	0	0	0	0	f = four
0	0	0	1	4	9	414	14	5	0	1	1	1	1	0	0	0	0	0	g = five
0	0	0	0	2	5	29	476	20	10	2	2	1	0	0	1	0	0	0	h = six
0	0	1	0	0	0	8	19	528	17	20	4	7	1	0	0	0	0	0	i = seven
0	0	0	0	0	0	11	35	31	1059	44	17	10	6	1	0	0	0	0	j = eight
0	0	0	0	0	0	4	6	11	45	1212	28	21	17	6	1	0	0	0	k = nine
0	0	0	0	0	0	2	0	4	23	49	1432	43	27	22	7	0	0	0	l = ten
0	0	0	0	0	0	3	2	9	38	41	81	1923	86	47	19	1	0	0	m = eleven
0	0	0	0	0	0	1	1	2	13	14	47	89	2588	120	42	6	0	0	n = twelve
0	0	0	0	0	0	1	1	1	2	2	23	43	111	3003	155	25	3	0	o = thirteen
0	0	0	0	0	0	0	0	0	1	0	7	19	47	118	3383	79	8	0	p = fourteen
0	0	0	0	0	0	0	0	0	0	1	0	3	7	30	84	1559	12	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	17	277	0	r = sixteen

## Four Objects per Leaf

```
Options: -U -M 4
```

```
J48 unpruned tree
```

```
Number of Leaves : 2148
```

```
Size of the tree : 4295
```

```
Time taken to build model: 10.03 seconds
```

```
Time taken to test model on training data: 0.43 seconds
```

```
=== Error on training (and test) data ===
```

Correctly Classified Instances	18609	82.5929 %
Incorrectly Classified Instances	3922	17.4071 %
Kappa statistic	0.8062	
Mean absolute error	0.0269	
Root mean squared error	0.116	
Relative absolute error	26.9983 %	
Root relative squared error	51.961 %	
Total Number of Instances	22531	

```
=== Confusion Matrix ===
```

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-------------------



```

1  0  1  0  9  17  339  33  17  5  6  11  5  7  0  0  0  0 | g = five
0  0  0  5  7  11  58  362  46  30  8  9  5  4  0  3  0  0 | h = six
0  0  4  1  4  7  28  33  371  65  36  22  17  14  1  2  0  0 | i = seven
0  0  0  0  1  1  9  40  79  833  114  68  30  31  7  1  0  0 | j = eight
0  0  0  0  1  0  3  6  44  134  911  112  56  67  12  4  1  0 | k = nine
0  0  0  0  0  0  8  10  27  70  85  1036  177  104  65  24  3  0 | l = ten
0  0  0  0  0  0  6  3  12  57  98  150  1527  224  105  63  5  0 | m = eleven
0  0  0  0  0  0  0  3  1  31  44  87  235  2049  322  131  20  0 | n = twelve
0  0  0  0  0  0  0  0  2  7  22  63  120  237  2442  426  47  4 | o = thirteen
0  0  0  0  0  0  0  0  0  2  3  14  18  105  277  2996  238  9 | p = fourteen
0  0  0  0  0  0  0  0  0  0  0  1  3  16  40  238  1350  48 | q = fifteen
0  0  0  0  0  0  0  0  0  0  0  0  0  0  3  11  57  227 | r = sixteen

```

## Sixteen Objects per Leaf

Options: -U -M 16

J48 unpruned tree

-----

Number of Leaves : 628

Size of the tree : 1255

Time taken to build model: 4.5 seconds

Time taken to test model on training data: 0.39 seconds

=== Error on training (and test) data ===

```

Correctly Classified Instances      15374      68.2349 %
Incorrectly Classified Instances    7157      31.7651 %
Kappa statistic                    0.6459
Mean absolute error                 0.0476
Root mean squared error             0.1543
Relative absolute error             47.7554 %
Root relative squared error         69.1067 %
Total Number of Instances          22531

```

=== Confusion Matrix ===

```

  a   b   c   d   e   f   g   h   i   j   k   l   m   n   o   p   q   r  <-- classified as
1918  0   0   6   0   3   0   0   0   0   0   0   0   0   0   0   0   0 | a = draw
  0  10   5  12   0   0   0   0   0   0   0   0   0   0   0   0   0   0 | b = zero
  0   0  61  13   0   1   0   0   3   0   0   0   0   0   0   0   0   0 | c = one
  0  10   5 212  15   2   1   0   1   0   0   0   0   0   0   0   0   0 | d = two
  0   0   0   2  43  17  12   5   2   0   0   0   0   0   0   0   0   0 | e = three
  0   0   0   1  11 137  26  10   6   0   4   0   0   0   0   0   0   0 | f = four
  0   0   3   2   5 24 316  39  18  17   5   9   5   8   0   0   0   0 | g = five
  0   0   2   5   3 14 63 333  53  40   8  13   4   4   4   2   0   0 | h = six
  0   0   4   4   3   1 42 55 295  96  46  19  16  15   5   4   0   0 | i = seven
  0   0   1   4   3   0 12 65 68 733 144  66  60  30  25   3   0   0 | j = eight
  0   0   0   3   5   0   5   8  31 156 797 109 114  90  23   9   1   0 | k = nine
  0   0   0   3   3   1 20 14 24 93 155 825 233 103 103  31   1   0 | l = ten

```



## Sixty four Objects per Leaf

Options: -U -M 64

J48 unpruned tree

-----  
Number of Leaves : 185

Size of the tree : 369

Time taken to build model: 2.66 seconds

Time taken to test model on training data: 0.33 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	12551	55.7055 %
Incorrectly Classified Instances	9980	44.2945 %
Kappa statistic	0.5057	
Mean absolute error	0.0626	
Root mean squared error	0.1769	
Relative absolute error	62.7248 %	
Root relative squared error	79.2007 %	
Total Number of Instances	22531	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
1918	0	0	6	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
0	0	0	22	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	24	25	0	0	1	25	0	3	0	0	0	0	0	0	0	0	0	c = one
0	0	9	221	0	0	9	7	0	0	0	0	0	0	0	0	0	0	0	d = two
0	0	5	19	0	2	23	5	17	4	0	1	5	0	0	0	0	0	0	e = three
0	0	4	13	0	26	34	74	4	5	0	5	30	0	0	0	0	0	0	f = four
0	0	3	67	0	21	142	116	46	30	3	9	14	0	0	0	0	0	0	g = five
0	0	6	11	0	24	85	195	109	67	9	16	15	5	6	0	0	0	0	h = six
0	0	3	16	0	2	54	25	223	134	36	41	50	10	10	1	0	0	0	i = seven
0	0	4	33	0	0	50	24	108	579	167	120	65	32	28	4	0	0	0	j = eight
0	0	3	20	0	0	27	21	37	204	616	111	179	78	47	8	0	0	0	k = nine
0	0	3	12	0	0	16	38	4	136	165	550	335	177	133	35	5	0	0	l = ten
0	0	0	6	0	0	15	25	6	120	98	214	1101	343	215	99	8	0	0	m = eleven
0	0	0	8	0	0	2	32	4	67	72	124	342	1336	586	306	44	0	0	n = twelve
0	0	0	2	0	0	3	7	0	6	35	78	157	251	2017	683	125	6	0	o = thirteen
0	0	0	0	0	0	1	12	0	0	14	19	54	88	528	2482	435	29	0	p = fourteen
0	0	0	0	0	0	0	9	0	0	6	2	3	1	36	569	982	88	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	13	145	139	0	r = sixteen

## Hundred twenty eight Objects per Leaf

Options: -U -M 128

J48 unpruned tree

-----

Number of Leaves : 95

Size of the tree : 189

Time taken to build model: 2.35 seconds

Time taken to test model on training data: 0.31 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	11362	50.4283 %
Incorrectly Classified Instances	11169	49.5717 %
Kappa statistic	0.4465	
Mean absolute error	0.0681	
Root mean squared error	0.1846	
Relative absolute error	68.2962 %	
Root relative squared error	82.6432 %	
Total Number of Instances	22531	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
1918	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	3	0	0	a = draw
0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	0	74	0	0	0	0	0	0	0	0	0	3	0	0	1	0	0	c = one
0	0	0	207	0	0	36	0	2	0	0	0	0	0	0	0	1	0	0	d = two
0	0	0	24	0	0	28	0	20	3	0	0	1	0	0	5	0	0	0	e = three
0	0	0	19	0	0	129	0	17	10	0	0	12	0	0	8	0	0	0	f = four
0	0	0	36	0	0	281	0	49	39	7	8	13	6	0	12	0	0	0	g = five
0	0	0	49	0	0	141	0	74	136	37	53	18	23	3	14	0	0	0	h = six
0	0	0	52	0	0	73	0	192	70	83	52	56	7	7	13	0	0	0	i = seven
0	0	0	50	0	0	100	0	130	319	231	158	146	39	25	16	0	0	0	j = eight
0	0	0	29	0	0	37	0	70	99	567	126	220	134	45	24	0	0	0	k = nine
0	0	0	27	0	0	44	0	80	69	206	414	363	127	125	143	11	0	0	l = ten
0	0	0	17	0	0	44	0	48	49	236	222	884	424	183	125	18	0	0	m = eleven
0	0	0	15	0	0	26	0	7	53	126	129	340	1353	463	356	55	0	0	n = twelve
0	0	0	9	0	0	5	0	2	17	20	114	245	335	1709	747	162	5	0	o = thirteen
0	0	0	1	0	0	2	0	0	1	0	30	66	208	427	2440	464	23	0	p = fourteen
0	0	0	0	0	0	0	0	0	0	0	1	7	28	20	599	963	78	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	10	171	115	0	r = sixteen

## All Attributes

### One Object per Leaf

Options: -U -M 1

J48 unpruned tree

-----

Number of Leaves : 4372

Size of the tree : 8675

Time taken to build model: 132.17 seconds

Time taken to test model on training data: 3.26 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	28044	99.9572 %
Incorrectly Classified Instances	12	0.0428 %
Kappa statistic	0.9995	
Mean absolute error	0.0001	
Root mean squared error	0.0057	
Relative absolute error	0.0661 %	
Root relative squared error	2.5702 %	
Total Number of Instances	28056	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
2796	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	c = one
0	0	0	246	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d = two
0	0	0	0	81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	e = three
0	0	0	0	0	198	0	0	0	0	0	0	0	0	0	0	0	0	0	f = four
0	0	0	0	0	0	471	0	0	0	0	0	0	0	0	0	0	0	0	g = five
0	0	0	0	0	0	0	592	0	0	0	0	0	0	0	0	0	0	0	h = six
0	0	0	0	0	0	0	0	683	0	0	0	0	0	0	0	0	0	0	i = seven
0	0	0	0	0	0	0	0	0	1433	0	0	0	0	0	0	0	0	0	j = eight
0	0	0	0	0	0	0	0	0	0	1712	0	0	0	0	0	0	0	0	k = nine
0	0	0	0	0	0	0	0	0	0	0	1985	0	0	0	0	0	0	0	l = ten
0	0	0	0	0	0	0	0	0	0	0	0	2854	0	0	0	0	0	0	m = eleven
0	0	0	0	0	0	0	0	0	0	0	0	0	63589	2	0	0	0	0	n = twelve
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4194	0	0	0	0	o = thirteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24551	0	0	0	p = fourteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22164	0	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	390	r = sixteen

## Two Objects per Leaf

Options: -U -M 2

J48 unpruned tree

-----

Number of Leaves : 3060

Size of the tree : 6078

Time taken to build model: 119.06 seconds

Time taken to test model on training data: 3.66 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	27113	96.6389 %
Incorrectly Classified Instances	943	3.3611 %
Kappa statistic	0.9625	
Mean absolute error	0.0056	
Root mean squared error	0.0527	
Relative absolute error	5.5845 %	
Root relative squared error	23.632 %	
Total Number of Instances	28056	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as	
2796	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
1	25	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	c = one
0	0	0	246	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d = two
0	0	0	2	77	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	e = three
0	0	0	0	2	193	3	0	0	0	0	0	0	0	0	0	0	0	0	0	f = four
0	0	1	0	3	3	460	1	2	1	0	0	0	0	0	0	0	0	0	0	g = five
0	0	0	0	3	3	13	545	16	5	3	1	2	1	0	0	0	0	0	0	h = six
0	0	2	1	1	0	4	12	637	14	7	4	1	0	0	0	0	0	0	0	i = seven
0	0	0	0	0	0	3	7	19	1373	21	9	1	0	0	0	0	0	0	0	j = eight
0	0	0	0	0	0	1	3	6	30	1635	22	12	2	1	0	0	0	0	0	k = nine
0	0	0	0	0	0	2	0	4	14	38	1885	32	8	2	0	0	0	0	0	l = ten
0	0	0	0	0	0	2	0	4	5	14	34	2717	57	16	4	1	0	0	0	m = eleven
0	0	0	0	0	0	1	0	2	1	4	13	46	3466	54	8	2	0	0	0	n = twelve
0	0	0	0	0	0	0	0	1	1	1	6	18	62	4038	66	1	0	0	0	o = thirteen
0	0	0	0	0	0	0	0	0	0	0	1	1	8	61	4453	29	0	0	0	p = fourteen
0	0	0	0	0	0	0	0	0	0	0	0	0	1	4	45	2110	6	0	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	10	379	0	0	r = sixteen

## Four Objects per Leaf

Options: -U -M 4

J48 unpruned tree

Number of Leaves : 2161

Size of the tree : 4289

Time taken to build model: 107.8 seconds

Time taken to test model on training data: 3.94 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	25818	92.0231 %
Incorrectly Classified Instances	2238	7.9769 %
Kappa statistic	0.9109	
Mean absolute error	0.0129	
Root mean squared error	0.0802	

```

Relative absolute error          12.9297 %
Root relative squared error      35.9585 %
Total Number of Instances        28056

```

```
=== Confusion Matrix ===
```

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
2796	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	a = draw
1	25	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b = zero
0	0	77	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	c = one
1	0	0	241	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	d = two
0	0	0	2	69	1	4	2	3	0	0	0	0	0	0	0	0	0	0	e = three
0	0	0	1	1	181	7	7	1	0	0	0	0	0	0	0	0	0	0	f = four
0	0	0	1	4	11	410	32	10	1	1	0	1	0	0	0	0	0	0	g = five
0	0	0	1	2	5	28	502	25	18	5	2	4	0	0	0	0	0	0	h = six
1	0	1	0	0	4	6	25	584	50	6	3	1	1	1	0	0	0	0	i = seven
0	0	0	0	0	1	8	12	41	1291	53	18	6	2	1	0	0	0	0	j = eight
0	0	0	0	0	0	2	3	12	66	1511	71	34	7	6	0	0	0	0	k = nine
0	0	0	0	0	1	6	2	13	37	57	1752	81	27	8	1	0	0	0	l = ten
0	0	0	0	0	1	2	0	8	11	24	81	2567	111	40	6	3	0	0	m = eleven
0	0	0	0	0	0	2	2	1	3	13	27	114	3302	110	21	2	0	0	n = twelve
0	0	0	0	0	2	2	0	3	2	5	20	25	149	3809	167	10	0	0	o = thirteen
0	0	0	0	0	0	1	0	0	0	1	5	1	22	146	4294	83	0	0	p = fourteen
0	0	0	0	0	0	0	0	0	0	0	0	0	2	5	101	2042	16	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	365	0	r = sixteen

## Eight Objects per Leaf

```
Options: -U -M 8
```

```
J48 unpruned tree
```

```
-----
```

```
Number of Leaves : 1329
```

```
Size of the tree : 2640
```

```
Time taken to build model: 108.24 seconds
```

```
Time taken to test model on training data: 3.32 seconds
```

```
=== Error on training (and test) data ===
```

```

Correctly Classified Instances      23903          85.1975 %
Incorrectly Classified Instances     4153          14.8025 %
Kappa statistic                     0.8347
Mean absolute error                  0.0233
Root mean squared error              0.108
Relative absolute error              23.4457 %
Root relative squared error          48.4216 %
Total Number of Instances           28056

```

```
=== Confusion Matrix ===
```

```

a   b   c   d   e   f   g   h   i   j   k   l   m   n   o   p   q   r   <-- classified as
2796 0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0 | a = draw
0   25  0   1   0   0   0   0   1   0   0   0   0   0   0   0   0 | b = zero
7   0  67   3   1   0   0   0   0   0   0   0   0   0   0   0   0 | c = one
0   0   0 239   2   2   0   0   3   0   0   0   0   0   0   0   0 | d = two
3   0   4   5  50   8   6   0   4   0   1   0   0   0   0   0   0 | e = three
0   2   1   0   0 165  19   6   2   0   0   0   0   3   0   0   0 | f = four
1   0   0   0  10  16 377  31  20   4   6   4   0   2   0   0   0 | g = five
0   1   0   2  18  20  27 439  34  21  20   4   1   1   3   1   0 | h = six
1   2   1   4   1   3  25  41 477  78  23  10   8   4   5   0   0 | i = seven
0   0   0   0   1   1   5  18  55 1160 122  46  15   6   4   0   0 | j = eight
0   0   0   1   0   0   0   6  17  112 1364 122  60  18  11   1   0 | k = nine
0   0   0   0   3   0   5  11  18  28  144 1525 174  62   9   5   1 | l = ten
0   2   1   0   0   0   5  10   5  15  67  152 2341 190  53  10   3 | m = eleven
0   3   0   0   0   0   3   0   6   1  23  71  204 3011 228  45   2 | n = twelve
0   0   0   0   0   0   0   0   7   3   9  17  66  281 3540 257  14 | o = thirteen
0   0   0   0   0   0   0   0   0   0   1   7  10  51  278 4057 147  2 | p = fourteen
0   0   0   0   0   0   0   0   0   0   0   0   1   6  10  198 1923 28 | q = fifteen
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   4  39 347 | r = sixteen

```

## Sixteen Objects per Leaf

Options: -U -M 16

J48 unpruned tree

Number of Leaves : 775

Size of the tree : 1539

Time taken to build model: 76.46 seconds

Time taken to test model on training data: 2.87 seconds

=== Error on training (and test) data ===

```

Correctly Classified Instances      21924          78.1437 %
Incorrectly Classified Instances    6132           21.8563 %
Kappa statistic                    0.7558
Mean absolute error                 0.0337
Root mean squared error             0.1298
Relative absolute error             33.8614 %
Root relative squared error         58.1916 %
Total Number of Instances          28056

```

=== Confusion Matrix ===

```

a   b   c   d   e   f   g   h   i   j   k   l   m   n   o   p   q   r   <-- classified as
2793 0   0   0   0   3   0   0   0   0   0   0   0   0   0   0   0   0 | a = draw
0   10  0   1   5   0   5   0   1   0   0   5   0   0   0   0   0 | b = zero
6   0  65   0   6   1   0   0   0   0   0   0   0   0   0   0   0 | c = one
0   9   5 225   2   0   0   2   3   0   0   0   0   0   0   0   0 | d = two

```

```

5  0  4  5  35  16  4  5  3  4  0  0  0  0  0  0  0  0 | e = three
0  0  2  0  4 152  24  11  2  0  0  0  0  3  0  0  0  0 | f = four
0  0  0  3  1  24 343  47  25  9  6  1  4  0  2  6  0  0 | g = five
0  0  5  3  5  18  44 410  36  31  20  6  5  3  3  3  0  0 | h = six
1  0  3  5  0  8  32  49 374 126  23  33  15  6  8  0  0  0 | i = seven
0  0  6  1  0  0  5  13  77 1013 167  87  42  13  6  3  0  0 | j = eight
0  0  3  1  0  2  4  7  23 115 1209 181 108  37  21  1  0  0 | k = nine
0  0  2  0  0  2  9  4  6  41 168 1301 294 117  23  17  1  0 | l = ten
0  0  1  0  0  2  4  5  0  7  72 202 2099 342 109  5  6  0 | m = eleven
0  0  0  0  0  0  0  0  2  1  25  64 289 2720 418  76  2  0 | n = twelve
0  0  0  0  0  0  0  0  1  1  2  25 123 374 3286 353  28  1 | o = thirteen
0  0  0  0  0  0  0  0  0  0  0  3  10  65 446 3717 302  10 | p = fourteen
0  0  0  0  0  0  0  0  0  0  0  0  2  8  20 222 1881  33 | q = fifteen
0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  8  90 291 | r = sixteen

```

## Thirty two Objects per Leaf

Options: -U -M 32

J48 unpruned tree

-----

Number of Leaves : 433

Size of the tree : 860

Time taken to build model: 57.79 seconds

Time taken to test model on training data: 2.97 seconds

=== Error on training (and test) data ===

```

Correctly Classified Instances      19903          70.9403 %
Incorrectly Classified Instances     8153          29.0597 %
Kappa statistic                     0.6753
Mean absolute error                  0.0439
Root mean squared error              0.1481
Relative absolute error              44.0794 %
Root relative squared error          66.3935 %
Total Number of Instances           28056

```

=== Confusion Matrix ===

```

   a  b  c  d  e  f  g  h  i  j  k  l  m  n  o  p  q  r  <-- classified as
2787 0  0  6  0  0  3  0  0  0  0  0  0  0  0  0  0  0 | a = draw
   0 10  6 11  0  0  0  0  0  0  0  0  0  0  0  0  0  0 | b = zero
   0  0 56 12  0  0  7  0  0  0  0  0  0  3  0  0  0  0 | c = one
   0  0 18 223 0  0  5  0  0  0  0  0  0  0  0  0  0  0 | d = two
   0  6  0  22  0 13  30  1  5  4  0  0  0  0  0  0  0  0 | e = three
   0  5  3  1  0 105  40  37  4  0  0  0  3  0  0  0  0  0 | f = four
   0  8  3  5  0  20 252  80  52  28  10  0  4  3  0  6  0  0 | g = five
   0  5  6  6  0  9  55 381  65  30  13  6  7  3  3  3  0  0 | h = six
   0  0  8 11  0  7  23  56 340 153  31  20 11 17  6  0  0  0 | i = seven
   0  0 10  3  0  0  2  59  50 913 248  91  29 15  9  4  0  0 | j = eight

```

```

0 0 7 0 0 0 2 7 50 176 1034 198 165 46 14 12 1 0 | k = nine
0 0 4 3 0 0 3 4 13 54 255 1136 312 155 31 14 1 0 | l = ten
0 0 1 0 0 0 5 7 0 21 150 292 1811 358 173 32 4 0 | m = eleven
0 0 0 0 0 0 0 0 5 11 38 109 366 2374 543 131 20 0 | n = twelve
0 0 0 0 0 0 0 0 0 4 4 31 106 464 3017 518 50 0 | o = thirteen
0 0 0 0 0 0 0 0 0 0 0 27 2 73 533 3517 381 20 | p = fourteen
0 0 0 0 0 0 0 0 0 0 0 0 0 1 49 377 1671 68 | q = fifteen
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 3 110 276 | r = sixteen

```

## Sixty four Objects per Leaf

Options: -U -M 64

J48 unpruned tree

-----

Number of Leaves : 227

Size of the tree : 452

Time taken to build model: 69.63 seconds

Time taken to test model on training data: 2.6 seconds

=== Error on training (and test) data ===

```

Correctly Classified Instances      17538          62.5107 %
Incorrectly Classified Instances    10518          37.4893 %
Kappa statistic                    0.5809
Mean absolute error                 0.054
Root mean squared error             0.1643
Relative absolute error             54.2515 %
Root relative squared error         73.6569 %
Total Number of Instances          28056

```

=== Confusion Matrix ===

```

  a  b  c  d  e  f  g  h  i  j  k  l  m  n  o  p  q  r  <-- classified as
2787 0  0  5  22 0  0  3  4  0  2  0  0  0  0  0  0  0 | a = draw
0  0  5  22 0  0  0  0  0  0  0  0  0  0  0  0  0  0 | b = zero
0  0  25 40 0  0  1  0  0  12 0  0  0  0  0  0  0  0 | c = one
0  0  5 209 0  0  26  4  0  2  0  0  0  0  0  0  0  0 | d = two
0  0  0  23 0  13 35  0  2  4  3  0  1  0  0  0  0  0 | e = three
0  0  0  12 0  111 36 29  4  3  0  0  0  3  0  0  0  0 | f = four
0  0  0  77 0  39 180 57 25 39 23 10 15  0  0  6  0  0 | g = five
0  0  0  10 0  54 113 175 87 58 27 43 22  0  0  3  0  0 | h = six
0  0  0  16 0  3  42 55 287 145 80 15 19 14  6  1  0  0 | i = seven
0  0  0  16 0  6  34 74 111 784 297 51 38 10  6  6  0  0 | j = eight
0  0  1  15 0  0  14  5  39 281 952 116 150 113 16 10  0  0 | k = nine
0  0  7  11 0  0  13 55  3 189 370 588 367 274 60 48  0  0 | l = ten
0  0  3  7  0  0  6  5  0  80 242 200 1509 480 264 51  7  0 | m = eleven
0  0  9  3  0  0  0  0  0  29 73 111 399 2011 764 188 10  0 | n = twelve
0  0  7  3  0  0  0  0  0  12 13 39 108 406 2843 715 48  0 | o = thirteen
0  0  4  0  0  0  0  0  0  0  2  3 11 93 679 3255 469 37 | p = fourteen
0  0  0  0  0  0  0  0  0  0  0  1  0  4  64 429 1530 138 | q = fifteen

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 10 84 292 | r = sixteen

## Hundred twenty eight Objects per Leaf

Options: -U -M 128

J48 unpruned tree

-----

Number of Leaves : 129

Size of the tree : 255

Time taken to build model: 51.57 seconds

Time taken to test model on training data: 2.97 seconds

=== Error on training (and test) data ===

Correctly Classified Instances	15872	56.5726 %
Incorrectly Classified Instances	12184	43.4274 %
Kappa statistic	0.5134	
Mean absolute error	0.0606	
Root mean squared error	0.1741	
Relative absolute error	60.9312 %	
Root relative squared error	78.0598 %	
Total Number of Instances	28056	

=== Confusion Matrix ===

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	<-- classified as
2787	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	3	0	0	a = draw
0	0	0	23	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	b = zero
0	0	0	59	0	0	12	0	0	3	0	3	0	0	0	1	0	0	0	c = one
0	0	0	188	0	0	3	0	0	19	0	35	0	0	0	1	0	0	0	d = two
0	0	0	3	0	12	25	4	0	28	0	3	1	0	0	5	0	0	0	e = three
0	0	0	6	0	66	39	57	6	13	0	0	0	3	0	8	0	0	0	f = four
0	0	0	20	0	40	244	23	31	58	0	22	15	0	0	18	0	0	0	g = five
0	0	0	19	0	24	139	87	102	113	10	52	32	0	0	14	0	0	0	h = six
0	0	0	19	0	8	82	23	213	136	81	66	22	14	6	13	0	0	0	i = seven
0	0	0	72	0	0	113	54	150	586	227	123	69	10	15	14	0	0	0	j = eight
0	0	0	66	0	0	28	11	63	256	650	191	302	96	26	23	0	0	0	k = nine
0	0	0	44	0	0	4	23	31	97	291	628	402	278	97	90	0	0	0	l = ten
0	0	0	19	0	0	3	0	13	32	203	370	1468	430	228	83	5	0	0	m = eleven
0	0	0	23	0	0	0	0	9	12	74	146	552	1734	819	213	15	0	0	n = twelve
0	0	0	18	0	0	0	0	5	9	24	16	246	448	2565	821	42	0	0	o = thirteen
0	0	0	15	0	0	0	0	0	0	4	0	88	187	661	3229	369	0	0	p = fourteen
0	0	0	1	0	0	0	0	0	0	0	0	3	14	88	706	1269	85	0	q = fifteen
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	11	217	158	0	r = sixteen

# Appendix B

## Script Source Code

We implemented a script in Ruby<sup>1</sup> to read automatically the primary database and transform it practically in the new databases. This script allows us to chose a particular attribute set and to specify some other options, mainly for test purpose. The implementation is based on libraries which could be easily reused in other programmes, if needed. The *Attribute Classes* contain the description of each attribute, and calculate their values based on the values returned by the *Position Class*. The attributes are handled as a combination by the *Attributes Class*. The *Position Class* computes all the distance values relying on the positions of the figures on the board. The *Hash Class* contains statistical functions which test the properties of the new computed database. The *Main File* reads the primary database and calls functions from the other classes, depending on what the *Option Class*, which deals with the command line options, specifies. Some *Utilities* are also included in the script, as well as *Another Main File* to test the attributes.

### Attribute Classes

```
1 #####
2 # Pieces files and rank
3 ##
4
5 # white king
6
7 class Attr_WK_file
8   def getHeader
9     return "@attribute whiteKingFile {a, b, c, d, e, f, g, h}"
```

---

<sup>1</sup><http://www.ruby-lang.org/>

```
10     end
11
12     def compute(position)
13         return position.wkf.chr
14     end
15
16     def name
17         return "white king file"
18     end
19 end
20
21 class Attr_WK_file_Num
22     def getHeader
23         return "@attribute whiteKingFile numeric"
24     end
25
26     def compute(position)
27         return position.wkf - "a"[0] + 1
28     end
29
30     def name
31         return "white king file as a number"
32     end
33 end
34
35 class Attr_WK_rank
36     def getHeader
37         return "@attribute whiteKingRank {1, 2, 3, 4, 5, 6, 7, 8}"
38     end
39
40     def compute(position)
41         return position.wkr.chr
42     end
43
44     def name
45         return "white king rank"
46     end
47 end
48
49 class Attr_WK_rank_Num
50     def getHeader
51         return "@attribute whiteKingRank numeric"
52     end
53
54     def compute(position)
55         return position.wkr.chr.to_i
56     end
```

```
57
58     def name
59         return "white king rank as a number"
60     end
61 end
62
63 # white rook
64
65 class Attr_WR_file
66     def getHeader
67         return "@attribute whiteRookFile {a, b, c, d, e, f, g, h}"
68     end
69
70     def compute(position)
71         return position.wrf.chr
72     end
73
74     def name
75         return "white rook file"
76     end
77 end
78
79 class Attr_WR_file_Num
80     def getHeader
81         return "@attribute whiteRookFile numeric"
82     end
83
84     def compute(position)
85         return position.wrf - "a"[0] + 1
86     end
87
88     def name
89         return "white rook file as a number"
90     end
91 end
92
93 class Attr_WR_rank
94     def getHeader
95         return "@attribute whiteRookRank {1, 2, 3, 4, 5, 6, 7, 8}"
96     end
97
98     def compute(position)
99         return position.wrr.chr
100    end
101
102    def name
103        return "white rook rank"
```

```
104     end
105 end
106
107 class Attr_WR_rank_Num
108     def getHeader
109         return "@attribute whiteRookRank numeric"
110     end
111
112     def compute(position)
113         return position.wrr.chr.to_i
114     end
115
116     def name
117         return "white rook rank as a number"
118     end
119 end
120
121 # black king
122
123 class Attr_BK_file
124     def getHeader
125         return "@attribute blackKingFile {a, b, c, d, e, f, g, h}"
126     end
127
128     def compute(position)
129         return position.bkf.chr
130     end
131
132     def name
133         return "black king file"
134     end
135 end
136
137 class Attr_BK_file_Num
138     def getHeader
139         return "@attribute blackKingFile numeric"
140     end
141
142     def compute(position)
143         return position.bkf - "a"[0] + 1
144     end
145
146     def name
147         return "black king file as a number"
148     end
149 end
150
```

```
151 class Attr_BK_rank
152   def getHeader
153     return "@attribute blackKingRank {1, 2, 3, 4, 5, 6, 7, 8}"
154   end
155
156   def compute(position)
157     return position.bkr.chr
158   end
159
160   def name
161     return "black king rank"
162   end
163 end
164
165 class Attr_BK_rank_Num
166   def getHeader
167     return "@attribute blackKingRank numeric"
168   end
169
170   def compute(position)
171     return position.bkr.chr.to_i
172   end
173
174   def name
175     return "black king rank as a number"
176   end
177 end
178
179
180 #####
181 # depth of win
182 ##
183
184 class Attr_Depth_of_win
185   def getHeader
186     return "@attribute depth_of_win {draw, zero, one, two, three, four, five, six,
187     seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen}"
188   end
189
190   def compute(position)
191     return position.dow
192   end
193
194   def name
195     return "depth of win"
196   end
197 end
```

```
198
199 class Attr_Depth_of_win_Num
200     def getHeader
201         return "@attribute depth_of_win_num numeric"
202     end
203
204     def compute(position)
205         return {"draw"=>17, "zero"=>0, "one"=>1, "two"=>2, "three"=>3, "four"=>4,
206             "five"=>5, "six"=>6, "seven"=>7, "eight"=>8, "nine"=>9, "ten"=>10,
207             "eleven"=>11, "twelve"=>12, "thirteen"=>13, "fourteen"=>14,
208             "fifteen"=>15, "sixteen"=>16}[position.dow]
209     end
210
211     def name
212         return "depth of win as number"
213     end
214 end
215
216
217 #####
218 # attributes: Adjacents
219 ##
220
221 # black king threatens white rook
222 class Attr_Adjacent_WR_BK
223     def getHeader
224         return "@attribute adjacent_WR_BK {true, false}"
225     end
226
227     def compute(position)
228         return(position.dist_WR_BK == 1)
229     end
230
231     def name
232         return "the black king threatens the white rook"
233     end
234 end
235
236 # white king protects white rook
237 class Attr_Adjacent_WK_WR
238     def getHeader
239         return "@attribute adjacent_WK_WR {true, false}"
240     end
241
242     def compute(position)
243         return(position.dist_WK_WR == 1)
244     end
```

```
245
246     def name
247         return "the white king protects the white rook"
248     end
249 end
250
251 # the two kings are adjacent: illegal
252 class Attr_Adjacent_WK_BK
253     def getHeader
254         return "@attribute adjacent_WK_BK {true, false}"
255     end
256
257     def compute(position)
258         return(position.dist_WK_BK == 1)
259     end
260
261     def name
262         return "the two kings are on adjacent squares"
263     end
264 end
265
266 #####
267 # attributes: Between
268 ##
269
270 # white rook divides the kings
271 class Attr_WR_between_WK_BK
272     def getHeader
273         return "@attribute WR_between_WK_BK {true, false}"
274     end
275
276     def compute(position)
277         return( smallerThan(position.wkf, position.wrf, position.bkf) or
278                 smallerThan(position.wkr, position.wrr, position.bkr) or
279                 smallerThan(position.bkf, position.wrf, position.wkf) or
280                 smallerThan(position.bkr, position.wrr, position.wkr) )
281     end
282
283     def name
284         return "the white rook divides the both kings"
285     end
286 end
287
288 # white king divides white rook and black king
289 class Attr_WK_between_WR_BK
290     def getHeader
291         return "@attribute WK_between_WR_BK {true, false}"
```

```

292     end
293
294     def compute(position)
295         return(smallerThan(position.wrf, position.wkf, position.bkf) or
296                smallerThan(position.wrr, position.wkr, position.bkr) or
297                smallerThan(position.bkf, position.wkf, position.wrf) or
298                smallerThan(position.bkr, position.wkr, position.wrr))
299     end
300
301     def name
302         return "the order of the pieces is white rook-white king-black king"
303     end
304 end
305
306 # black king divides white king and white rook
307 class Attr_BK_between_WK_WR
308     def getHeader
309         return "@attribute BK_between_WK_WR {true, false}"
310     end
311
312     def compute(position)
313         return(smallerThan(position.wkf, position.bkf, position.wrf) or
314                smallerThan(position.wkr, position.bkr, position.wrr) or
315                smallerThan(position.wrf, position.bkf, position.wkf) or
316                smallerThan(position.wrr, position.bkr, position.wkr))
317     end
318
319     def name
320         return "the order of the pieces is white king-black king-white rook"
321     end
322 end
323
324
325 #####
326 # attributes: Pieces distances
327 ##
328
329 # distance between the kings
330 class Attr_Distance_WK_BK
331     def getHeader
332         return "@attribute distance_WK_BK {1, 2, 3, 4, 5, 6, 7}"
333     end
334
335     def compute(position)
336         return position.dist_WK_BK
337     end
338

```

```
339     def name
340         return "the distance between the kings is"
341     end
342 end
343
344 # distance between the kings (numeric)
345 class Attr_Distance_WK_BK_Num
346     def getHeader
347         return "@attribute distance_WK_BK_num numeric"
348     end
349
350     def compute(position)
351         return position.dist_WK_BK
352     end
353
354     def name
355         return "the distance between the kings as number is"
356     end
357 end
358
359 # distance between white rook and black king
360 class Attr_Distance_WR_BK
361     def getHeader
362         return "@attribute distance_WR_BK {1, 2, 3, 4, 5, 6, 7}"
363     end
364
365     def compute(position)
366         return position.dist_WR_BK
367     end
368
369     def name
370         return "the distance between the white rook and the black king is"
371     end
372 end
373
374 # distance between white rook and black king (numeric)
375 class Attr_Distance_WR_BK_Num
376     def getHeader
377         return "@attribute distance_WR_BK_num numeric"
378     end
379
380     def compute(position)
381         return position.dist_WR_BK
382     end
383
384     def name
385         return "the distance between the white rook and the black king as a number is"
```

```
386     end
387 end
388
389 # distance between white king and white rook
390 class Attr_Distance_WK_WR
391     def getHeader
392         return "@attribute distance_WK_WR {0, 1, 2, 3, 4, 5, 6, 7}"
393     end
394
395     def compute(position)
396         return position.dist_WK_WR
397     end
398
399     def name
400         return "the distance between the white king and the white rook is"
401     end
402 end
403
404 # distance between white king and white rook (numeric)
405 class Attr_Distance_WK_WR_Num
406     def getHeader
407         return "@attribute distance_WK_WR_num numeric"
408     end
409
410     def compute(position)
411         return position.dist_WK_WR
412     end
413
414     def name
415         return "the distance between the white king and the white rook as a number is"
416     end
417 end
418
419 #####
420 # attributes: Pieces file and rank distances
421 ##
422
423 # file distance between the kings
424 class Attr_File_distance_WK_BK
425     def getHeader
426         return "@attribute file_distance_WK_BK {0, 1, 2, 3, 4, 5, 6, 7}"
427     end
428
429     def compute(position)
430         return position.fileDist_WK_BK
431     end
432
```

```
433     def name
434         return "the file distance between the kings is"
435     end
436 end
437
438 # file distance between the kings (numeric)
439 class Attr_File_distance_WK_BK_Num
440     def getHeader
441         return "@attribute file_distance_WK_BK_num numeric"
442     end
443
444     def compute(position)
445         return position.fileDist_WK_BK
446     end
447
448     def name
449         return "the file distance between the kings as number is"
450     end
451 end
452
453 # rank distance between the kings
454 class Attr_Rank_distance_WK_BK
455     def getHeader
456         return "@attribute rank_distance_WK_BK {0, 1, 2, 3, 4, 5, 6, 7}"
457     end
458
459     def compute(position)
460         return position.rankDist_WK_BK
461     end
462
463     def name
464         return "the rank distance between the kings is"
465     end
466 end
467
468 # rank distance between the kings (numeric)
469 class Attr_Rank_distance_WK_BK_Num
470     def getHeader
471         return "@attribute rank_distance_WK_BK_num numeric"
472     end
473
474     def compute(position)
475         return position.rankDist_WK_BK
476     end
477
478     def name
479         return "the rank distance between the kings as number is"
```

```
480     end
481 end
482
483 # file distance between white rook and black king
484 class Attr_File_distance_WR_BK
485   def getHeader
486     return "@attribute file_distance_WR_BK {0, 1, 2, 3, 4, 5, 6, 7}"
487   end
488
489   def compute(position)
490     return position.fileDist_WR_BK
491   end
492
493   def name
494     return "the file distance between the white rook and the black king is"
495   end
496 end
497
498 # file distance between white rook and black king (numeric)
499 class Attr_File_distance_WR_BK_Num
500   def getHeader
501     return "@attribute file_distance_WR_BK_num numeric"
502   end
503
504   def compute(position)
505     return position.fileDist_WR_BK
506   end
507
508   def name
509     return "the file distance between the white rook and the black king as a number is"
510   end
511 end
512
513 # rank distance between white rook and black king
514 class Attr_Rank_distance_WR_BK
515   def getHeader
516     return "@attribute rank_distance_WR_BK {0, 1, 2, 3, 4, 5, 6, 7}"
517   end
518
519   def compute(position)
520     return position.rankDist_WR_BK
521   end
522
523   def name
524     return "the rank distance between the white rook and the black king is"
525   end
526 end
```

```
527
528 # rank distance between white rook and black king (numeric)
529 class Attr_Rank_distance_WR_BK_Num
530   def getHeader
531     return "@attribute rank_distance_WR_BK_num numeric"
532   end
533
534   def compute(position)
535     return position.rankDist_WR_BK
536   end
537
538   def name
539     return "the rank distance between the white rook and the black king as a number is"
540   end
541 end
542
543 # file distance between white king and white rook
544 class Attr_File_distance_WK_WR
545   def getHeader
546     return "@attribute file_distance_WK_WR {0, 1, 2, 3, 4, 5, 6, 7}"
547   end
548
549   def compute(position)
550     return position.fileDist_WK_WR
551   end
552
553   def name
554     return "the file distance between the white king and the white rook is"
555   end
556 end
557
558 # file distance between white king and white rook (numeric)
559 class Attr_File_distance_WK_WR_Num
560   def getHeader
561     return "@attribute file_distance_WK_WR_num numeric"
562   end
563
564   def compute(position)
565     return position.fileDist_WK_WR
566   end
567
568   def name
569     return "the file distance between the white king and the white rook as a number is"
570   end
571 end
572
573 # rank distance between white king and white rook
```

```
574 class Attr_Rank_distance_WK_WR
575   def getHeader
576     return "@attribute rank_distance_WK_WR {0, 1, 2, 3, 4, 5, 6, 7}"
577   end
578
579   def compute(position)
580     return position.rankDist_WK_WR
581   end
582
583   def name
584     return "the rank distance between the white king and the white rook is"
585   end
586 end
587
588 # rank distance between white king and white rook (numeric)
589 class Attr_Rank_distance_WK_WR_Num
590   def getHeader
591     return "@attribute rank_distance_WK_WR_num numeric"
592   end
593
594   def compute(position)
595     return position.rankDist_WK_WR
596   end
597
598   def name
599     return "the rank distance between the white king and the white rook as a number is"
600   end
601 end
602
603 #####
604 # attributes: Board distances
605 ##
606
607 # distance from the black king to the closest edge
608 class Attr_Distance_BK_ce
609   def getHeader
610     return "@attribute Distance_WK_ce {0, 1, 2, 3}"
611   end
612
613   def compute(position)
614     return position.dist_BK_ce
615   end
616
617   def name
618     return "the distance from the black king to the closest edge is"
619   end
620 end
```

```
621
622 # distance from the black king to the closest edge (numeric)
623 class Attr_Distance_BK_ce_Num
624     def getHeader
625         return "@attribute Distance_WK_ce_num numeric"
626     end
627
628     def compute(position)
629         return position.dist_BK_ce
630     end
631
632     def name
633         return "the distance from the black king to the closest edge as a number is"
634     end
635 end
636
637 # distance of the black king to the closest corner
638 class Attr_Distance_BK_co
639     def getHeader
640         return "@attribute Distance_BK_ce {0, 1, 2, 3}"
641     end
642
643     def compute(position)
644         return position.dist_BK_co
645     end
646
647     def name
648         return "the distance from the black king to the closest corner is"
649     end
650 end
651
652 class Attr_Distance_BK_co_Num
653     def getHeader
654         return "@attribute Distance_BK_ce_num numeric"
655     end
656
657     def compute(position)
658         return position.dist_BK_co
659     end
660
661     def name
662         return "the distance from the black king to the closest corner as a number is"
663     end
664 end
665
666 # distance of the white to the central cross
667 class Attr_Distance_WK_cr
```

```
668     def getHeader
669         return "@attribute Distance_WK_cc {0, 1, 2, 3}"
670     end
671
672     def compute(position)
673         return position.dist_WK_cr
674     end
675
676     def name
677         return "the distance from the white king to the central cross is"
678     end
679 end
680
681 class Attr_Distance_WK_cr_Num
682     def getHeader
683         return "@attribute Distance_WK_cc_num numeric"
684     end
685
686     def compute(position)
687         return position.dist_WK_cr
688     end
689
690     def name
691         return "the distance from the white king to the central cross as a number is"
692     end
693 end
694
695 # distance of the white to the closest edge
696 class Attr_Distance_WK_ce
697     def getHeader
698         return "@attribute Distance_WK_ce {0, 1, 2, 3}"
699     end
700
701     def compute(position)
702         return position.dist_WK_ce
703     end
704
705     def name
706         return "the distance from the white king to the closest edge is"
707     end
708 end
709
710 class Attr_Distance_WK_ce_Num
711     def getHeader
712         return "@attribute Distance_WK_ce_num numeric"
713     end
714
```

```
715     def compute(position)
716         return position.dist_WK_ce
717     end
718
719     def name
720         return "the distance from the white king to the closest edge as a number is"
721     end
722 end
723
724 # distance of the white to the closest corner
725 class Attr_Distance_WK_co
726     def getHeader
727         return "@attribute Distance_WK_co {0, 1, 2, 3}"
728     end
729
730     def compute(position)
731         return position.dist_WK_co
732     end
733
734     def name
735         return "the distance from the white king to the closest corner is"
736     end
737 end
738
739 class Attr_Distance_WK_co_Num
740     def getHeader
741         return "@attribute Distance_WK_co_num numeric"
742     end
743
744     def compute(position)
745         return position.dist_WK_co
746     end
747
748     def name
749         return "the distance from the white king to the closest corner is"
750     end
751 end
752
753 #####
754 # attributes: oriented characteristics
755 ##
756
757 # Is the vertical distance between WR and BBK equal 1?
758 class Attr_Vertical_distance_WR_BK
759     def getHeader
760         return "@attribute vertical_distance_WR_BK_1 {not_one_square, one_square}"
761     end
```

```
762
763 def compute(position)
764     choosenBottom = position.bottom
765
766     case choosenBottom
767     when "a"
768         if position.wrf == position.bkf + 1 then return "one_square"
769         else return "not_one_square" end
770     when "h"
771         if position.wrf == position.bkf - 1 then return "one_square"
772         else return "not_one_square" end
773     when "1"
774         if position.wrr == position.bkr + 1 then return "one_square"
775         else return "not_one_square" end
776     when "8"
777         if position.wrr == position.bkr - 1 then return "one_square"
778         else return "not_one_square" end
779     end
780 end
781
782 def name
783     return "the vertical distance (1) between the black king and the rook is"
784 end
785 end
786
787 # Is the vertical distance between WK and BK equal 2?
788 class Attr_Vertical_distance_WK_BK
789     def getHeader
790         return "@attribute vertical_distance_WK_BK_1 {not_two_squares, two_squares}"
791     end
792
793     def compute(position)
794         choosenBottom = position.bottom
795
796         case choosenBottom
797         when "a"
798             if position.wkf == position.bkf + 2 then return "two_squares"
799             else return "not_two_squares" end
800         when "h"
801             if position.wkf == position.bkf - 2 then return "two_squares"
802             else return "not_two_squares" end
803         when "1"
804             if position.wkr == position.bkr + 2 then return "two_squares"
805             else return "not_two_squares" end
806         when "8"
807             if position.wkr == position.bkr - 2 then return "two_squares"
808             else return "not_two_squares" end
```

```
809     end
810   end
811
812   def name
813     return "the vertical distance between the both kings is"
814   end
815 end
816
817 # How is the horizontal distance between WK and BK
818 class Attr_Horizontal_distance_WK_BK
819   def getHeader
820     return "@attribute horizontal_distance_WK_BK_1 {odd, even, zero}"
821   end
822
823   def compute(position)
824     choosenBottom = position.bottom
825
826     case choosenBottom
827     when "a", "h"
828       if position.rankDist_WK_BK == 0
829         return "zero"
830       elsif position.rankDist_WK_BK % 2 == 0
831         return "even"
832       else
833         return "odd"
834       end
835
836     when "1", "8"
837       if position.fileDist_WK_BK == 0
838         return "zero"
839       elsif position.fileDist_WK_BK % 2 == 0
840         return "even"
841       else
842         return "odd"
843       end
844
845     end
846   end
847
848   def name
849     return "the horizontal distance between the both kings is"
850   end
851 end
852
853
854 #####
855 # attributes: special positions
```

```
856  ##
857
858  # kings are in opposition
859  class Attr_K_in_opp
860    def getHeader
861      return "@attribute K_in_opp {true, false}"
862    end
863
864    def compute(position)
865      return( (position.fileDist_WK_BK == 0 and position.rankDist_WK_BK == 2) or
866              (position.fileDist_WK_BK == 2 and position.rankDist_WK_BK == 0) )
867    end
868
869    def name
870      return "the kings are in opposition"
871    end
872  end
873
874  # kings almost in opposition
875  class Attr_K_almost_in_opp
876    def getHeader
877      return "@attribute K_almost_in_opp {true, false}"
878    end
879
880    def compute(position)
881      return( (position.fileDist_WK_BK == 2 and position.rankDist_WK_BK == 1) or
882              or (position.fileDist_WK_BK == 1 and position.rankDist_WK_BK == 2) )
883    end
884
885    def name
886      return "the both kings are almost in opposition"
887    end
888  end
889
890  # white king, black king and white rook form a 'L' pattern
891  class Attr_L_patt
892    def getHeader
893      return "@attribute L_patt {true, false}"
894    end
895
896    def compute(position)
897      return( (position.fileDist_WK_BK == 0 and position.rankDist_WR_BK == 0 and
898              position.rankDist_WK_BK == 2) or (position.rankDist_WK_BK == 0 and
899              position.fileDist_WR_BK == 0 and position.fileDist_WK_BK == 2))
900    end
901
902    def name
```

```
903     return "there is a L-pattern"
904   end
905 end
906
907
908 #####
909 # attributes: special positions on the board
910 ##
911
912 # the white rook divides the 2 kings toward the closest edge
913 class Attr_WR_divides_K_toward_ce
914   def getHeader
915     return "@attribute WR_divides_K_toward_ce {true, false}"
916   end
917
918   def compute(position)
919     return ((smallerThan(position.wkf, position.wrf, position.bkf) and
920             position.dist_BK_h == position.dist_BK_ce) or
921            (smallerThan(position.wkr, position.wrr, position.bkr) and
922             position.dist_BK_8 == position.dist_BK_ce) or
923            (smallerThan(position.bkf, position.wrf, position.wkf) and
924             position.dist_BK_a == position.dist_BK_ce) or
925            (smallerThan(position.bkr, position.wrr, position.wkr) and
926             position.dist_BK_1 == position.dist_BK_ce))
927   end
928
929   def name
930     return "the white rook divides the two kings towards the closest edge"
931   end
932 end
933
934 # the white rook holds the black king towards the closest edge
935 class Attr_WR_holds_BK_toward_ce
936   def getHeader
937     return "@attribute WR_holds_BK_toward_ce {true, false}"
938   end
939
940   def compute(position)
941     return
942     ((position.wrf == position.bkf + 1 and position.bk_closest_edge.include? "a") or
943      (position.wrf == position.bkf - 1 and position.bk_closest_edge.include? "h") or
944      (position.wrr == position.bkf + 1 and position.bk_closest_edge.include? "1") or
945      (position.wrr == position.bkr - 1 and position.bk_closest_edge.include? "8"))
946   end
947
948   def name
949     return "the white rook holds the black king towards the closest edge"
```

```

950     end
951 end
952
953 # the kings are in opposition that forces the black king towards the closest edge
954 class Attr_K_in_opp_toward_ce
955     def getHeader
956         return "@attribute K_in_opp_toward_ce {true, false}"
957     end
958
959     def compute(position)
960         return ((position.wkf == position.bkf and position.wkr == position.bkr + 2 and
961             position.bk_closest_edge.include? "1") or (position.wkf == position.bkf and
962             position.wkr == position.bkr - 2 and position.bk_closest_edge.include? "8") or
963             (position.wkf == position.bkf + 2 and position.wkr == position.bkr and
964             position.bk_closest_edge.include? "a") or (position.wkf == position.bkf - 2 and
965             position.wkr == position.bkr and position.bk_closest_edge.include? "h"))
966     end
967
968     def name
969         return "the both kings are in opposition " +
970             "so that the black king is held towards the closest edge"
971     end
972 end
973
974 # the white rook divides the 2 kings toward the closest edge and
975 # the it holds the black king toward the closest edge
976 class Attr_WR_squeeze_BK
977     def getHeader
978         return "@attribute WR_squeeze_BK {true, false}"
979     end
980
981     def compute(position)
982         a = Attr_WR_divides_K_toward_ce.new
983         b = Attr_K_in_opp_toward_ce.new
984         return (a.compute(position) and b.compute(position))
985     end
986
987     def name
988         return "the black king is squeezed towards the closest edge"
989     end
990 end
991
992 #####
993 # attributes: Other
994 ##
995
996 # free space around the black king as let free by the rook

```

```
997 class Attr_BK_free_space
998   def getHeader
999     return "@attribute BK_free_space {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14,
1000       15, 16, 18, 20, 21, 24, 25, 28, 30, 35, 36, 42, 49}"
1001   end
1002
1003   def compute(position)
1004     return position.rook_free_space
1005   end
1006
1007   def name
1008     return "the free space of the black king as let by the rook is"
1009   end
1010 end
1011
1012 class Attr_BK_free_space_Num
1013   def getHeader
1014     return "@attribute BK_free_space_num numeric"
1015   end
1016
1017   def compute(position)
1018     return position.rook_free_space
1019   end
1020
1021   def name
1022     return "the free space of the black king as let by the rook as a number is"
1023   end
1024 end
1025
1026 # true free space around the black king
1027 class Attr_BK_true_free_space
1028   def getHeader
1029     return "@attribute BK_true_free_space {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
1030       12, 14, 15, 16, 18, 20, 21, 24, 25, 28, 30, 35, 36, 42, 49}"
1031   end
1032
1033   def compute(position)
1034     return position.free_space
1035   end
1036
1037   def name
1038     return "the number of fields where the black king can go"
1039   end
1040 end
1041
1042 class Attr_BK_true_free_space_Num
1043   def getHeader
```

```

1044     return "@attribute BK_true_free_space_num numeric"
1045 end
1046
1047 def compute(position)
1048     return position.free_space
1049 end
1050
1051 def name
1052     return "the number of fields where the black king can go"
1053 end
1054 end
1055
1056 # WR and BK are in the "same zone"
1057 class Attr_Same_zone_WR_BK_1
1058     def getHeader
1059         return "@attribute same_zone_WR_BK_1 {true, false}"
1060     end
1061
1062     def compute(position)
1063         choosenBottom = position.bottom
1064
1065         case choosenBottom
1066         when "a", "h"
1067             return ((position.wrr <= "3"[0] and position.bkr <= "3"[0]) or
1068                 (position.wrr >= "3"[0] and position.wrr <= "6"[0] and
1069                 position.bkr >= "3"[0] and position.bkr <= "6"[0]) or
1070                 (position.wrr >= "6"[0] and position.bkr >= "6"[0]))
1071         when "1", "8"
1072             return ((position.wrf <= "c"[0] and position.bkf <= "c"[0]) or
1073                 (position.wrf >= "c"[0] and position.wrf <= "f"[0] and
1074                 position.bkf >= "c"[0] and position.bkf <= "f"[0]) or
1075                 (position.wrf >= "f"[0] and position.bkf >= "f"[0]))
1076         end
1077     end
1078
1079     def name
1080         return "the black king and the rook are in the same zone: "
1081     end
1082 end
1083
1084 #####
1085 # attributes: mine
1086 ##
1087
1088 # file or rank WR - BK distance
1089 class Attr_Oriented_distance_WR_BK
1090     def getHeader

```

```
1091     return "@attribute oriented_dist_WR_BK {0,1,2,3,4,5,6,7}"
1092   end
1093
1094   def compute(position)
1095     return [position.fileDist_WR_BK, position.rankDist_WR_BK].min
1096   end
1097
1098   def name
1099     return "the distance between the black king and the file or the rank of the rook is "
1100   end
1101 end
1102
1103 # file or rank WR - BK distance num
1104 class Attr_Oriented_distance_WR_BK_Num
1105   def getHeader
1106     return "@attribute oriented_dist_WR_BK_num numeric"
1107   end
1108
1109   def compute(position)
1110     return [position.fileDist_WR_BK, position.rankDist_WR_BK].min
1111   end
1112
1113   def name
1114     return "the distance between the black king and the file or the rank of the rook is "
1115   end
1116 end
1117
1118
1119
```

## Attributes Class

```
1  class Attributes
2    attr_accessor :list, :mode
3    def initialize(mode = "enum")
4      @list = Array.new
5      @mode = mode
6    end
7
8    # add one or more attributes to the list
9    def add(*attrArray)
10     attrArray.each do |at|
11       if at.class == Array then
12         at.each { |att| self.addOne(att) }
13       else
```

```
14         self.addOne(at)
15     end
16 end
17 end
18
19 def addOne(attr)
20     if @mode == "numeric" and
21         Module.constants.include?(attr.name.to_s + "_Num")
22         @list.push Module.const_get(attr.name.to_s + "_Num").new
23     else
24         @list.push attr.new
25     end
26 end
27
28 #return the header line for all the attributes
29 def getHeader
30     ret = ""
31     @list.each { |attr| ret += attr.getHeader + "\n"}
32     ret += Attr_Depth_of_win.new.getHeader + "\n"
33     return ret
34 end
35
36 def values(position)
37     vals = Array.new
38     @list.each { |attr| vals.push attr.compute(position).to_s }
39     return vals
40 end
41
42 def names
43     vals = Array.new
44     @list.each { |attr| vals.push attr.name }
45     return vals
46 end
47
48 def summary(position)
49     vals = Array.new
50     @list.each { |attr|
51         vals.push attr.name + ": " + attr.compute(position).to_s
52     }
53     return vals
54 end
55 end
```

## Position Class

```

1  class Position
2    attr_accessor :wkf, :wkr, :wrf, :wrr, :bkf, :bkr, :dow
3    def initialize
4      @wkf = @wkr = @wrf = @wrr = @bkf = @bkr = 0
5      @dow = ""
6    end
7
8    # define the position with the 7 first elements of the array
9    def readArray(array)
10     if array.class == Array and array.size >= 7 then
11       @wkf = array[0][0]; @wkr = array[1][0]
12       @wrf = array[2][0]; @wrr = array[3][0]
13       @bkf = array[4][0]; @bkr = array[5][0]
14       @dow = array[6]
15     else
16       puts array.inspect
17       puts "Position::readArray() invalid argument"
18     end
19   end
20
21   def to_s
22     @wkf.chr + @wkr.chr + "," + @wrf.chr + @wrr.chr +
23     "," + @bkf.chr + @bkr.chr
24   end
25   #####
26   # all distances
27   ##
28
29   def dist(a, b)
30     case a.downcase
31     when "wk" then return self.dist_WK(b)
32     when "wr" then return self.dist_WR(b)
33     when "bk" then return self.dist_BK(b)
34     end
35   end
36
37   def dist_WK(a)
38     case a.downcase
39     when "wk" then return 0
40     when "wr" then return self.dist_WK_WR
41     when "bk" then return self.dist_WK_BK
42     when "a" then return self.dist_WK_a
43     when "h" then return self.dist_WK_h
44     when "1" then return self.dist_WK_1

```

```

45     when "8" then return self.dist_WK_8
46     when "ce" then return self.dist_WK_ce
47     when "cr" then return self.dist_WK_cr
48     when "co" then return self.dist_WK_co
49   end
50 end
51
52 def dist_WR(a)
53   case a.downcase
54     when "wk" then return self.dist_WK_WR
55     when "wr" then return 0
56     when "bk" then return self.dist_WR_BK
57     when "a" then return self.dist_WR_a
58     when "h" then return self.dist_WR_h
59     when "1" then return self.dist_WR_1
60     when "8" then return self.dist_WR_8
61     when "ce" then return self.dist_WR_ce
62     when "cr" then return self.dist_WR_cr
63     when "co" then return self.dist_WR_co
64   end
65 end
66
67 def dist_BK(a)
68   case a.downcase
69     when "wk" then return self.dist_WK_BK
70     when "wr" then return self.dist_WR_BK
71     when "bk" then return 0
72     when "a" then return self.dist_BK_a
73     when "h" then return self.dist_BK_h
74     when "1" then return self.dist_BK_1
75     when "8" then return self.dist_BK_8
76     when "ce" then return self.dist_BK_ce
77     when "cr" then return self.dist_BK_cr
78     when "co" then return self.dist_BK_co
79   end
80 end
81
82 #####
83 # file distances between pieces
84 ##
85
86 def fileDist_WK_WR
87   return (@wkf - @wrf).abs
88 end
89
90 def fileDist_WK_BK
91   return (@wkf - @bkf).abs

```

```
92     end
93
94     def fileDist_WR_BK
95         return (@wrf - @bkf).abs
96     end
97
98     #####
99     # rank distances between pieces
100    ##
101
102    def rankDist_WK_WR
103        return (@wkr - @wrr).abs
104    end
105
106    def rankDist_WK_BK
107        return (@wkr - @bkr).abs
108    end
109
110    def rankDist_WR_BK
111        return (@wrr - @bkr).abs
112    end
113
114    #####
115    # distances between pieces
116    ##
117    def dist_WK_WR
118        return [self.fileDist_WK_WR, self.rankDist_WK_WR].max
119    end
120
121    def dist_WK_BK
122        return [self.fileDist_WK_BK, self.rankDist_WK_BK].max
123    end
124
125    def dist_WR_BK
126        return [self.fileDist_WR_BK, self.rankDist_WR_BK].max
127    end
128
129    #####
130    # distances between WK and edges, central cross
131    ##
132
133    # first column
134    def dist_WK_a
135        return (@wkf - 'a'[0]).abs
136    end
137
138    #8th column
```

```
139     def dist_WK_h
140         return (@wkf - 'h'[0]).abs
141     end
142
143     #first row
144     def dist_WK_1
145         return (@wkr - '1'[0]).abs
146     end
147
148     # 8th row
149     def dist_WK_8
150         return (@wkr - '8'[0]).abs
151     end
152
153     #closest edge
154     def dist_WK_ce
155         return [self.dist_WK_a, self.dist_WK_h, self.dist_WK_1, self.dist_WK_8].min
156     end
157
158     #central cross
159     def dist_WK_cr
160         return [ [(@wkf - 'd'[0]).abs, (@wkf - 'e'[0]).abs].min,
161                 [(@wkr - '4'[0]).abs, (@wkr - '5'[0]).abs].min ].max
162     end
163
164     #closest corner
165     def dist_WK_co
166         return [ [(@wkf - 'a'[0]).abs, (@wkf - 'h'[0]).abs].min,
167                 [(@wkr - '1'[0]).abs, (@wkr - '8'[0]).abs].min ].max
168     end
169
170     #####
171     # distances between WR and edges
172     ##
173
174     # first column
175     def dist_WR_a
176         return (@wrf - 'a'[0]).abs
177     end
178
179     #8th column
180     def dist_WR_h
181         return (@wrf - 'h'[0]).abs
182     end
183
184     #first row
185     def dist_WR_1
```

```
186     return (@wrr - '1'[0]).abs
187 end
188
189 # 8th row
190 def dist_WR_8
191     return (@wrr - '8'[0]).abs
192 end
193
194 #closest edge
195 def dist_WR_ce
196     return [self.dist_WR_a, self.dist_WR_h, self.dist_WR_1, self.dist_WR_8].min
197 end
198
199 #central cross
200 def dist_WR_cr
201     return [ [(@wrf - 'd'[0]).abs, (@wrf - 'e'[0]).abs].min,
202             [(@wrr - '4'[0]).abs, (@wrr - '5'[0]).abs].min ].max
203 end
204
205 #closest corner
206 def dist_WR_co
207     return [ [(@wrf - 'a'[0]).abs, (@wrf - 'h'[0]).abs].min,
208             [(@wrr - '1'[0]).abs, (@wrr - '8'[0]).abs].min ].max
209 end
210
211 #####
212 # distances between BK and edges, corners
213 ##
214
215 # first column
216 def dist_BK_a
217     return (@bkf - 'a'[0]).abs
218 end
219
220 #8th column
221 def dist_BK_h
222     return (@bkf - 'h'[0]).abs
223 end
224
225 #first row
226 def dist_BK_1
227     return (@bkr - '1'[0]).abs
228 end
229
230 # 8th row
231 def dist_BK_8
232     return (@bkr - '8'[0]).abs
```

```

233     end
234
235     #closest edge
236     def dist_BK_ce
237         return [self.dist_BK_a, self.dist_BK_h, self.dist_BK_1, self.dist_BK_8].min
238     end
239
240     #closest corner
241     def dist_BK_co
242         return [ [self.dist_BK_a, self.dist_BK_h].min,
243                 [self.dist_BK_1, self.dist_BK_8].min ].max
244     end
245
246     #central cross
247     def dist_BK_cr
248         return [ (@wrf - 'd'[0]).abs, (@wrf - 'e'[0]).abs].min,
249                 [(@wrr - '4'[0]).abs, (@wrr - '5'[0]).abs].min ].max
250     end
251
252     #####
253     # closest edges
254     ##
255
256     # return an array with the WK's closest edge(s)
257     def wk_closest_edge
258         ce = Array.new
259         ce.push "a" if self.dist_WK_a == self.dist_WK_ce
260         ce.push "h" if self.dist_WK_h == self.dist_WK_ce
261         ce.push "1" if self.dist_WK_1 == self.dist_WK_ce
262         ce.push "8" if self.dist_WK_8 == self.dist_WK_ce
263         return ce
264     end
265
266     # return an array with the WR's closest edge(s)
267     def wr_closest_edge
268         ce = Array.new
269         ce.push "a" if self.dist_WR_a == self.dist_WR_ce
270         ce.push "h" if self.dist_WR_h == self.dist_WR_ce
271         ce.push "1" if self.dist_WR_1 == self.dist_WR_ce
272         ce.push "8" if self.dist_WR_8 == self.dist_WR_ce
273         return ce
274     end
275
276     # return an array with the BK's closest edge(s)
277     def bk_closest_edge
278         ce = Array.new
279         ce.push "a" if self.dist_BK_a == self.dist_BK_ce

```

```

280     ce.push "h" if self.dist_BK_h == self.dist_BK_ce
281     ce.push "1" if self.dist_BK_1 == self.dist_BK_ce
282     ce.push "8" if self.dist_BK_8 == self.dist_BK_ce
283     return ce
284   end
285
286 #####
287 # vertical and horizontal detection
288 ##
289
290   def bottom
291     ces = self.bk_closest_edge
292     case ces.size
293     when 1 then return ces[0]
294     when 2
295       if self.dist_WK(ces[0]) <= self.dist_WK(ces[1])
296         return ces[0]
297       else
298         return ces[1]
299       end
300     end
301   end
302
303
304   def rook_free_space
305     if @wrf < @bkf then width = 'h'[0] - @wrf
306     elsif @wrf > @bkf then width = @wrf - 'a'[0]
307     else width = ['h'[0] - @wrf, @wrf - 'a'[0]].max end
308
309     if @wrr < @bkr then height = '8'[0] - @wrr
310     elsif @wrr > @bkr then height = @wrr - '1'[0]
311     else height = ['8'[0] - @wrr, @wrr - '1'[0]].max end
312
313     return(width * height)
314   end
315
316   def free_space
317     d = false
318
319
320     pos = @bkf.chr + @bkr.chr
321     board = ('a'..'h').to_a.collect{|a| ((a+'1')..(a+'8')).to_a}.flatten
322     grasp = ((@bkf-1).chr..(@bkf+1).chr).to_a.collect{ |a|
323       (a+(@bkr-1).chr..a+(@bkr+1).chr).to_a
324     }.flatten
325     wkgrasp = ((@wkf-1).chr..(@wkf+1).chr).to_a.collect{ |a|
326       (a+(@wkr-1).chr..a+(@wkr+1).chr).to_a

```

```

327     }.flatten
328
329     poss = (grasp & board) - Array[pos] - wkgrasp
330     fs = poss.size
331
332     for a in poss
333         i, j = a[0], a[1]
334         if i==@wrf and j!=@wrr and
335             !(j<@wkr and @wkr<@wrr and @wkr==@wrf) and
336             !(@wrr<@wkr and @wkr<j and @wkr==@wrf)
337
338             puts i.chr + "," + j.chr + ": rook file" if d; fs -= 1
339         next
340     end
341
342     if i!=@wrf and j==@wrr and
343         !(i<@wkr and @wkr<@wrf and @wkr==@wrr) and
344         !(@wrf<@wkr and @wkr<i and @wkr==@wrr)
345
346         puts i.chr + "," + j.chr + ": rook rank" if d
347         fs-=1
348     next
349 end
350
351     puts i.chr + "," + j.chr + " ok" if d
352 end
353 return fs
354 end
355
356 end

```

## Hash Class

```

1  class MyHash
2      attr_reader :numberOfDifferentEntries, :numberOfEntries
3      attr_reader :numberOfAmbiguousEntries, :summary
4      attr_reader :hashContainer, :numbers
5
6      def initialize
7          @debug = false
8          @hashTrigger = false
9          @hashContainer = Hash.new
10         @hashContainerSummed = Hash.new
11         @dowContainer = Hash.new
12         @dowStatContainer = Hash.new

```

```
13     @dowContainerSummed = Hash.new
14
15     @numberOfDifferentEntries = 0
16     @numberOfEntries = 0
17     @numberOfAmbiguousEntries = 0
18     @summaries = Hash.new
19     @matrix1 = Array.new
20     @graph1 = Array.new
21     @numbers = ["draw", "zero", "one", "two", "three", "four",
22               "five", "six", "seven", "eight", "nine", "ten", "eleven",
23               "twelve", "thirteen", "fourteen", "fifteen", "sixteen"]
24 end
25
26 def addEntry(attributes, value)
27   if @hashTrigger
28     index = attributes.join.hash
29   else
30     index = attributes
31   end
32   @dowContainer[value] = Array.new if !@dowContainer[value]
33   @dowContainer[value].push attributes.join.hash
34   @hashContainer[index] = Array.new if !@hashContainer[index]
35   @hashContainer[index].push value
36
37   @dowContainerSummed[value] = Hash.new if !@dowContainerSummed[value]
38   @dowContainerSummed[value][index] = 0 if !@dowContainerSummed[value][index]
39   @dowContainerSummed[value][index] += 1
40   @hashContainerSummed[index] = Hash.new if !@hashContainerSummed[index]
41   @hashContainerSummed[index][value] = 0 if !@hashContainerSummed[index][value]
42   @hashContainerSummed[index][value] += 1
43 end
44
45 def myCompare(a, b)
46   a = a.sort {|x, y| @numbers.index(x[0]) <=> @numbers.index(y[0])}
47   b = b.sort {|x, y| @numbers.index(x[0]) <=> @numbers.index(y[0])}
48   puts "MyHash::myCompare(" + a.class.to_s + ", " + b.class.to_s + ")" if @debug
49   puts "| comparing " + a.inspect + " and " + b.inspect + " ..." if @debug
50
51   if a.size > b.size then res = 1
52   elsif a.size < b.size then res = -1
53   else res = self.recurs(a, b)
54   end
55   puts "| result: " + res.to_s if @debug
56   puts "-----" if @debug
57   return res
58 end
59
```

```

60   def recurs(a, b)
61     a = a.to_a if a.class == Hash
62     b = b.to_a if b.class == Hash
63     if @numbers.index(a[0][0]) > @numbers.index(b[0][0])
64       return 1
65     elsif @numbers.index(a[0][0]) < @numbers.index(b[0][0]) then return -1
66     else
67       if a.size == 1 then return 0
68       else return self.recurs(a[1..a.size], b[1..b.size])
69     end
70   end
71 end
72
73 def computeInfos
74   @numberOfDifferentEntries = @container.length
75   @numberOfEntries = 0
76   @numberOfAmbiguousEntries = 0
77   @hashContainer.each do |attributesSet, dowSet|
78     positionsOfTheSet = 0
79     summaryOfTheSet = ""
80     keyForSummaries = ("A"[0] + dowSet.length).chr
81
82     dowOfTheMax = @numbers.index(dowSet.max { |a, b| a[1] <=> b[1] }[0])
83     amplitudeOfTheSet = @numbers.index(dowSet.max{ |a, b|
84       @numbers.index(a[0]) <=> @numbers.index(b[0])
85     }[0]) - @numbers.index(dowSet.min{ |a, b|
86       @numbers.index(a[0]) <=> @numbers.index(b[0])
87     }[0])
88
89     @matrix1[dowOfTheMax] = Array.new if @matrix1[dowOfTheMax].class != Array
90     dowSet.sort{ |a, b| @numbers.index(a[0]) <=> @numbers.index(b[0]) }.each do
91       |dow, quantityOfPositions|
92
93       positionsOfTheSet += quantityOfPositions
94       summaryOfTheSet += (dow + "=>" + quantityOfPositions.to_s + ",")
95       keyForSummaries += ("A"[0] + @numbers.index(dow)).chr
96
97       if !@matrix1[dowOfTheMax][@numbers.index(dow)]
98         @matrix1[dowOfTheMax][@numbers.index(dow)] = quantityOfPositions
99       else
100         @matrix1[dowOfTheMax][@numbers.index(dow)] += quantityOfPositions
101       end
102
103       if !@graph1[@numbers.index(dow)] or
104         @graph1[@numbers.index(dow)] < amplitudeOfTheSet
105
106         @graph1[@numbers.index(dow)] = amplitudeOfTheSet

```

```
107         end
108
109     end
110     while @summaries.has_key? keyForSummaries do
111         keyForSummaries += "A"
112     end
113     @summaries[keyForSummaries] = "[" + attributesSet.join(",") +
114         "]" => " + positionsOfTheSet.to_s +
115         " {" + summaryOfTheSet[0..-2] + "}"
116     @numberOfEntries += positionsOfTheSet
117     @numberOfAmbiguousEntries += positionsOfTheSet if dowSet.length > 1
118 end
119 end
120
121 def summary
122     summary = ""
123     print "computing the summary."
124     @hashContainerSummed.sort{ |a,b|
125         self.myCompare(a[1], b[1])
126     }.each { |attributes, values|
127         summary += "["
128
129         if @hashTrigger
130             summary += attributes.to_s
131         else
132             summary += attributes.join(",")
133         end
134
135         summary += "]" => {"
136         values.sort{ |a,b|
137             @numbers.index(a[0]) <=> @numbers.index(b[0])
138         }.each { |dow, quantity|
139             summary += dow + ":" + quantity.to_s + ","
140         }
141         summary[summary.size-1] = '}'
142         summary += "\n"
143         print "." if @debug
144     }
145     print "\n"
146     return summary
147 end
148
149 def load(hash)
150     @hashContainer = hash
151 end
152
153 def amplitude
```

```

154     quartiles = true
155
156     if @debug
157         print "first loop size: ",
158             @dowContainerSummed.size,
159             "; second loop average size: ",
160             Kernel.eval( "(" +
161                 @dowContainerSummed.to_a.collect { |a|
162                     a[1].size
163                 }.join("+")+")/" +
164                 @dowContainerSummed.size.to_s),
165             "; second loop iterations sum: ",
166             Kernel.eval( "(" + @dowContainerSummed.to_a.collect{ |a|
167                 a[1].size
168             }.join("+")+")"),
169             "\n"
170     end
171     print "calculating the amplitude."
172
173     graph = "#dow\tmin\tmed\tmax\tfirstQ\tthirdQ\tnb\tnbfalseNb\n"
174     @dowContainerSummed.sort{ |a,b|
175         @numbers.index(a[0]) <=> @numbers.index(b[0])
176     }.each { |dow, hashVals|
177         falseNb = 0
178         nb, min, max, sum, sum2 = 0, 10000000000, 0, 0, 0
179         allValues = Hash.new if quartiles
180         hashVals.each { |attrs, number|
181             @hashContainerSummed[attrs].each { |ndow, renumber|
182                 nb += number * renumber
183                 falseNb += renumber if dow != ndow
184                 d = @numbers.index(ndow)
185                 min = d if min > d
186                 max = d if max < d
187                 sum += d * renumber * number
188                 sum2 += d**2 * number * renumber
189                 if quartiles
190                     allValues[ndow] = 0 if !allValues[ndow]
191                     allValues[ndow] += number*renumber
192                 end
193             }
194         }
195
196         size = 0
197         firstQ = 0
198         thirdQ = 0
199         if quartiles
200             allValues.each { |a, b| size += b}

```

```
201     fq = (size.to_f / 4).ceil
202     tq = (3 * size.to_f / 4).floor
203
204     allValues.sort{ |a,b|
205         @numbers.index(a[0]) <=> @numbers.index(b[0])
206     }.each{ |idow, quant|
207
208         fq -= quant
209         tq -= quant
210         firstQ = @numbers.index(idow) if fq <= 0 and firstQ == 0
211         thirdQ = @numbers.index(idow) if tq <= 0 and thirdQ == 0
212         break if fq < 0 and tq < 0
213     }
214 end
215 med = sum.to_f / nb
216 std = Math.sqrt((sum2.to_f / nb) - ((sum**2).to_f / (nb**2)))
217 print "."
218
219 graph += @numbers.index(dow).to_s + "\t" + min.to_s +
220 graph += "\t" + med.to_s + "\t" + max.to_s
221 if quartiles
222     graph += "\t" + firstQ.to_s + "\t" + thirdQ.to_s + "\t" + size.to_s
223 end
224 graph += "\t" + falseNb.to_s + "\n"
225 }
226 print "\n"
227 return graph
228 end
229
230 def graph2
231     graph = ""
232     for i in 0..17
233         graph += i.to_s + "\t" + @graph1[i].to_s + "\n"
234     end
235     return graph
236 end
237
238 def size
239     @hashContainer.size
240 end
241
242 def newDb
243     db = String.new
244     errors = Array.new
245     if @hashTrigger
246         raise "cannot print new database with hashed attributes"
247     end
```

```
248
249     @hashContainerSummed.sort{ |a,b|
250         self.myCompare(a[1], b[1])
251     }.each {|attributes, dows|
252         if dows.size == 1
253             db += attributes.join(",") + ', ' + dows.keys[0] + "\n"
254         else
255             errors.push attributes
256         end
257     }
258     return db, errors
259 end
260 end
```

## Main File

```
1  #!/usr/bin/ruby
2
3  require "utils.rb"
4  require "position.rb"
5  require "attributes.rb"
6  require "attribute.rb"
7  require "myhash.rb"
8  require "options.rb"
9
10 file = "./krkopt.data"
11 attributes = Attributes.new
12 options = Options.new
13 options.read ARGV
14
15 if options.attributes.length > 0 then
16     if options.numeric then attributes.mode = "numeric" end
17     attributes.add options.attributes
18 else
19     raise "error: no attribute given"
20 end
21
22 newdb = "@relation krk\n\n"
23 newdb << attributes.getHeader
24 newdb << "\n@data\n"
25
26 aFile = File.new(file, "r")
27 i = 0
28 hashTable = MyHash.new
29
```

```
30 t0 = Time.new
31 puts t0.strftime("[%H:%M:%S] calculating new db...")
32 aFile.each_line do |line|
33
34   if line.chomp.split(',').size < 7
35     raise "invalid line " + i.to_s + ": " + line
36   end
37   position = Position.new
38   position.readArray line.chomp.split(',')
39
40   b = attributes.values(position)
41   if b.include? "irrelevant" then next end
42
43   hashTable.addEntry(b, position.dow)
44
45   i += 1
46   break if options.maxIterations > 0 and i >= options.maxIterations
47 end
48 aFile.close
49
50 puts Time.now.strftime("[%H:%M:%S]") + " " +
51       i.to_s + " lines calculated, analysing results..."
52
53 ##
54 # saving all produced lines
55 #
56 repartitionFile = File.new(options.resultsFileName + ".rep",
57                             File::CREAT|File::TRUNC|File::WRONLY)
58 repartitionFile << "Stats\n "
59 repartitionFile << "size: " << hashTable.size << "\n"
60 repartitionFile << hashTable.summary
61 repartitionFile.close
62 puts "writing repartition into " + options.resultsFileName + ".rep"
63
64
65 ##
66 # saving the amplitudes for gnuplot to draw a graphic
67 ##
68 amplitudeFile = File.new(options.resultsFileName + ".amp",
69                             File::CREAT|File::TRUNC|File::WRONLY)
70 amplitudeFile << hashTable.amplitude
71 amplitudeFile.close
72 puts "writing amplitudes into " + options.resultsFileName + ".amp"
73
74 ##
75 # new database
76 ##
```

```

77 dbFile = File.new(options.resultsFileName + ".arff",
78     File::CREAT|File::TRUNC|File::WRONLY)
79 dbFile << newdb
80 dbFile << (db, errors = hashTable.newDb; db)
81 dbFile.close
82 puts Time.now.strftime("[%H:%M:%S]") +
83     " writing new database into " +
84     options.resultsFileName + ".arff"
85 errorsFile = File.new(options.resultsFileName + ".dber",
86     File::CREAT|File::TRUNC|File::WRONLY)
87
88 aFile = File.new(file, "r")
89 aFile.each_line do |line|
90     if line.chomp.split(',').size < 7
91         raise "invalid line " + i.to_s + ": " + line
92     end
93     position = Position.new
94     position.readArray line.chomp.split(',')
95
96     b = attributes.values(position)
97     if b.include? "irrelevant" then next end
98     if errors.include? b then errorsFile << line end
99     break if options.maxIterations > 0 and i >= options.maxIterations
100 end
101 aFile.close
102 errorsFile.close
103
104
105 puts Time.now.strftime("[%H:%M:%S]") +
106     " done after " + printTime(Time.new - t0)

```

## Option Class

```

1 class Options
2     attr_reader :maxIterations, :attributes, :resultsFileName,
3     attr_reader :position, :numeric
4
5     def initialize
6         @maxIterations = 0
7         @attributes = Array.new
8         @resultsFileName = "results"
9         @position = Array.new
10        @numeric = false
11    end
12

```

```
13 def read(argv)
14   argv.each do |option|
15
16     # searching for option --max-lines=20 and -l=20
17     if option[0,12] == "--max-lines=" or option[0,3] == "-l=" then
18       _start = (0..option.size - 1).find {|x| option[x] == "="[0]} + 1
19       @maxIterations = option[_start..option.size].to_i
20       puts "reading only the " + @maxIterations.to_s +
21           " first lines of the input file"
22
23     # searching for option --mode=morales and -m=s
24     elsif option[0,7] == "--mode=" or option[0,3] == "-m=" then
25       _start = (0..option.size - 1).find {|x| option[x] == "="[0]} + 1
26       _cut = (0..@resultsFileName.size - 1).find {|x|
27         @resultsFileName[x..@resultsFileName.length - 1] == ".arff"
28       }
29
30     case option[_start..option.size].downcase
31     when "m", "morales"
32       push("morales")
33       @resultsFileName = "morales"
34       puts "adding Morales' attributes"
35
36     when "t", "torres"
37       push "torres"
38       @resultsFileName = "torres"
39       puts "adding Torres y Quevedos' attributes"
40
41     when "b", "bain"
42       push "bain"
43       @resultsFileName = "bain"
44       puts "adding Bain's attributes"
45
46     when "s", "sadikov"
47       push "sadikov"
48       @resultsFileName = "sadikov"
49       puts "adding Sadikov's attributes"
50
51     when "a", "all"
52       push("bain"); push("morales"); push("sadikov")
53       push("torres"); push("gab")
54       @resultsFileName = "bmst"
55       puts "setting mode All (bmst)"
56
57     when "d", "debug"
58       push "debug"
59       @resultsFileName = "debug"
```

```

60         puts "adding debug's attributes"
61
62         when "g", "gab", "gabriel"
63             push "gab"
64             @resultsFileName = "gabriel"
65             puts "adding gabriel's special attributes"
66         end
67
68         # searching for option --out-file=results
69         elsif option[0,11] == "--out-file=" or option[0..2] == "-f=" then
70             _start = (0..option.size - 1).find {|x| option[x] == "="[0]} + 1
71             @resultsFileName = option[_start..option.size]
72             puts "setting results file name to " + @resultsFileName
73
74         # searching for option --position=a,1,b,2,c,3
75         elsif option[0,11] == "--position=" or option[0..2] == "-p=" then
76             _start = (0..option.size - 1).find {|x| option[x] == "="[0]} + 1
77             @position = option[_start..option.size].split(",")
78             @position.push ""
79
80     #     searching for option --numeric or -n
81     elsif option[0,9] == "--numeric" or option[0..1] == "-n" then
82         @numeric = true
83         puts "setting numeric on"
84     end
85 end
86 end
87
88 def push(name)
89     case name
90     when "bain"
91         @attributes.push(Attr_File_distance_WK_BK, Attr_Rank_distance_WK_BK,
92             Attr_File_distance_WR_BK, Attr_Rank_distance_WR_BK,
93             Attr_File_distance_WK_WR, Attr_Rank_distance_WK_WR,
94             Attr_WK_between_WR_BK, Attr_WR_between_WK_BK,
95             Attr_BK_between_WK_WR, Attr_Adjacent_WK_WR,
96             Attr_Adjacent_WK_BK, Attr_Adjacent_WR_BK)
97     when "morales"
98         @attributes.push(Attr_Adjacent_WR_BK, Attr_WR_between_WK_BK,
99             Attr_K_almost_in_opp, Attr_L_patt, Attr_WK_between_WR_BK)
100    when "sadikov"
101        @attributes.push(Attr_Distance_WK_BK, Attr_Distance_WR_BK,
102            Attr_K_in_opp, Attr_Distance_BK_ce, Attr_BK_free_space,
103            Attr_WR_between_WK_BK, Attr_WR_divides_K_toward_ce,
104            Attr_WR_holds_BK_toward_ce, Attr_K_in_opp_toward_ce,
105            Attr_WR_squeeze_BK, Attr_Distance_BK_co, Attr_Distance_WK_cr)
106    when "torres"

```

```

107     @attributes.push(Attr_Vertical_distance_WR_BK,
108         Attr_Same_zone_WR_BK_1, Attr_Vertical_distance_WK_BK,
109         Attr_Horizontal_distance_WK_BK)
110     when "gab"
111         @attributes.push(Attr_Distance_WR_BK, Attr_Distance_WK_WR,
112             Attr_Distance_WK_BK, Attr_Distance_BK_co,
113             Attr_Oriented_distance_WR_BK, Attr_BK_true_free_space,
114             Attr_BK_free_space, Attr_K_in_opp_toward_ce, Attr_Distance_WK_cr,
115             Attr_Distance_WK_co, Attr_K_almost_in_opp)
116     when "debug"
117         @attributes.push(Attr_WK_file, Attr_WK_rank, Attr_WR_file,
118             Attr_WR_rank, Attr_BK_file, Attr_BK_rank)
119     else
120         raise "no valable attributes group"
121     end
122     @attributes.uniq!
123 end
124 end

```

## Utilities

```

1  def smallerThan(*int)
2    cond = true
3    if int.length > 1 then
4      for i in 0.. int.length - 2 do
5        cond = (cond and int[i] < int[i+1])
6      end
7    end
8    return cond
9  end
10
11 def printTime(timeInS)
12   if timeInS < 60.0
13     return timeInS.to_s + "s"
14   elsif timeInS < 3600.0
15     timeM = (timeInS / 60.0).floor
16     return timeM.to_s + "m" + (timeInS - 60.0 * timeM).to_s + "s"
17   elsif timeInS < 86400.0
18     timeH = (timeInS / 3600.0).floor
19     timeM = ((timeInS - 3600.0 * timeH) / 60.0).floor
20     return timeH.to_s + "h" + timeM.to_s + "m" +
21         (timeInS - 3600.0 * timeH - 60.0 * timeM).to_s + "s"
22   end
23 end
24

```

## Another Main File to test the Attributes

```
1  #!/usr/bin/ruby
2
3  require "utils.rb"
4  require "position.rb"
5  require "attributes.rb"
6  require "attribute.rb"
7  require "attributetype.rb"
8  require "myhash.rb"
9  require "options.rb"
10
11 position = Position.new
12 attributes = Attributes.new
13 options = Options.new
14 options.read ARGV
15
16 attributes.add options.attributes
17
18 if options.position.empty? then
19   puts "error: no position given to test attributes"
20   exit
21 else
22   position.readArray options.position
23   i = 0
24   attributes.summary(position).each { |a| print i, "-", a, "\n"; i+= 1}
25   puts
26 end
27
28
```

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